Constructing the model of humanoid robot based on the hyper-redundant manipulator with bracing

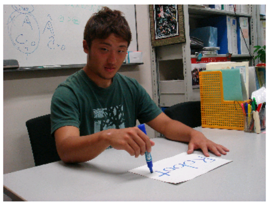
Geng Wang, Mamoru Minami, Tomohide Maeba, Fujia Yu, Akira Yanou

Abstract— **Today the humanoid robots prefer to behave without contacting with environment and avoid the surroundings that being recognized as obstacles along such as body and links. However some particular complex situations to be not simply avoided cause that the humanoid robot can not complete the task given accurately and also has high energy consumption. In contrast the humanoid robot is able to accomplish a great task accurately on contacting with environment, even if it is necessary to consume much energy. For example human being can write characters by contacting with table on hand for a long time and keep doing some accurate task for sitting on chair.**

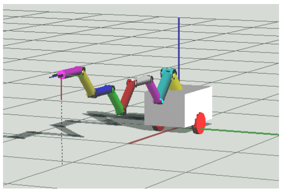
keywords— **humanoid robot;**

I. INTRODUCTION

Hindrances interfering realistic and practical utilization of hyper-redundant manipulator is thought to be the facts that the higher redundant degrees make the weight of the structure heavier, resulting in some difficulties in the controlling, accuracy and stability including a possibility of the hyper-redundant mobile manipulator overturning. For solving this problem we have come up with some inspirations about effective motion control strategies by observing human's handwriting motion. Writing characters on a paper with contacting one's elbow as shown Fig.1(a) is one of the examples of human's skillful behavior thought to be exploiting the contact constraint of the elbow with the table for reducing inputting energy by countering gravity effects with reaction forces.



(a) Human's writing motion



(b) Contacting strategy of mobile manipulator

Fig. 1. The sketch picture of Hyper-Redundant mobile Manipulator with elbows

Therefore up to now there has been several researches discussing effectiveness and accuracy of the hyper-redundant manipulator with constraint due to contact with the environment. West and Asada [1] presented a general kinematics contact model for the design of hybrid position/force controllers for constrained manipulator. And then a multi-contact kinematic model to control manipulator's contact motion was also presented in [2], [3], in which they assumed the contact environment as a spring model. However actually the contact environment is naturally thought to be rigid since the deformation of contacting surface of objects needs unusually large contacting force. Therefore we think the spring model to represent environments is not natural. Moreover the contact point of manipulator may oscillate with respect to the contact environment due to the assumed spring model and exerting contacting force. So this spring contact environment model is somewhat not practical approach to represent contacting nature. Contrarily in this paper we will discuss a model purely without contacting deformation of environment.

In this research, we propose a new dynamical model of manipulator with multi-elbow and basement which is shown in Fig.1(b) depicting 10-links redundant manipulator whose plural intermediate links contact to the ground, comprising manipulator's dynamics and geometrical constraint conditions, realized through the synthesization of multi-constraint condition of elbows and equation of manipulator's motion. Moveover after designing a mobile basement for the hyper-redundant manipulator, the whole dynamical model has some characters resembling a humanoid robot, such as the two wheels of mobile robot can be replaced by the dynamics of feet of humoid robot and the contacting of manipulator with ground is similar with the hands of humanoid robot touching ground. Therefore it is supposed that the proposed dynamical model of hyper-redundant mobile manipulator will be foundation of discussing constraint motion of humanoid robot to environments.

Finally as the first step we design a model of humanoid robot based on the hyper-redundant manipulator without considering the constraint condition and using the motor. In the last section we also have done some simulations to check this model of humanoid robot.

