

# Grinding Experiment by Direct Position / Force Control with On-line Constraint Estimation

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**Abstract:** Based on the analyses of the interaction between a manipulator's hand and a working object, a model representing the constrained dynamics of the robot is first discussed. The constrained forces are expressed by an algebraic function of states, input generalized forces, and constraint condition, and then direct position / force controller without force sensor is proposed based on the algebraic relation. To give the grinding system the ability to adapt to any object shape being changed by itself, we added new estimating function of time-varying constraint condition in real time for the adaptive position / force control. Evaluations through simulations and real grinding experiments by fitting the changing constraint surface with spline functions, indicate that reliable position / force control and shape-grinding can be achieved by the proposed methodology.

**Keywords:** Constraint condition, Direct position / force controller without force sensor, Estimating function

## 1. INTRODUCTION

Many researches have discussed on the force control of robots for contacting tasks. Most force control strategies are to use force sensors [1] to obtain force information, where their reliability and accuracy are limited since the work-sites of the robot are filled with noise and thermal disturbances. Force sensors could lead to the falling of the structure stiffness of manipulators, which is one of the most essential defects for manipulators executing grinding tasks. To solve the problem, some approaches without any force sensor have been presented [3]. To ensure the stabilities of the constrained motion, force and position control have utilized Lyapunov's stability analysis under the inverse dynamic compensation. Their force control strategies have been explained intelligibly in books [5]-[7].

However, insofar as we survey the controllers introduced in the books or papers don't base on the algebraic function of states and input generalized forces derived from the relation between the constraint condition and the equation of dynamics. So we discuss first a strategy for simultaneous control of the position and force without any force sensors, where the equation of dynamics in reference to the constrained force has been reformulated [8]. The constrained force is derived from the equation of dynamics and the constrained equation as an explicit algebraic function of states and input generalized forces, which means force information can be obtained by calculation rather than by force sensing. Equation (1), which has been pointed out by Hemami [10] in the analysis of biped walking robot, denotes also the kinematical algebraic relation of the controller, when robot's end-effector being in touch with a surface in 3-D space:

$$F_n = a(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{A}(\mathbf{q})\boldsymbol{\tau}, \quad (1)$$

where,  $F_n$  is exerting force on the constrained surface.  $\mathbf{q}$  and  $\dot{\mathbf{q}}$  are state variables.  $a(\mathbf{q}, \dot{\mathbf{q}})$  and  $\mathbf{A}(\mathbf{q})$  are scalar function and vector defined in following section.  $\boldsymbol{\tau}$  is input torque. This algebraic equation has been known, but

it was the first time in robotics to be applied to the sensing function of exerting force by Peng [4]. As a new control law, the controller doesn't include any force feedback sensors but realizes simultaneous control of position and force in the constrained motions and is different from the traditional ones[1].

A strategy to control force and position proposed in this paper is also based on (1). Contrarily to Peng's Method to use (1) as a force sensor, we used the equation for calculating  $\boldsymbol{\tau}$  to achieve a desired exerting force  $F_{nd}$ . Actually, the strategy is based on two facts of (1) that have been ignored for a long time. The first fact is that the force transmission process is an immediate process being stated clearly by (1) providing that the manipulator's structure is rigid. Contrarily, the occurrence of velocity and position is a time-consuming process. By using this algebraic relation, it's possible to control the exerting force to the desired one without time lag. Another important fact is the input generalized forces have some redundancy against the constrained generalized forces in the constrained motion. Based on the above analysis, we had confirmed our force / position control method can realize the grinding task through real grinding robot [8].

The problem to be solved in our approach is that the mathematical expression of algebraic constraint condition should be predefined in the controller instead of the merit of not using force sensor. Grinding task requires on-line estimation of changing constraint condition since the grinding is the action to change the constraint condition. In this presentation, we estimate the object's surface using the grinder as a touch sensor. In order to give the system the ability to grind any working object into any shape, we focus on how to update the constraint condition in real time. Based on the above preparation we constructed a simulator to evaluate the proposed shape-grinding system, which indicates the validity of our system to have the performance to adapt for grinding desired-shape without force sensor [11]. Also, we constructed a grinding robot to complete the grinding

experiment by using this proposed constraint condition estimation method when the constraint surface is flat, the experiment results will be discussed in the end of this paper.

