Multi-Elbows Bracing Dynamical Model of Hyper-redundant Mobile Manipulator

Geng Wang, Fujia Yu, Mamoru Minami, Akira Yanou, Mingcong Deng

***Abstract*— To overcome the conflict between the required high-redundancy for dexterous manipulation and heavy weight stemming from the high redundancy structure, we discuss a realizability of whether the contacting and bracing motion of intermediate links with environment may simultaneously prevent from overturning and reduce energy-consumption, inspired by human's handwriting motion with the elbow or wrist contacting to a table. Moreover considering the availability of the hyper-redundant manipulator, we design the basement of hyper-redundant manipulator as a mobile robot to give the mobile manipulator an ability to move to where a given task is convenient to be done. Thus in this paper we present a practical dynamical model of hyper-redundant mobile manipulator whose plural elbows of intermediate links are being braced with environment.**

***keywords*— hyper-redundant manipulator; bracing motion; overturning;**

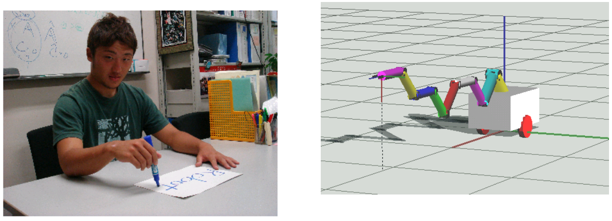
I. INTRODUCTION

Hindrances interfering realistic and practical utilization of hyper-redundant manipulator is thought to be the facts that the higher redundant degrees make the weight of the structure heavier, resulting in some difficulties in the controlling, accuracy and stability including a possibility of the hyper-redundant mobile manipulator overturning. For solving this problem we have come up with some inspirations about effective motion control strategies by observing human's handwriting motion. Writing characters on a paper with contacting one's elbow as shown Fig.1(a) is one of the examples of human's skillful behavior thought to be exploiting the contact constraint of the elbow with the table for reducing inputting energy by countering gravity effects with reaction forces.

Hyper-redundant manipulator had been researched intensively and those efforts had been introduced by Chirikjian and Burdick[1] more than decades ago, where the structure of the discussed hyper-redundant manipulator could move in 3D space but the hand's load has been restricted tiny things meaninglessly. Though considerable researches have discussed how to utilize the redundancy [2]-[5], for example avoiding obstacles [6]-[9] or optimizing the configuration concerning practical criteria [10],[11], etc.. However it seems that the merits of the hyper-redundancy has not been utilized enough effectively and practically. The reason is we think

This work was not supported by any organizatio

Geng Wang, Fujia Yu, Mamoru Minami, Akira Yanou, Mingcong Deng are with Graduate School of Natural Science and Technology, Okayama University Tsushimanaka3-1-1 , Okayama, JAPAN. {owg_cyo, yufujia, minami, yanou, deng}@suri.sys.okayama-u.ac.jp



(a) Human's writing motion

(b) Contacting strategy of mobile manipulator

Fig. 1. The sketch picture of Hyper-Redundant Manipulator with elbows

higher redundant degree causes heavier weight of structure, accordingly its end effector falls down by gravity influence, which has the control precision of the end-effector getting worse.

Therefore up to now there has been several researches discussing effectiveness and accuracy of the hyper-redundant manipulator with constraint due to contact with the environment. West and Asada [12] presented a general kinematics contact model for the design of hybrid position/force controllers for constrained manipulator. And then a multi-contact kinematic model to control manipulator's contact motion was also presented in [13], [14], in which they assumed the contact environment as a spring model. However actually the contact environment is naturally thought to be rigid since the deformation of contacting surface of objects needs unusually large contacting force. Therefore we think the spring model to represent environments is not natural. Moreover the contact point of manipulator may oscillate with respect to the contact environment due to the assumed spring model and exerting contacting force. So this spring contact environment model is somewhat not practical approach to represent contacting nature. Contrarily in this paper we will discuss a model purely without contacting deformation of environment.