II. MODELLING OF HYPER-REDUNDANT MOBILE MANIPULATOR WITH CONSTRAINT

A. *Manipulator's Model with Hand's Constraint*

To make the explanation of constraint motion with multi-elbows be easily understandable, we discuss firstly about the model of the manipulator whose end-effector is contacing with rigid environment without elasticity. Equation of motion

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of manipulator is composed of rigid structure of s links, and also contact relation between manipulator's end-effector and definition of constraint surface should be introduced firstly. L represents Lagrangian, $\boldsymbol{q} \in R^s$ represents the general coordinate, $\boldsymbol{\tau} \in R^s$ represents the general input. u is the unknown constant of lagrange, f_t is the friction. Manipulator hand's Lagrange equation can be expressed as follows

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}) - (\frac{\partial L}{\partial \boldsymbol{q}}) = \boldsymbol{\tau} + (\frac{\partial C}{\partial \boldsymbol{q}^T})^T u - (\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^T})^T \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|} f_t \quad (1)$$

Here according to the kinematic relation, manipulator hand's position/posture vector $\boldsymbol{r} \in R^s$ and scalar function, a single constraint condition C that is used to express the hypersurface can be expressed as

$$\boldsymbol{r} = \boldsymbol{r}(\boldsymbol{q}) \quad (2)$$

$$C(\boldsymbol{r}(\boldsymbol{q})) = 0 \quad (3)$$

Here Eq.(2) and Eq.(3) represent constraint is undeformed.

To move freely in the directions without constraint the freedom of manipulator's end-effctor is left to be more than one, so here $s > 1$. If we set f_n to indicate the constraint force of manipulator hand, then the relation of u and f_n can be expressed as

$$u = f_n / \| \frac{\partial C}{\partial \boldsymbol{r}^T} \| \quad (4)$$

$\|\partial C/\partial \boldsymbol{r}^T\|$ shows Euclidean norm of vector $\partial C/\partial \boldsymbol{r}^T$. Then manipulator's equation of motion can be derived by combining Eq(1) with Eq(4) with viscous friction of joints [4].

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ &= \boldsymbol{\tau} + \{(\frac{\partial C}{\partial \boldsymbol{q}^T})^T / \| \frac{\partial C}{\partial \boldsymbol{r}^T} \| \} f_n - (\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^T})^T \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|} f_t \end{aligned} \quad (5)$$

$\boldsymbol{M}$ is inertia matrix of $s \times s$, $\boldsymbol{h}$ and $\boldsymbol{g}$ are $s \times 1$ vectors which indicate the effects from coriolis force, centrifugal force and gravity, $\boldsymbol{D}$ is a $s \times s$ matrix which indicates the coefficient of joints' viscous friction, expressed as $\boldsymbol{D} = diag[D_1, D_2, \cdots, D_s]$. $\boldsymbol{q}$ is the joint angle and $\boldsymbol{\tau}$ is the input torque.

B. Model with Multiple Constraints

Here we consider a motion of a manipulator having s links whose elbows are contact at p points with environments defined as

$$C_i(\boldsymbol{r}_i(\boldsymbol{q})) = 0, \quad (i = 1, 2, \cdots, p) \quad (6)$$

where $\boldsymbol{r}_i$ is the equation of position and posture of link i contacting with constraint, like Eq(2).

$$\boldsymbol{r}_i = \boldsymbol{r}_i(\boldsymbol{q}) \quad (7)$$

The Eq(5) describes a motion of the manipulator whose hand is constrained. Under the situation with the i-th link contacting, then we can define two vectors concerning i-th constraint condition C_i as follows,

$$(\frac{\partial C_i}{\partial \boldsymbol{q}^T})^T / \| \frac{\partial C_i}{\partial \boldsymbol{r}^T} \| = \boldsymbol{j_c}_i^T \quad (8)$$

$$(\frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{q}^T})^T \frac{\dot{\boldsymbol{r}}_i}{\|\dot{\boldsymbol{r}}_i\|} = \boldsymbol{j_t}_i^T \quad (9)$$

Accumulating all the above vectors ($i = 1, 2, \cdots, p$) where p is the number of contact point, so the next relations are redefined.