## 2. MODELING

### 2.1 Constrained Dynamic Systems

Hemami and Wyman have addressed the issue of control of a moving robot according to constraint condition and examined the problem of the control of the biped locomotion constrained in the frontal plane. Their purpose was to control the position coordinates of the biped locomotion rather than generalized forces of constrained dynamic equation involved the item of generalized forces of constraints. And the constrained force is used as a determining condition to change the dynamic model from constrained motion to free motion of the legs. In this paper, the grinding manipulator shown in Fig.1, whose end-point is in contact with the constrained surface, is modelled as following (2) with Lagrangian equations of motion in term of the constraint forces, referring to what Hemami and Arimoto have done:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\mathbf{q}}} \right) - \left( \frac{\partial L}{\partial \mathbf{q}} \right) = \boldsymbol{\tau} + \mathbf{J}_c^T(\mathbf{q}) F_n - \mathbf{J}_r^T(\mathbf{q}) F_t, \quad (2)$$

where,  $\mathbf{J}_c$  and  $\mathbf{J}_r$  satisfy,

$$\mathbf{J}_c = \frac{\partial C}{\partial \mathbf{q}} / \left\| \frac{\partial C}{\partial \mathbf{r}} \right\| = \frac{\partial C}{\partial \mathbf{r}} \tilde{\mathbf{J}}_r / \left\| \frac{\partial C}{\partial \mathbf{r}} \right\|,$$

$$\tilde{\mathbf{J}}_r = \frac{\partial \mathbf{r}}{\partial \mathbf{q}}, \quad \mathbf{J}_r^T = \tilde{\mathbf{J}}_r^T \dot{\mathbf{r}} / \left\| \dot{\mathbf{r}} \right\|,$$

$\mathbf{r}$  is the  $l$  position vector of the hand and can be expressed as a kinematic equation ,

$$\mathbf{r} = \mathbf{r}(\mathbf{q}).$$

$L$  is the Lagrangian function,  $\mathbf{q}$  is  $l(\geq 2)$  generalized coordinates,  $\boldsymbol{\tau}$  is  $l$  inputs. The discussing robot system does not have kinematical redundancy.  $C$  is a scalar function of constraint, and expressed as an equation of constraints

$$C(\mathbf{r}(\mathbf{q})) = 0, \quad (3)$$

$F_n$  is the constrained force associated with  $C$  and  $F_t$  is the tangential disturbance force.

Equation (2) can be derived to be

$$\begin{aligned} & \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) \\ & = \boldsymbol{\tau} + \mathbf{J}_c^T(\mathbf{q}) F_n - \mathbf{J}_r^T(\mathbf{q}) F_t, \end{aligned} \quad (4)$$

where  $\mathbf{M}$  is an  $l \times l$  matrix,  $\mathbf{H}$  and  $\mathbf{G}$  are  $l$  vectors.  $F_n$  can be expressed as

$$F_n = a(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{A}(\mathbf{q}) \mathbf{J}_r^T F_t - \mathbf{A}(\mathbf{q}) \boldsymbol{\tau}, \quad (5)$$

where,  $a(\mathbf{q}, \dot{\mathbf{q}})$  is a scalar representing the first term in the expression of  $F_n$ , and  $\mathbf{A}(\mathbf{q})$  is an  $l$  vector to represent the coefficient vector of  $\boldsymbol{\tau}$  in the same expression. Equations (4) and (5) compose a constrained system that can be controlled, if  $F_n = 0$ , describing the unconstrained motion of the system.

Substituting the (5) into (4), the state equation of the system including the constrained force (as  $F_n > 0$ ) can be derived.