In this research, we propose a new dynamical model of mobile manipulator with multi-elbow and basement which is shown in Fig.1(b) depicting 10-links redundant manipulator whose plural intermediate links contact to the ground, comprising manipulator's dynamics and geometrical constraint conditions, realized through the synthesization of multi-constraint condition of elbows and equation of manipulator's motion. Moveover after designing a mobile basement for the hyper-redundant manipulator, the whole dynamical model has some characters resembling a humanoid robot, such as the two wheels of mobile robot can be replaced by the dynamics of feet of humoid robot and the contacting of manipulator with ground is similar with the hands of humanoid robot touching ground. Therefore it is supposed that the proposed dynamical model of hyper-redundant mobile manipulator will be foundation of discussing constraint motion of humanoid robot to environments.

II. MODELLING OF HYPER-REDUNDANT MOBILE MANIPULATOR WITH CONSTRAINT

A. Manipulator's Model with Hand's Constraint

To make the explanation of constraint motion with multi-elbows be easily understandable, we discuss firstly about the model of the manipulator whose end-effector is contacing with rigid environment without elasticity. Equation of motion of manipulator is composed of rigid structure of s links, and also contact relation between manipulator's end-effector and definition of constraint surface should be introduced firstly. L represents Lagrangian, $q \in R^s$ represents the general coordinate, $\tau \in R^s$ represents the general input. u is the unknown constant of lagrange, f_t is the friction. Manipulator hand's Lagrange equation can be expressed as follows

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}) - (\frac{\partial L}{\partial q}) = \tau + (\frac{\partial C}{\partial q^T})^T u - (\frac{\partial r}{\partial q^T})^T \frac{\dot{r}}{\|\dot{r}\|} f_t \quad (1)$$

Here according to the kinematic relation, manipulator hand's position/posture vector $r \in R^s$ and scalar function, a single constraint condition C that is used to express the hypersurface can be expressed as

$$r = r(q) \quad (2)$$

$$C(r(q)) = 0 \quad (3)$$

Here Eq.(2) and Eq.(3) represent constraint is undeformed.

To move freely in the directions without constraint the freedom of manipulator's end-effctor is left to be more than one, so here $s > 1$. If we set f_n to indicate the constraint force of manipulator hand, then the relation of u and f_n can be expressed as

$$u = f_n / \| \frac{\partial C}{\partial r^T} \| \quad (4)$$

$\|\partial C/\partial r^T\|$ shows Euclidean norm of vector $\partial C/\partial r^T$. Then manipulator's equation of motion can be derived by combining Eq(1) with Eq(4) with viscous friction of joints [15].

$$\begin{aligned} & M(q)\ddot{q} + h(q,\dot{q}) + g(q) + D\dot{q} \\ = \quad & \tau + \{(\frac{\partial C}{\partial q^T})^T / \| \frac{\partial C}{\partial r^T} \| \} f_n - (\frac{\partial r}{\partial q^T})^T \frac{\dot{r}}{\|\dot{r}\|} f_t \quad (5) \end{aligned}$$

M is inertia matrix of $s \times s$, h and g are $s \times 1$ vectors which indicate the effects from coriolis force, centrifugal force and gravity, D is a $s \times s$ matrix which indicates the coefficient of joints' viscous friction, expressed as $D = diag[D_1, D_2, \cdots, D_s]$. q is the joint angle and τ is the input torque.

B. Model with Multiple Constraints

Here we consider a motion of a manipulator having s links whose elbows are contact at p points with environments defined as

$$C_i(r_i(q)) = 0, \quad (i = 1, 2, \cdots, p) \quad (6)$$

where r_i is the equation of position and posture of link i contacting with constraint, like Eq(2).