$$\boldsymbol{J_c}^T = [\boldsymbol{j_c}_1^T, \ \boldsymbol{j_c}_2^T, \ \cdots, \ \boldsymbol{j_c}_p^T] \quad (10)$$

$$\boldsymbol{J_t}^T = [\boldsymbol{j_t}_1^T, \ \boldsymbol{j_t}_2^T, \ \cdots, \ \boldsymbol{j_t}_p^T] \quad (11)$$

$$\boldsymbol{f_n} = [f_{n1}, \ f_{n2}, \ \cdots, \ f_{np}]^T \quad (12)$$

$$\boldsymbol{f_t} = [f_{t1}, \ f_{t2}, \ \cdots, \ f_{tp}]^T \quad (13)$$

$\boldsymbol{J_c}^T, \boldsymbol{J_t}^T$ are $s \times p$ matrices, $\boldsymbol{f_n}, \boldsymbol{f_t}$ are $p \times 1$ vectors. Considering about p constraints of the intermediate links, the manipulator's equation of motion can be expressed as

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ &= \boldsymbol{\tau} + \sum_{i=1}^{p}(\boldsymbol{j_c}_i^T f_{ni}) - \sum_{i=1}^{p}(\boldsymbol{j_t}_i^T f_{ti}) \\ &= \boldsymbol{\tau} + \boldsymbol{J_c}^T \boldsymbol{f_n} - \boldsymbol{J_t}^T \boldsymbol{f_t} \end{aligned} \quad (14)$$

Moreover, Eq (6) is differentiated by time t two times, then we can derive the constraint condition of $\ddot{q}$.

$$\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_i}{\partial \boldsymbol{q}^T})\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}} + (\frac{\partial C_i}{\partial \boldsymbol{q}^T})\ddot{\boldsymbol{q}} = 0 \quad (15)$$

To make sure that manipulator hand is contact with the undeformed constraint surface all the time, value of $\boldsymbol{q}(t)$ in Eq(14) always has to satisfy Eq(6) which has no relationship with time t, if value of $\ddot{\boldsymbol{q}}$ in Eq(15) have the same value with $\ddot{\boldsymbol{q}}$ in Eq(14), then value of $\boldsymbol{q}(t)$ in Eq(14) and Eq(6) always keeps the same value regardless of time.

C. Robot's Dynamics Including Motors

In this research, we want to evaluate the effects to increase the trajectory tracking accuracy and reduce the energy consumption used for countering gravity force and other effects by bracing the intermediate links. Even though there is no robot's motion –robot is stop– the energy is kept to be consumed since motors of joints have to generate torques to maintain the required configuration of robot against gravity influence. When the robot is in motion, other effects of dynamics will be added more to the gravity effect. To evaluate this kind of wasted energy consumption, we included the effects of electronic circuit of servo motor into the equation of motion of the manipulator to represent explicitely that the robot consumes energy even while stopping.

Here v_i represents motor's voltage, R_i does resistance, L_i and i_i do the inductance and electric current, θ_i does the angular phase of motor, τ_{gi} does the motor ouput torque, τ_{Li} does the load torque, v_{gi} does electromotive force, I_{mi}

does the inertia moment of motor, K_{Ei} does the constant of electrmotive force, K_{Ti} does the constant of torque, d_{mi} does the viscous friction's coefficient of speed reducer. The relations of those variables are shown hereunder.

$$\begin{aligned} v_i(t) &= L_i\dot{i}_i + R_i i_i(t) + v_{gi}(t) &(16)\\ v_{gi}(t) &= K_{Ei}\dot{\theta}_i(t) &(17)\\ I_{mi}\ddot{\theta}_i &= \tau_{gi}(t) - \tau_{Li}(t) - d_{mi}\dot{\theta}_i &(18)\\ \tau_g(t) &= K_{Ti} i_i(t) &(19)\end{aligned}$$

From the relations of magnetic field and the coefficients above, $K_{Ti} = K_{Ei}(= K)$ holds for motors used. Combining Eq (17) with Eq (16), and also Eq (19) with Eq (18), we can derive

$$\begin{aligned} v_i &= L_i\dot{i}_i + R_i i_i + K_i\dot{\theta}_i &(20)\\ I_{mi}\ddot{\theta}_i &= K_i i_i - \tau_{Li} - d_{mi}\dot{\theta}_i &(21)\end{aligned}$$