### 2.2 Modeling of Grinding Process

In the past, we did the experiment when working surface was flat, so we can just do flat grinding. Now we want to grind the work-piece into the one with different kinds of shapes, for example, grinding the flat surface into a curved one, just like Fig.2. In Fig.2, we can find that the desired working surface is prescribed (it can be decided by us), which means the desired constrained condition  $C_d$  is known, so

$$C_d = y - f_d(x) = 0 \quad (6)$$

where  $\mathbf{r} = [\mathbf{x}, \mathbf{y}]$  means the hand position given by (3). But the constrained condition  $C^{(j)}$  ( $j = 1, 2, \dots, d-1$ ) changed by the iterative grinding as shown in Fig. 1 and Fig. 2 is defined,

$$C^{(j)} = y - f^{(j)}(x) = 0 \quad (7)$$

We assume  $C^{(1)}$  is known, that is to say,  $f^{(1)}(x)$  is initially defined.  $f^{(j)}(x)$  is the working surface remained by  $i$ -th grinding. And  $f^{(j)}(x)$  is a function passing through all points,  $(x_1, f^{(j)}(x_1)), (x_2, f^{(j)}(x_2)), \dots, (x_p, f^{(j)}(x_p))$ , these observed points representing the  $(j)$ -th constraint condition obtained through the grinding tip position used as a touching sensor of ground newly surface. Here we assume  $f^{(j)}(x)$  could be represented by a polynomial of  $(p-1)$ -th order of  $x$ . Given the above  $p$  points, we can easily decide the parameters of polynomial function  $y = f^{(j)}(x)$ . If the current constrained condition can be got successfully, which means the current working surface  $f^{(j)}(x)$  can be detected correctly, the depth from the current working surface to the desired working surface which is expressed as  $\Delta h^{(j)}$  shown in Fig. 2 can be obtained easily.

$$\Delta h^{(j)}(x_i) = f^{(j)}(x) \Big|_{x=x_i} - f_d(x) \Big|_{x=x_i} \quad (8)$$

In this case, we can obviously find that the desired constrained force should not be a constant. It should be changed while  $\Delta h^{(j)}$  changes. So we redefine the desired constrained force  $F_{nd}^{(j)}$  as a function of  $\Delta h^{(j)}$ , shown as follows:

$$F_{nd}^{(j)}(x_i) = k \Delta h^{(j)}(x_i) \quad (9)$$

where,  $k$  is a constant.

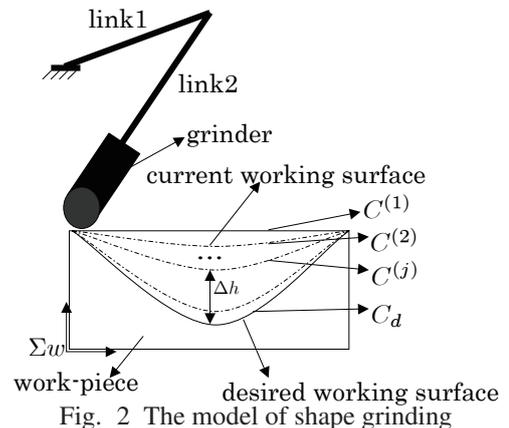


Fig. 2 The model of shape grinding

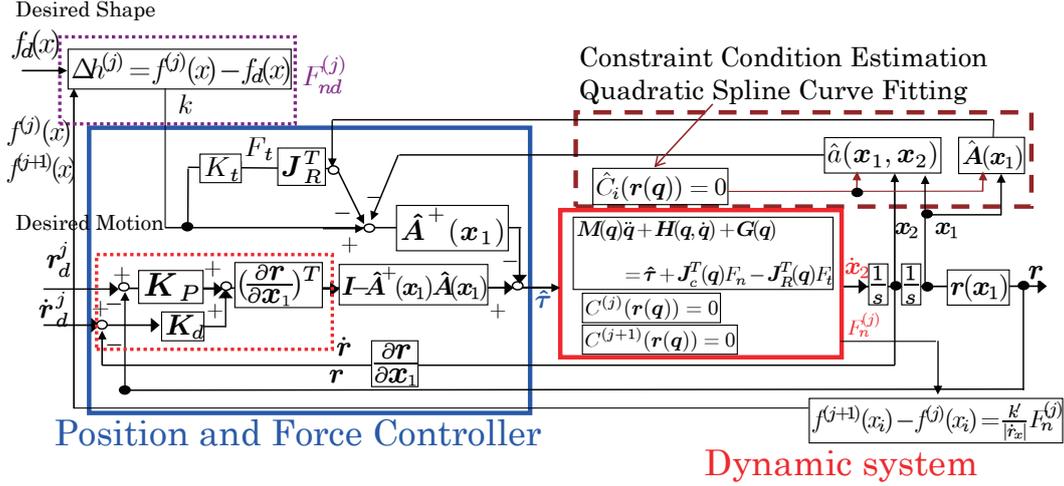


Fig. 1 Shape-grinding position / force control system

$F_{nd}^{(j)}(x_i)$  is given to the controller(16), then the exerted force  $F_n^{(j)}(x_i)$  is determined by(5). New-ground surface  $f^{(j+1)}(x_i)$  can be represented through exerted force  $F_n^{(j)}(x_i)$  and previous constraint  $f^{(j)}(x_i)$  as