$$r_i = r_i(q) \quad (7)$$

The Eq(5) describes a motion of the manipulator whose hand is constrained. Under the situation with the i-th link contacting, then we can define two vectors concerning i-th constraint condition C_i as follows,

$$(\frac{\partial C_i}{\partial q^T})^T / \| \frac{\partial C_i}{\partial r^T} \| = j_{ci}^T \quad (8)$$

$$(\frac{\partial r_i}{\partial q^T})^T \frac{\dot{r}_i}{\|\dot{r}_i\|} = j_{ti}^T \quad (9)$$

Accumulating all the above vectors $(i = 1, 2, \cdots, p)$ where p is the number of contact point, so the next relations are redefined.

$$J_c^T = [j_{c1}^T, \ j_{c2}^T, \ \cdots, \ j_{cp}^T] \quad (10)$$

$$J_t^T = [j_{t1}^T, \ j_{t2}^T, \ \cdots, \ j_{tp}^T] \quad (11)$$

$$f_n = [f_{n1}, \ f_{n2}, \ \cdots, \ f_{np}]^T \quad (12)$$

$$f_t = [f_{t1}, \ f_{t2}, \ \cdots, \ f_{tp}]^T \quad (13)$$

J_c^T, J_t^T are $s \times p$ matrices, f_n, f_t are $p \times 1$ vectors. Considering about p constraints of the intermediate links, the manipulator's equation of motion can be expressed as

$$\begin{aligned} & M(q)\ddot{q} + h(q,\dot{q}) + g(q) + D\dot{q} \\ = \quad & \tau + \sum_{i=1}^{p}(j_{ci}^T f_{ni}) - \sum_{i=1}^{p}(j_{ti}^T f_{ti}) \\ = \quad & \tau + J_c^T f_n - J_t^T f_t \quad (14) \end{aligned}$$

Moreover, Eq (6) is differentiated by time t two times, then we can derive the constraint condition of $\ddot{q}$.

$$\left[\frac{\partial}{\partial q}(\frac{\partial C_i}{\partial q^T})\dot{q}\right]\dot{q} + (\frac{\partial C_i}{\partial q^T})\ddot{q} = 0 \quad (15)$$

To make sure that manipulator hand is contact with the undeformed constraint surface all the time, value of $q(t)$ in Eq(14) always has to satisfy Eq(6) which has no relationship with time t, if value of $\ddot{q}$ in Eq(15) have the same value with $\ddot{q}$ in Eq(14), then value of $q(t)$ in Eq(14) and Eq(6) always keeps the same value regardless of time.

C. Robot's Dynamics Including Motors

In this research, we want to evaluate the effects to increase the trajectory tracking accuracy and reduce the energy consumption used for countering gravity force and other effects by bracing the intermediate links. Even though there is no robot's motion –robot is stop– the energy is kept to be consumed since motors of joints have to generate torques to maintain the required configuration of robot against gravity influence. When the robot is in motion, other effects of dynamics will be added more to the gravity effect. To evaluate this kind of wasted energy consumption, we included the effects of electronic circuit of servo motor into the equation of motion of the manipulator to represent explicitely that the robot consumes energy even while stopping.

Here v_i represents motor's voltage, R_i does resistance, L_i and i_i do the inductance and electric current, θ_i does the angular phase of motor, τ_{gi} does the motor ouput torque, τ_{Li} does the load torque, v_{gi} does electromotive force, I_{mi} does the inertia moment of motor, K_{Ei} does the constant of electrmotive force, K_{Ti} does the constant of torque, d_{mi} does the viscous friction's coefficient of speed reducer. The relations of those variables are shown hereunder.

$$v_i(t) = L_i\dot{i}_i + R_i i_i(t) + v_{gi}(t) \tag{16}$$
$$v_{gi}(t) = K_{Ei}\dot{\theta}_i(t) \tag{17}$$
$$I_{mi}\ddot{\theta}_i = \tau_{gi}(t) - \tau_{Li}(t) - d_{mi}\dot{\theta}_i \tag{18}$$
$$\tau_g(t) = K_{Ti} i_i(t) \tag{19}$$