In the situation with motor and gear whose reduction ratio is k_i are installed onto manipulator,

$$\begin{aligned} \theta_i &= k_i q_i &(22)\\ \tau_{Li} &= \frac{\tau_i}{k_i} &(23)\end{aligned}$$

Then Eq (22) and Eq (23) are substituted into Eq (20) and Eq (21) , following equations are obtained as follows,

$$\begin{aligned} L_i\dot{i}_i &= v_i - R_i i_i - K_i k_i \dot{q}_i &(24)\\ \tau_i &= -I_{mi}k_i^2\ddot{q}_i + K_i k_i i_i - d_{mi}k_i^2\dot{q}_i &(25)\end{aligned}$$

Then using vector and matrix to indicate Eq (24) and Eq (25),

$$\begin{aligned} \boldsymbol{L}\dot{\boldsymbol{i}} &= \boldsymbol{v} - \boldsymbol{R}\boldsymbol{i} - \boldsymbol{K_m}\dot{\boldsymbol{q}} &(26)\\ \boldsymbol{\tau} &= -\boldsymbol{J_m}\ddot{\boldsymbol{q}} + \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{D_m}\dot{\boldsymbol{q}} &(27)\end{aligned}$$

$$\begin{aligned} \boldsymbol{v} &= [v_1, v_2, \cdots, v_s]^T\\ \boldsymbol{i} &= [i_1, i_2, \cdots, i_s]^T\end{aligned}$$

and the definitions are shown as follows, which always have positive value.

$$\begin{aligned} \boldsymbol{L} &= diag[L_1, L_2, \cdots, L_s]\\ \boldsymbol{R} &= diag[R_1, R_2, \cdots, R_s]\\ \boldsymbol{K_m} &= diag[K_{m1}, K_{m2}, \cdots, K_{ms}]\\ \boldsymbol{J_m} &= diag[J_{m1}, J_{m2}, \cdots, J_{ms}]\\ \boldsymbol{D_m} &= diag[D_{m1}, D_{m2}, \cdots, D_{ms}]\\ K_{mi} &= K_i k_i,\ J_{mi} = I_{mi}k_i^2,\ D_{mi} = d_{mi}k_i^2\end{aligned}$$

Now substitute Eq (27) into Eq (14), we get

$$\begin{aligned} &(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{J_m})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}}\\ &= \boldsymbol{K_m}\boldsymbol{i} + \boldsymbol{J_c}^T\boldsymbol{f_n} - \boldsymbol{J_t}^T\boldsymbol{f_t} \qquad (28)\end{aligned}$$

Similar to the same relation between Eq (14) and Eq (15), the value of $\ddot{\boldsymbol{q}}$ in Eq (28) have to be identical to the value of $\ddot{\boldsymbol{q}}$ in Eq (15) representing constraint condition.

D. Robot/Motor Equation with Contact Constraint

To make sure that $\ddot{\boldsymbol{q}}$ in Eq (28) and Eq (15) are identical, constraint force $\boldsymbol{f_n}$ is subordinately decided by simultaneous equation. Then Eq (28) and Eq (15) should be transformed as follows

$$\begin{aligned} &(\boldsymbol{M} + \boldsymbol{J_m})\ddot{\boldsymbol{q}} - \boldsymbol{J_c}^T\boldsymbol{f_n}\\ &\quad = \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T\boldsymbol{f_t} \qquad (29)\\ &(\frac{\partial C_i}{\partial \boldsymbol{q}^T})\ddot{\boldsymbol{q}} = -\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_i}{\partial \boldsymbol{q}})\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}}\\ &\qquad = -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_i}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}} \qquad (30)\end{aligned}$$

Then Eq (29), Eq (30) and Eq (24) can be combined as Eq.(31).