$$f^{(j+1)}(x_i) - f^{(j)}(x_i) = \frac{k'}{|\dot{r}_x|} F_n^{(j)}(x_i) \quad (10)$$

where,  $k'$  is a constant,  $|\dot{r}_x|$  is the real velocity of grinder in  $x$ -axis, which is output from Dynamic system. Here is why we set the coefficient of  $F_n^{(j)}(x_i)$  with both  $k'$  and  $|\dot{r}_x|$ . According to the fact of grinding process, we all know that with a same constrained force, the bigger grinder's velocity will cause thinner ground depth. Therefore, coefficient of  $F_n^{(j)}(x_i)$  should be modeled as to be divided by velocity term  $|\dot{r}_x|$ . Then  $k'$  will be set along with  $|\dot{r}_x|$  to make the influence of  $F_n^{(j)}(x_i)$  more reasonable.

A condition that the new object shape  $f^{(j+1)}(x_i)$  have to satisfy, i.e.,

$$y = f^{(j+1)}(x_i) \quad (11)$$

Then  $C^{(j+1)}$  can also be known:

$$C^{(j+1)} = y - f^{(j+1)}(x) = 0 \quad (12)$$

So, starting from  $C^{(1)}$ , all of  $C^{(j)}$  can be decided. What we want to emphasize is  $C_i$  represents the resulted ground shape of the object defined in the shape-grinding simulator.

### 3. ADAPTIVE CONTROLLER

#### 3.1 Controller using predicted constraint condition

Reviewing the dynamic equation (2) and constraint condition (3), it can be found that as  $l > 1$ , the number of input generalized forces is more than that of the constrained forces. From this point and (5) we can claim that there is some redundancy of constrained force between the input torque  $\boldsymbol{\tau}$ , and the constrained force  $F_n$ . This condition is much similar to the kinematical redundancy of redundant manipulator. Based on the above argument and assuming that, the parameters of the (5) are known

and its state variables could be measured, and  $a(\mathbf{q}, \dot{\mathbf{q}})$  and  $\mathbf{A}(\mathbf{q})$  could be calculated correctly, which means that the constraint condition  $C = 0$  is prescribed. As a result, a control law is derived and can be expressed as

$$\boldsymbol{\tau} = -\mathbf{A}^+(\mathbf{q}) \left\{ F_{nd} - a(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{A}(\mathbf{q}) \mathbf{J}_R^T F_t \right\} + (\mathbf{I} - \mathbf{A}^+(\mathbf{q}) \mathbf{A}(\mathbf{q})) \mathbf{k}, \quad (13)$$

where  $\mathbf{I}$  is an identity matrix of  $l \times l$ ,  $F_{nd}$  is the desired constrained forces,  $\mathbf{A}(\mathbf{q})$  is defined in (5) and  $\mathbf{A}^+(\mathbf{q})$  is the pseudoinverse matrix of it,  $a(\mathbf{q}, \dot{\mathbf{q}})$  is also defined in (5) and  $\mathbf{k}$  is an arbitrary vector which is defined as

$$\mathbf{k} = \tilde{\mathbf{J}}_r^T(\mathbf{q}) \left\{ \mathbf{K}_p(\mathbf{r}_d - \mathbf{r}) + \mathbf{K}_d(\dot{\mathbf{r}}_d - \dot{\mathbf{r}}) \right\}, \quad (14)$$