From the relations of magnetic field and the coefficients above, $K_{Ti} = K_{Ei}(= K)$ holds for motors used. Combining Eq (17) with Eq (16), and also Eq (19) with Eq (18), we can derive

$$v_i = L_i\dot{i}_i + R_i i_i + K_i\dot{\theta}_i \tag{20}$$
$$I_{mi}\ddot{\theta}_i = K_i i_i - \tau_{Li} - d_{mi}\dot{\theta}_i \tag{21}$$

In the situation with motor and gear whose reduction ratio is k_i are installed onto manipulator,

$$\theta_i = k_i q_i \tag{22}$$
$$\tau_{Li} = \frac{\tau_i}{k_i} \tag{23}$$

Then Eq (22) and Eq (23) are substituted into Eq (20) and Eq (21) , following equations are obtained as follows,

$$L_i\dot{i}_i = v_i - R_i i_i - K_i k_i \dot{q}_i \tag{24}$$
$$\tau_i = -I_{mi}k_i^2\ddot{q}_i + K_i k_i i_i - d_{mi}k_i^2\dot{q}_i \tag{25}$$

Then using vector and matrix to indicate Eq (24) and Eq (25),

$$L\dot{i} = v - Ri - K_m\dot{q} \tag{26}$$
$$\tau = -J_m\ddot{q} + K_m i - D_m\dot{q} \tag{27}$$

$$v = [v_1, v_2, \cdots, v_s]^T$$
$$i = [i_1, i_2, \cdots, i_s]^T$$

and the definitions are shown as follows, which always have positive value.

$$L = diag[L_1, L_2, \cdots, L_s]$$
$$R = diag[R_1, R_2, \cdots, R_s]$$
$$K_m = diag[K_{m1}, K_{m2}, \cdots, K_{ms}]$$
$$J_m = diag[J_{m1}, J_{m2}, \cdots, J_{ms}]$$
$$D_m = diag[D_{m1}, D_{m2}, \cdots, D_{ms}]$$
$$K_{mi} = K_i k_i,\ J_{mi} = I_{mi}k_i^2,\ D_{mi} = d_{mi}k_i^2$$

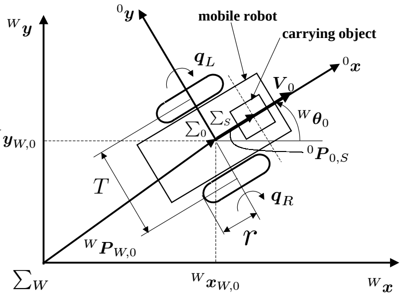


Fig. 2. Power-Wheeled-Steeling(PWS) mobile robot

Now substitute Eq (27) into Eq (14), we get

$$(M(q) + J_m)\ddot{q} + h(q,\dot{q}) + g(q) + (D + D_m)\dot{q} = K_m i + {J_c}^T f_n - {J_t}^T f_t \tag{28}$$

Similar to the same relation between Eq (14) and Eq (15), the value of $\ddot{q}$ in Eq (28) have to be identical to the value of $\ddot{q}$ in Eq (15) representing constraint condition.

D. The basement model of hyper-redundant mobile manipulator

In this research we consider the link 0 as the basement of hyper-redundant mobile manipulator, so the effect to the mobile robot is same as to link 0 due to the motion of hyper-redundant manipulator.

Translational velocity ${}^W\dot{P}_0$ and angular velocity ${}^W\omega_0$ of link 0 representing mobile robot in Σ_W are

$${}^W\dot{P}_0 = {}^W R_0\,{}^0 V_0 = {}^W R_0[V_0, 0, 0]^{\mathrm{T}} \tag{29}$$

$${}^W\omega_0 = [0, 0, {}^W\omega_0]^{\mathrm{T}} \tag{30}$$

To consider gravity acceleration into dynamical model, the gravity vector is defined as ${}^W g = [0,\ 0,\ -g]^{\mathrm{T}}$. Then the translational acceleration ${}^W\ddot{P}_0$ including ${}^W g$ and the angular acceleration ${}^W\dot{\omega}_0$ of link 0 are obtained from Eq (29) and Eq (30) as

$${}^W\ddot{P}_0 = {}^W\ddot{\bar{P}}_0 - {}^W g = {}^W\dot{R}_0\,{}^0 V_0 + {}^W R_0\,{}^0\dot{V}_0 - {}^W g \tag{31}$$
$${}^W\dot{\omega}_0 = [0, 0, {}^W\dot{\omega}_0]^{\mathrm{T}} \tag{32}$$