The inertia term $(\boldsymbol{M}+\boldsymbol{J_m})$ is a $s\times s$ matrix, the coefficient vector of constraint force $\boldsymbol{j_{c}}_i^T$ is $s \times 1$ vertical vector, $\partial C_i/\partial \boldsymbol{q}^T$ is $1 \times s$ horizontal vector, inductance term $\boldsymbol{L}$ is $s \times s$ diagonal matrix, therefore, the matrix of the first term in left side in Eq (31) is a matrix of $(2s + p) \times (2s + p)$.

$$\begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -\boldsymbol{j_{c1}}^T & \cdots & -\boldsymbol{j_{cp}}^T & 0 & \cdots & 0\\ \frac{\partial C_1}{\partial \boldsymbol{q}^T} & 0 & \cdots & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ \frac{\partial C_p}{\partial \boldsymbol{q}^T} & 0 & \cdots & 0 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0 & L_1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0 & 0 & \cdots & L_s \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}\\ f_{n1}\\ \vdots\\ f_{np}\\ \dot{i}_1\\ \vdots\\ \dot{i}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T\boldsymbol{f_t}\\ -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_1}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}}\\ \vdots\\ -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_p}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}}\\ v_1 - R_1 i_1 - K_{m1}\dot{q}_1\\ \vdots\\ v_i - R_s i_s - K_{ml}\dot{q}_s \end{bmatrix} \qquad (31)$$

Here $\ddot{\boldsymbol{q}_L}, \ddot{\boldsymbol{q}_R}$ can be included in $\ddot{\boldsymbol{q}}$ and τ_L, τ_R can be also obtained by the current i input from the motors of two wheels. So then Eq (31) can be rewritten concisely using the definitions of Eq (10), Eq (12) and Eq (26) as follows,

$$\begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -\boldsymbol{J_c}^T & \boldsymbol{0}\\ \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}\\ \boldsymbol{f_n}\\ \dot{\boldsymbol{i}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T\boldsymbol{f_t}\\ -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}}\\ \boldsymbol{v} - \boldsymbol{R}\boldsymbol{i} - \boldsymbol{K_m}\dot{\boldsymbol{q}} \end{bmatrix} \qquad (32)$$

where, $\boldsymbol{C}$ is a vector of $\boldsymbol{C} = [C_1, C_2, \cdots, C_p]^T$. Furthermore by redefining as

$$M^* = \begin{bmatrix} M + J_m & -J_c{}^T & 0 \\ \frac{\partial C}{\partial q^T} & 0 & 0 \\ 0 & 0 & L \end{bmatrix} \quad (33)$$

$$b = \begin{bmatrix} K_m i - h - g - (D + D_m)\dot{q} - J_t{}^T f_t \\ -\dot{q}^T \left[\frac{\partial}{\partial q} \left(\frac{\partial C}{\partial q^T} \right) \right] \dot{q} \\ v - Ri - K_m \dot{q} \end{bmatrix} \quad (34)$$

Then Eq (32) can be expressed as

$$M^* \begin{bmatrix} \ddot{q} \\ f_n \\ \dot{i} \end{bmatrix} = b \quad (35)$$

Here M^* has been confirmed to be nonsingular matrix before by us, and then calculate the inverse of M^*, finally the unknown value of $\ddot{q}$,f_n,$\dot{i}$ can be determined based on the above simultaneous equation.

III. FORWARD DYNAMICS CALCULATION

To calculate M^* and b in Eq (35), we need to first calculate M, h, g. Here we can notice that M, h and g are included in Eq (28) that describes the dynamics of non-constraint, and those can be calculated numerically and recursively through forward dynamics calculation [5] by exploiting the inverse dynamics calculation called ''Newton Euler'' Method [6]. Because M is 12×12 matrix when the hyper-redundant mobile manipulator including 10 links and 2 wheels of mobile robot, resulting in a large amount of computation to calculate each element of M by using Lagrange method. This implies that analytical deriving Eq (28) is almost impossible by hand writing calculation, then we introduce Newton-Euler method as follows.