Through the experimental experiences we noticed that owing to the characteristic of kinematics, the oscillation of  $\Delta \dot{\mathbf{r}} = \dot{\mathbf{r}}_d - \dot{\mathbf{r}}$  exerted during grinding procedure will effect torque  $\boldsymbol{\tau}$ 's result. Therefore, here  $\Delta \dot{\mathbf{r}}_i = \dot{\mathbf{r}}_i(i \times \Delta t)$ , ( $\Delta t = 0.0007s$ ) will be treated as:

$$\Delta \dot{\mathbf{r}}_i = \left( \sum_{j=i-100}^i \Delta \dot{\mathbf{r}}_j \right) / 100 \quad (15)$$

where  $\mathbf{K}_p = \text{diag}[k_{p1}, k_{p2}]$  and  $\mathbf{K}_d = \text{diag}[k_{d1}, k_{d2}]$  are coefficient matrices applied to the position and the velocity control by the redundant degree of freedom of  $\mathbf{A}(\mathbf{q})$ ,  $\mathbf{r}_d(\mathbf{q})$  is the desired position vector of the end-effector along the constrained surface and  $\mathbf{r}(\mathbf{q})$  is the real position vector of it. The controller presented by (13) and (14) assumes that the constraint condition  $C = 0$  be known precisely even though the grinding operation is a task to change the constraint condition. This looks like to be a contradiction, so we need to observe time-varying constraint conditions in real time by using grinding tip as a touch sensor.

The time-varying condition is estimated as an approximate constrained function by position of the manipulator hand, which based on the estimated constrained surface. The estimated condition is denoted by  $\hat{C} = 0$ . Hence,  $a(\mathbf{q}, \dot{\mathbf{q}})$  and  $\mathbf{A}(\mathbf{q})$  including  $\partial \hat{C} / \partial \mathbf{q}$  and  $\partial / \partial \mathbf{q} (\partial \hat{C} / \partial \mathbf{q})$

are changed to  $\hat{a}(\mathbf{q}, \dot{\mathbf{q}})$  and  $\hat{\mathbf{A}}(\mathbf{q})$  as shown in (17), (18). They were used in the later simulations of the unknown constrained condition. As a result, a controller based on the estimated constrained condition is given as

$$\hat{\tau} = -\hat{\mathbf{A}}^+(\mathbf{q}) \left\{ F_{nd} - \hat{a}(\mathbf{q}, \dot{\mathbf{q}}) - \hat{\mathbf{A}}(\mathbf{q}) \mathbf{J}_R^T F_t \right\} + (\mathbf{I} - \hat{\mathbf{A}}^+(\mathbf{q}) \hat{\mathbf{A}}(\mathbf{q})) \mathbf{k}, \quad (16)$$

$$m_c^{-1} \left\| \frac{\partial \hat{C}}{\partial \mathbf{r}} \left\{ - \left[ \frac{\partial}{\partial \mathbf{q}} \left( \frac{\partial \hat{C}}{\partial \mathbf{q}} \right) \dot{\mathbf{q}} + \left( \frac{\partial \hat{C}}{\partial \mathbf{q}} \right) \mathbf{M}^{-1} (\mathbf{h} + \mathbf{g}) \right] \right\} \right\| \triangleq \hat{a}(\mathbf{q}, \dot{\mathbf{q}}) \quad (17)$$

$$m_c^{-1} \left\| \frac{\partial \hat{C}}{\partial \mathbf{r}} \left\{ \left( \frac{\partial \hat{C}}{\partial \mathbf{q}} \right) \mathbf{M}^{-1} \right\} \right\| \triangleq \hat{\mathbf{A}}(\mathbf{q}) \quad (18)$$

Figure 1 illustrates a control system constructed according to the above control law that consists of a position feedback control loop and a force feedforward control. It can be found from (5) and (16) that the constrained force always equals to the desired one explicitly if the estimated constraint condition equals to the real one, i.e.,  $C = \hat{C}$  and  $F_t = 0$ . This is based on the fact that force transmission is an instant process. In the next section, we will introduce several prediction methods which are used to get  $\hat{C}_i$  in current time.