Let ${}^0 S_0$ be the gravity center of link 0 with respect to Σ_0 as shown in Fig.2. By using ${}^0 S_0$ and ${}^W R_0$, the position vector ${}^W S_0$ from the origin of Σ_0 to the gravity center of link 0 with respect to Σ_W is expressed as ${}^W S_0 = {}^W R_0\,{}^0 S_0$. The acceleration ${}^W\ddot{P}_{G0}$ at the gravity center of link 0 is given with ${}^W S_0$ by

$${}^W\ddot{P}_{G0} = {}^W\ddot{P}_0 + {}^W\dot{\omega}_0 \times {}^W S_0 + {}^W\omega_0 \times ({}^W\omega_0 \times {}^W S_0) \tag{33}$$

and then ${}^W\ddot{P}_{Gi}$ at the gravity center of link i can be calculated by

$${}^W\ddot{P}_{Gi} = {}^W\ddot{P}_i + {}^W\dot{\omega}_i \times {}^W S_i + {}^W\omega_i \times ({}^W\omega_i \times {}^W S_i) \tag{34}$$

The movement of i-th link of manipulator should be given force and torque, so they can be obtained by

$$
\begin{aligned}
{}^W f_i &= {}^W f_{i+1} + \mathrm{m}_i {}^W \ddot{P}_{W,Gi} && (35)\\
{}^W n_i &= {}^W n_{i+1} + {}^W I_i {}^W \dot{\omega}_i + {}^W \omega_i \times ({}^W I_i {}^W \omega_i)\\
&+ {}^W S_i \times \mathrm{m}_i {}^W \ddot{P}_{W,Gi} + {}^W P_{i,i+1} \times {}^W f_{i+1} && (36)\\
\tau_i &= ({}^W n_i^T)\, {}^W z_i + I_{ai}\ddot{q}_i + C_i \dot{q}_i && (37)
\end{aligned}
$$

So the equation of motion of mobile robot can be derived as,

$$
\begin{aligned}
{}^W f_0 &= {}^W f_1 + \mathrm{m}_0 {}^W \ddot{P}_{G0} && (38)\\
{}^W n_0 &= {}^W n_1 + {}^W I_0 {}^W \dot{\omega}_0 + {}^W \omega_0 \times ({}^W I_0 {}^W \omega_0)\\
&+ {}^W S_0 \times \mathrm{m}_0 {}^W \ddot{P}_{G0} + {}^W P_{0,1} \times {}^W f_1 && (39)
\end{aligned}
$$

Link 0 can be actuated by two variables above to get motions which are translation along the direction ${}^W x_0$ and rotation around axis ${}^W z_0$. However, by using non-holonomic constraint, it has three degrees of freedom in the traveling motion. Then the driving force and rotational torque of link 0 are determined by summation and subtraction of left and right wheels' driving torques $\widehat{\tau}_L$ and $\widehat{\tau}_R$, which are required to make desired rotations of each driving wheel. Taking these relations into consideration, we have two equations between those torques and ${}^W f_0$, ${}^W n_0$ as follows:

$$
\begin{aligned}
\frac{\widehat{\tau}_R}{\mathrm{r}} + \frac{\widehat{\tau}_L}{\mathrm{r}} &= {}^W f_0{}^{\mathrm{T}}\ {}^W x_0 = ({}^W R_0 {}^0 f_0)^{\mathrm{T}}\, {}^W R_0 {}^0 x_0\\
&= ({}^0 f_0{}^{\mathrm{T}})^0 x_0 = f_0 && (40)\\
\frac{\mathrm{T}}{2}(\frac{\widehat{\tau}_R}{\mathrm{r}} - \frac{\widehat{\tau}_L}{\mathrm{r}}) &= {}^W n_0{}^{\mathrm{T}}\ {}^W z_0 = ({}^W R_0 {}^0 n_0)^{\mathrm{T}}\, {}^W R_0 {}^0 z_0\\
&= ({}^0 n_0{}^{\mathrm{T}})^0 z_0 = \tau_0 && (41)
\end{aligned}
$$