First of all, Eq (28) should be set as hereunder.

$$M_J \ddot{q} + b_J = \tilde{\tau} \quad (36)$$

Here

$$\begin{aligned} M_J &= M(q) + J_m \\ b_J &= h(q, \dot{q}) + g(q) + (D + D_m)\dot{q} \\ \tilde{\tau} &= K_m i + J_c{}^T f_n - J_t{}^T f_t \end{aligned}$$

With forward motion analysis, Eq (36) should be calculated by Newton-Euler method from the bottom link to upper link until the manipulator's hand, and also with the motion analysis of backward calculation, we get equation of motion of i-th link Eq (37).

$$\tilde{\tau}_i = {}^i z_i^{T\,i} n_i + J_{mi}\ddot{q}_i + (D_i + D_{mi})\dot{q}_i \quad (37)$$

Therefore, the motion Eq (36) can be used to inverse dynamics calculation $\tilde{\tau} = [\tilde{\tau}_1, \tilde{\tau}_2, \cdots, \tilde{\tau}_n, \tilde{\tau}_L, \tilde{\tau}_R]^T$ in Eq (37). This inverse calculation can be described as $\tilde{\tau} = p(q, \dot{q}, \ddot{q}, g)$. Then considering Eq (36) and Eq (37),

$$M_J \ddot{q} + b_J = p(q, \dot{q}, \ddot{q}, g) \quad (38)$$

Substitute $\ddot{q} = 0$ into Eq (38):

$$b_J = p(q, \dot{q}, 0, g) \quad (39)$$

so b_J can be calculated. Next substitute $g = 0, \dot{q} = 0$ and $\ddot{q} = e_i (i = 1, 2, \cdots, s)$ into Eq (38), then the $b_J = 0$:

$$m_i = M_J e_i = p(q, 0, e_i, 0) \quad (40)$$

here we can calculate m_i defined as the component vector of the i-th column in inertia matrix M, e_i is a $(l + 2) \times 1$ matrix in which the i-th element is 1 and others are all 0 like $e_i = [0, 0, \cdots, 1_{(i)}, \cdots, 0, 0]^T$. So with Eq (40) $M_J = [m_1, m_2, \cdots, m_l, m_L, m_R]$ can be calculated one by one separately.

Thus up to now, we have calculated the M_J and b_J. Back to the Eq (33), the M^* can be calculated while the constraint condition is given. Moreover, the inverse of M^* can be also calculated due to invertible for M^*.

IV. THE MODEL OF HUMANOID ROBOT

A. Modeling the humanoid robot

The model of humanoid robot can be designed based on the model of hyper-redundant manipulator that is presented three sections above as Fig.2. This model of humanoid robot without fingers has 33 degrees of freedom which are divided into 33 links. From Fig.2, the green link is head of humanoid robot, the red links without length are joints of humanoid robot.

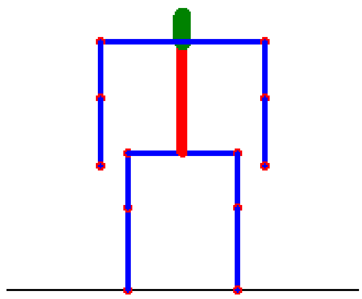


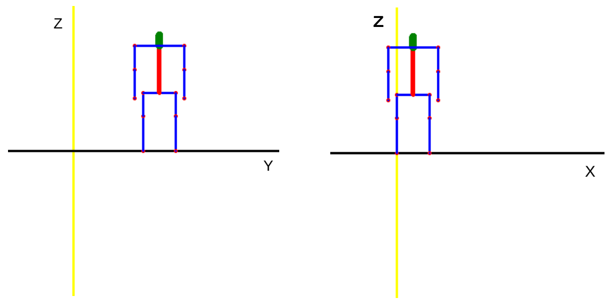
Fig. 2. The model of humanoid robot

B. Simulation

In this simulation we consider the right foot of humanoid robot is just fixed on the ground. The acceleration of gravity is $g = 9.8$, The coefficient of viscose friction among each link is $D = 1$. The length of each link is shown as Fig.2. And the intention of this simulation is to check the model of humanoid robot without considering the constraint condition and motor. The results are shown as follows.