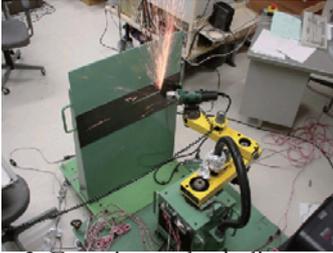


Fig. 3 Experimental grinding system

### 3.2 On-line Estimation of Constraint

When the constraint surface of the manipulator is unknown, we fit respectively the constraint surface with linear function, quadratic function, and spline curve. Three simulations have been done basing on different constraint conditions. Here, an unknown constrained condition is estimated as following,

(Assumptions)

1. The end point position of the manipulator during performing the grinding task can be surely measured and updated.
2. The grinding task is defined in  $x - y$  plane.
3. When beginning to work, the initial condition of the end-effector is known and it has touched the work object.
4. The chipped and changed constraint condition can be approximated by connections of minute sections.

Three methods which are fitting by linear function, quadratic function and spline function had been used to get the online estimation of the unknown constrained condition and results of spline function is most accurate, we adopt it in our final experiment. Here we just introduce the spline curve fitting method.

#### 3.2.1 Fitting by quadratic spline curve

The unknown constrained condition, which is in Fig. 1, is estimated and expressed as,

$$\hat{C}_{i+1} = y - [a_i(x - x_{i-1})^2 + b_i(x - x_{i-1}) + c_i] \quad (19)$$

The end-effector position at time  $(i-1)\Delta t$ ,  $i\Delta t$  are denoted respectively as  $(x_{i-1}, y_{i-1})$ ,  $(x_i, y_i)$ .

The quadratic spline curve denoted as

$$S_i(x) = a_i(x - x_{i-1})^2 + b_i(x - x_{i-1}) + c_i, \quad x \in [x_{i-1}, x_i] (i = 1, 2, 3 \dots n) \quad (20)$$

The constrained condition  $\hat{C}_{i+1} = y - (a_i(x - x_{i-1})^2 + b_i(x - x_{i-1}) + c_i)$  can be determined. And we can get the coefficients of the spline curve uniquely as follows.

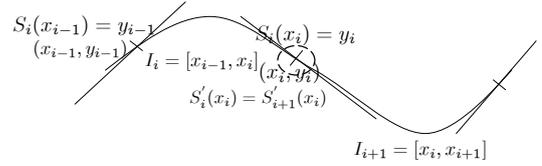


Fig. 4 Fitting by quadratic spline curve

Firstly, let  $S_i(x)$  satisfy the following conditions shown in Fig. 4.

(A) Go through two ends of the interval

$$y_{i-1} = S_i(x_{i-1}) \quad (21)$$

$$y_i = S_i(x_i) \quad (22)$$

(B) First-order differential of the spline polynomials are equal at the end-point of adjoined function.

$$\left. \frac{dS_{i+1}(x)}{dx} \right|_{x=x_i} = \left. \frac{dS_i(x)}{dx} \right|_{x=x_i} \quad (23)$$

Inputting (20) into (21), (22) and (23), we can obtain:

$$c_i = y_{i-1}, (i = 1, 2, \dots, n) \quad (24)$$

$$b_{i+1} = 2u_i - b_i, (i = 1, 2, \dots, n-1) \quad (25)$$

$$a_i = \frac{b_{i+1} - b_i}{2h_i}, (i = 1, 2, \dots, n-1) \quad (26)$$

where,  $h_i = x_i - x_{i-1}$ ,  $u_i = (y_i - y_{i-1})/h_i$ . Here to avoid the oscillation, coefficients  $a_i$  and  $b_i$  are treated as:

$$\alpha_i = \left( \sum_{j=i-100}^i \alpha_j \right) / 100, (\alpha_i = a_i, b_i) \quad (27)$$

From the above-mentioned result, the constrained conditional expression  $\hat{C}_{i+1}$  can be updated step by step.

## 4. EXPERIMENT

The experiment when constraint surface is being estimated by quadratic spline curve method has been done. Parameters of the grinding robot manipulator used in the experiment are: length of link 1 is 0.3[m], length of link 2 is 0.5[m], and the mass of link 1 is 12.28[kg], the mass of link 2 is 7.64[kg]. The specifications of first and second joints are as follows, the first joint: AC Servo Motor, 200V, 400W, 2.6A; the second joint: AC Servo Motor, 200V, 200W, 2.0A; both are made by YASKAWA ELECTRIC Co.. The desired constrained force is set as  $F_{nd} = 10$ [N]. Here the force control performance that