Here r is the radius of each wheel and T is the distance between the center of two wheels. $\widehat{\tau}_R$ and $\widehat{\tau}_L$ are obtained by solving Eq (40) and Eq (41). Let $\mathrm{C}_R, \mathrm{C}_L$ and $\mathrm{I}_{aR}, \mathrm{I}_{aL}$ be respectively, viscous damping and inertia moment of right and left driving systems, then the torque τ_R and τ_L, which make the traveling motion, are calculated as

$$
\begin{aligned}
\tau_L &= \widehat{\tau}_L + \mathrm{I}_{aL}\ddot{q}_L + \mathrm{C}_L \dot{q}_L && (42)\\
\tau_R &= \widehat{\tau}_R + \mathrm{I}_{aR}\ddot{q}_R + \mathrm{C}_R \dot{q}_R && (43)
\end{aligned}
$$

$\widehat{\tau}_R$ and $\widehat{\tau}_L$, which are calculated from ${}^W f_0$ and ${}^W n_0$, contain $V_0, \dot{V}_0, \omega_0$ and $\dot{\omega}_0$ as variables in them.

E. Robot/Motor Equation with Contact Constraint

To make sure that $\ddot{q}$ in Eq (28) and Eq (15) are identical, constraint force f_n is subordinately decided by simultaneous equation. Then Eq (28) and Eq (15) should be transformed as follows

$$
\begin{aligned}
&(M + J_m)\ddot{q} - J_c{}^T f_n\\
&\quad = K_m i - h - g - (D + D_m)\dot{q} - J_t{}^T f_t && (44)\\
&(\frac{\partial C_i}{\partial q^T})\ddot{q} = -\left[\frac{\partial}{\partial q}(\frac{\partial C_i}{\partial q})\dot{q}\right]\dot{q}\\
&\quad = -\dot{q}^T\left[\frac{\partial}{\partial q}(\frac{\partial C_i}{\partial q^T})\right]\dot{q} && (45)
\end{aligned}
$$

Then Eq (44), Eq (45) and Eq (24) can be combined as Eq.(46). Here we assume that friction force f_{ti} is dynamic friction and define it as $f_t = 0.1 f_n (i = 1, 2, \cdots, p)$.

The inertia term $(M + J_m)$ is a $(s+2)\times(s+2)$ matrix, the coefficient vector of constraint force $j_{c_i}^T$ is $s \times 1$ vertical vector, $\partial C_i / \partial q^T$ is $1 \times s$ horizontal vector, inductance term L is $s \times s$ diagonal matrix, therefore, the matrix of the first term in left side in Eq (46) is a matrix of $(2s+p+2)\times(2s+p+2)$.

$$
\begin{bmatrix}
M + J_m & -j_{c1}^T & \cdots & -j_{cp}^T & 0 & \cdots & 0\\
\frac{\partial C_1}{\partial q^T} & 0 & \cdots & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
\frac{\partial C_p}{\partial q^T} & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & 0 & \cdots & 0 & L_1 & \cdots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & 0 & 0 & \cdots & L_s
\end{bmatrix}
\begin{bmatrix}
\ddot{q}\\ \ddot{q}_L\\ \ddot{q}_R\\ f_{n1}\\ \vdots\\ f_{np}\\ \dot{i}_1\\ \vdots\\ \dot{i}_s
\end{bmatrix}
= \begin{bmatrix}
K_m i - h - g - (D + D_m)\dot{q} - J_t{}^T f_t\\
\tau_L\\
\tau_R\\
-\dot{q}^T\left[\frac{\partial}{\partial q}(\frac{\partial C_1}{\partial q^T})\right]\dot{q}\\
\vdots\\
-\dot{q}^T\left[\frac{\partial}{\partial q}(\frac{\partial C_p}{\partial q^T})\right]\dot{q}\\
v_1 - R_1 i_1 - K_{m1}\dot{q}_1\\
\vdots\\
v_i - R_s i_s - K_{ml}\dot{q}_s
\end{bmatrix}
\qquad (46)
$$