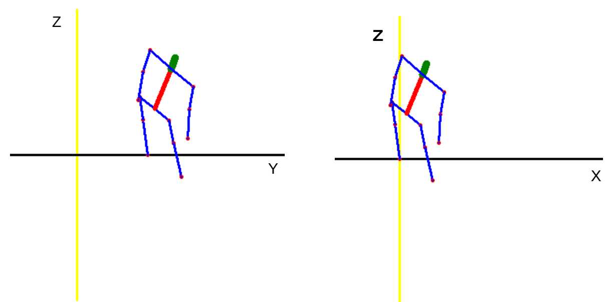
V. CONCLUSION

We first propose a dynamical model of hyper-redundant manipulator whose plural intermediate links are contacting with environment, second the model of humanoid robot can be designed according to the hyper-rudundnat manipulator is proved to be correct without considering the constraint condition and motor. Next we will consider the method using the constraint condition how to input the suitable torque and force to control the humanoid robot to stand and walk.



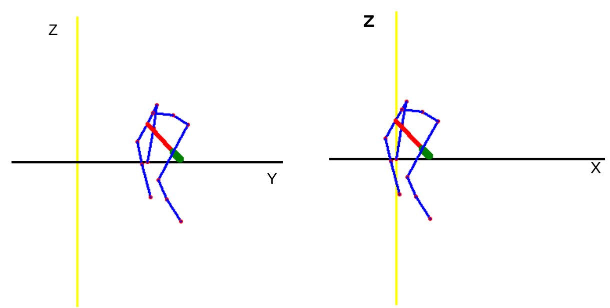
(a) on yz space (b) on xz space

Fig. 3. The shape of humanoid robot at t=0s



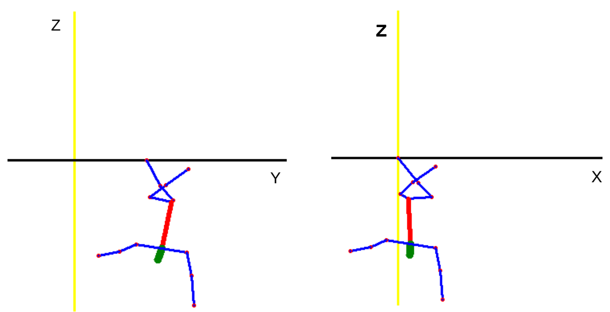
(a) on yz space (b) on xz space

Fig. 4. The shape of humanoid robot at t=0.3s



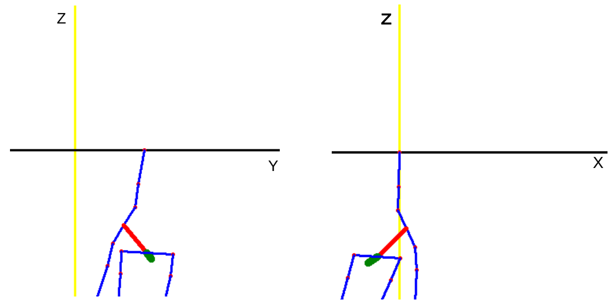
(a) on yz space (b) on xz space

Fig. 5. The shape of humanoid robot at t=0.6s



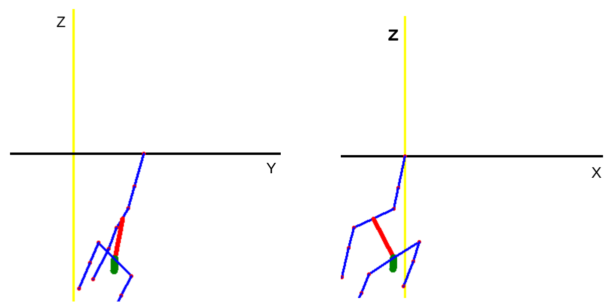
(a) on yz space (b) on xz space

Fig. 6. The shape of humanoid robot at t=1s



(a) on yz space (b) on xz space

Fig. 7. The shape of humanoid robot at t=5s



(a) on yz space (b) on xz space

Fig. 8. The shape of humanoid robot at t=10s

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