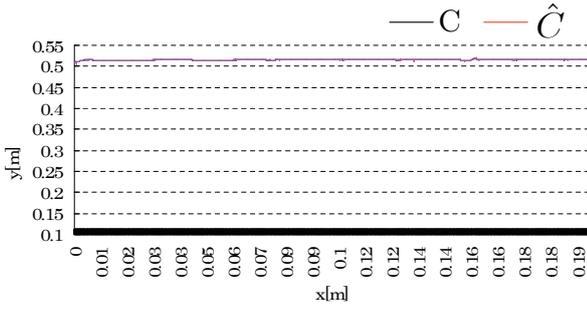


Fig. 5 Trajectory of end of grinder

the exerting force  $F_n$  be equal to the desired one,  $F_{nd}$ , is expected to be realized by compensating the effect generated by friction force  $F_t$  in (16) by  $F_t = K_t F_n$ . We had confirmed this force control accuracy improvement by real grinding experiments[9], so we will not discuss the influence of  $F_t$  in this paper, thus setting  $F_t = 0[N]$ .

The desired constrained surface is denoted as

$$f(x) = 0.51 \quad (28)$$

The known constraint condition  $C = y - 0.51$  has been recorded in Fig.5, at the same time, unknown constraint condition  $\hat{C}$  which was estimated by quadratic spline curve fitting method through (19), has also been recorded in Fig.5. From this comparison figure, we can find that using the proposed estimation method, the shape of constraint surface can be estimated very well. And we consider this kind of coincidence has provided an initial proof for the operability of this proposed controlling method.

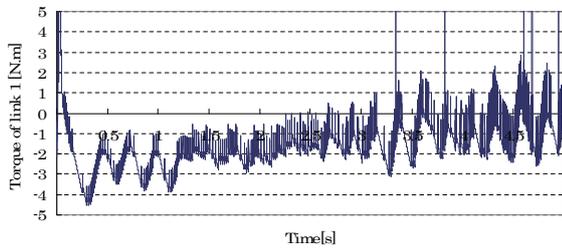


Fig. 6 Change of  $\tau_1$

Then let's take a look at the torque resulted in grinding experiment through Fig.6 to Fig.9.  $\tau_1$  and  $\tau_2$  which had been calculated by (13), meaning with known constraint condition, are shown in Fig.6 and Fig.7,  $\hat{\tau}_1$  and  $\hat{\tau}_2$  which had been calculated by (16), meaning with estimated constraint condition, are shown in Fig.8 and Fig.9.  $\tau_i (i = 1, 2)$  and  $\hat{\tau}_i (i = 1, 2)$  are different from each other, as we discussed in section 4.1. When  $\tau_i (i = 1, 2)$  being calculated, the already known constraint condition  $C = 0$  is used, but when  $\hat{\tau}_i (i = 1, 2)$  being calculated, the estimated constraint condition  $\hat{C} = 0$  is used, which means that  $\hat{\tau}_i (i = 1, 2)$  is the calculation result from the estimation of constraint surface using quadratic spline curve fitting method.

And in this experiment, we use  $\hat{\tau}_i (i = 1, 2)$  to drive the grinding robot which is shown in Fig.3 to complete the grinding experiment. Constraint force  $F_n$  exerted on the tip of grinder has been measured by a force sensor. On