Here $\ddot{q_L}, \ddot{q_R}$ can be included in $\ddot{q}$ and τ_L, τ_R can be also obtained by the current i input from the motors of two wheels. So then Eq (46) can be rewritten concisely using the definitions of Eq (10), Eq (12) and Eq (26) as follows,

$$
\begin{bmatrix}
M + J_m & -J_c{}^T & 0\\
\frac{\partial C}{\partial q^T} & 0 & 0\\
0 & 0 & L
\end{bmatrix}
\begin{bmatrix}
\ddot{q}\\ f_n\\ \dot{i}
\end{bmatrix}
= \begin{bmatrix}
K_m i - h - g - (D + D_m)\dot{q} - J_t{}^T f_t\\
-\dot{q}^T\left[\frac{\partial}{\partial q}(\frac{\partial C}{\partial q^T})\right]\dot{q}\\
v - Ri - K_m\dot{q}
\end{bmatrix}
\qquad (47)
$$

where, C is a vector of $C = [C_1, C_2, \cdots, C_p]^T$. Furthermore by redefining as

$$
M^* = \begin{bmatrix}
M + J_m & -J_c{}^T & 0\\
\frac{\partial C}{\partial q^T} & 0 & 0\\
0 & 0 & L
\end{bmatrix}
\qquad (48)
$$

$$
b = \begin{bmatrix}
K_m i - h - g - (D + D_m)\dot{q} - J_t{}^T f_t\\
-\dot{q}^T\left[\frac{\partial}{\partial q}(\frac{\partial C}{\partial q^T})\right]\dot{q}\\
v - Ri - K_m\dot{q}
\end{bmatrix}
\qquad (49)
$$

Then Eq (47) can be expressed as

$$
M^* \begin{bmatrix}\ddot{q}\\ f_n\\ \dot{i}\end{bmatrix} = b \qquad (50)
$$

Here M^* has been confirmed to be nonsingular matrix before by us, and then calculate the inverse of M^*, finally the unknown value of $\ddot{q}$,f_n,$\dot{i}$ can be determined based on the above simultaneous equation.

III. FORWARD DYNAMICS CALCULATION

To calculate M^* and b in Eq (50), we need to first calculate M,h,g. Here we can notice that M,h and g are included in Eq (28) that describes the dynamics of non-constraint, and those can be calculated numerically and recursively through forward dynamics calculation [16] by exploiting the inverse dynamics calculation called ''Newton Euler" Method [17]. Because M is 12×12 matrix when the hyper-redundant mobile manipulator including 10 links and 2 wheels of mobile robot, resulting in a large amount of computation to calculate each element of M by using Lagrange method. This implies that analytical deriving Eq (28) is almost impossible by hand writing calculation, then we introduce Newton-Euler method as follows.

First of all, Eq (28) should be set as hereunder.

$$M_J\ddot{q}+b_J=\tilde{\tau} \quad (51)$$

Here

$$\begin{aligned} M_J &= M(q)+J_m \\ b_J &= h(q,\dot{q})+g(q)+(D+D_m)\dot{q} \\ \tilde{\tau} &= K_m i+{J_c}^T f_n-{J_t}^T f_t \end{aligned}$$

With forward motion analysis, Eq (51) should be calculated by Newton-Euler method from the bottom link to upper link until the manipulator's hand, and also with the motion analysis of backward calculation, we get equation of motion of i-th link Eq (52).