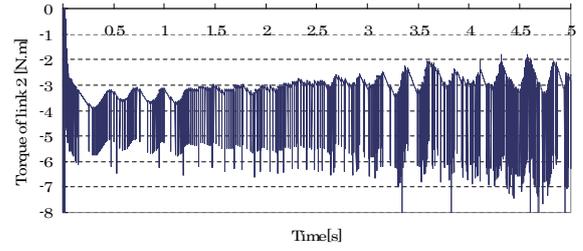


Fig. 7 Change of  $\tau_2$

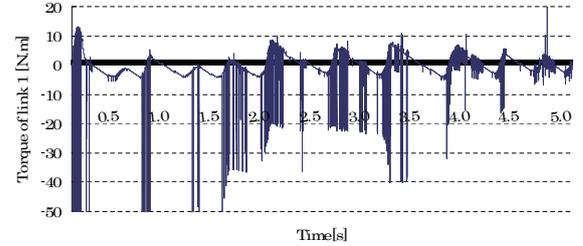


Fig. 8 Change of  $\hat{\tau}_1$

the case that controller is (16) using estimated constraint condition  $\hat{C} = 0$ , the measured force denoted as  $\hat{F}_n$  is shown in Fig.10. Also, for comparing the performance of this experiment, the one without using quadratic spline curve fitting method, by means of  $\tau_i (i = 1, 2)$  is used to drive the grinding robot, has also been done and constraint force  $F_n$ 's value is shown in Fig.11. By comparing these two figures, we can see that constraint force  $F_n$  in these two different experiments are almost the same value, which can be thought as an proof to show that the proposed controlling method has the same performance with the controller using known constraint condition.

To judge this experiment is successful or not, another view point of the important evaluation methods is to see whether the constraint force  $F_n$  be equalling to  $F_{nd}$  or not, since constraint force  $F_n$  is deduced from desired constraint force  $F_{nd} = 10[N]$  through (5) and (16). In Fig.10, until about 0.2s, constraint force  $F_n$  was spike-like bigger than desired constraint force  $F_{nd}$ , this phenomenon doesn't testify that the proposed controlling method can be utilized effectively, but until 0.2s the grinder hadn't been touching with constraint surface. So we think this phenomenon came from the instability of the manipulator when it was trying to touch the constraint surface. From about 0.2s to 5s, value of  $F_n$  fluctuated above and below the value of desired constraint force  $F_{nd}$ . Of course some of other noises such as the unevenness of the constraint surface influenced the con-

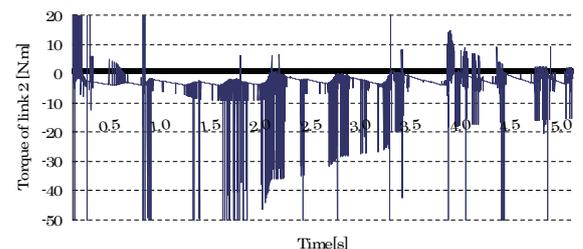


Fig. 9 Change of  $\hat{\tau}_2$

straint force  $F_n$ , but substantially, value of  $F_n$  generated during this experiment is believable and proposed controlling method is available in shape-grinding field.

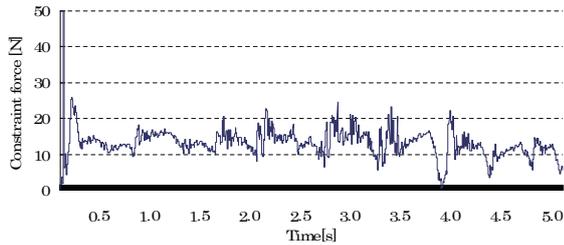


Fig. 10 Measured force  $F_n$  with on-line constraint estimation

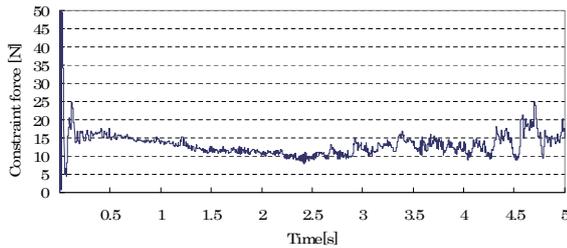


Fig. 11 Measured force  $F_n$  with known constraint

## 5. CONCLUSION

The constraint dynamical equations of a manipulator are derived and the constraint forces are expressed as an explicit function of the state and inputs. The presented methodology allows computation of the forces, as an alternative to sensing. Hence, the system is controlled with no force sensor. The control law presented is constructed by using the dynamical redundancy of constraint systems. The controller designed with this control law can be used for simultaneous control of force and position. In this paper, we present three methods for estimating the constrained condition to attain time-varying unknown constraint information. The simulations indicate the quadratic spline curve fitting for unknown constrained surface is the most closed to the known constraint surface, but from the final experiment result, quadratic spline curve fitting can not be thought as the best unknown constraint condition estimation method, although it is the best one among those three estimation methods theoretically. As a result, we will consider the cubic spline curve fitting as the unknown constraint condition estimation method and try to test it better or not for increasing the performance of controller through the real grinding experiment.

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