$$\tilde{\tau}_i = {}^i z_i^{T\,i} n_i+J_{mi}\ddot{q}_i+(D_i+D_{mi})\dot{q}_i \quad (52)$$

Therefore, the motion Eq (51) can be used to inverse dynamics calculation $\tilde{\tau}=[\tilde{\tau}_1,\tilde{\tau}_2,\cdots,\tilde{\tau}_n,\tilde{\tau}_L,\tilde{\tau}_R]^T$ in Eq (52). This inverse calculation can be described as $\tilde{\tau}=p(q,\dot{q},\ddot{q},g)$. Then considering Eq (51) and Eq (52),

$$M_J\ddot{q}+b_J=p(q,\dot{q},\ddot{q},g) \quad (53)$$

Substitute $\ddot{q}=0$ into Eq (53):

$$b_J=p(q,\dot{q},0,g) \quad (54)$$

so b_J can be calculated. Next substitute $g=0,\dot{q}=0$ and $\ddot{q}=e_i(i=1,2,\cdots,s)$ into Eq (53), then the $b_J=0$:

$$m_i=M_Je_i=p(q,0,e_i,0) \quad (55)$$

here we can calculate m_i defined as the component vector of the i-th column in inertia matrix M, e_i is a $(l+2)\times 1$ matrix in which the i-th element is 1 and others are all 0 like $e_i=[0,0,\cdots,1_{(i)},\cdots,0,0]^T$. So with Eq (55) $M_J=[m_1,m_2,\cdots,m_l,m_L,m_R]$ can be calculated one by one separately.

Thus up to now, we have calculated the M_J and b_J. Back to the Eq (48), the M^* can be calculated while the constraint condition is given. Moreover, the inverse of M^* can be also calculated due to invertible for M^*.

IV. SIMULATION

In this section we will introduce the simulation to check the model desired by us right or not. Simulation's condition has been set as: mobile's mass is $m_0=10[kg]$, width is $W=0.6[m]$, length is $S=0.8[m]$, height is $H=0.3[m]$, the radius of two wheels is $r=0.1[m]$, each link's mass is $m_i=0.1[kg]$, length is $l_1=0[m],l_j=0.3[m]$,radius of cylindrical link is $r_i=0.01[m]$,proportional gain is $k_{pi}=500$,velocity gain is $k_{di}=20$,viscous friction coefficient of joint is $D_i=0.5$, torque constant is $K_i=0.203$, resistance is $R_i=1.1[\Omega]$, inductance is $L_i=0.0017[H]$,inertia moment of motor is $I_{mi}=0.000164$,reduction ratio is $k_i=3.0$, viscous friction coefficient of reducer is $d_{mi}=0.01$ and these parameters are given by actucal motor's specifications. The gravity acceleration is $g=0[m/s^2]$. The link1 is input a torque to turn around z-axis like $\tau_1=cos(2\pi t/T)$, T is the period, and the torque of other links is zero. The mobile robot has no extra-input and friction force. Initial condition of each link: $q_1(0)=0.5\pi,q_2(0)=0.25\pi,q_3(0)=0.5\pi,q_4(0)=-0.5\pi,q_5(0)=0.25\pi,q_6(0)=0.25\pi$, $q_7(0)=-0.5\pi,q_8(0)=0.25\pi,q_9(0)=-0.25\pi,q_{10}(0)=0.25\pi$, $\dot{q}_i=0$. The constraint condition is set as $C=z=0$. Simulation has been done under three situations shown as Fig.3 which we view the model from different direction. The mobile robot is moving along x-axis. Because the gravity is not considered in this simulation now, the results of three situations are same each other. Then here we just show the one group as follows.

1) trajectory tracking motion with two elbows contacting at joint 4 and joint 7;
2) trajectory tracking motion with just one elbow contacting at joint 7;
3) trajectory tracking motion with just one elbow contacting at joint 4;
4) trajectory tracking motion with no elbow contacting at all.

Simulation results are shown in Fig.4∼Fig.6. Fig.4 shows

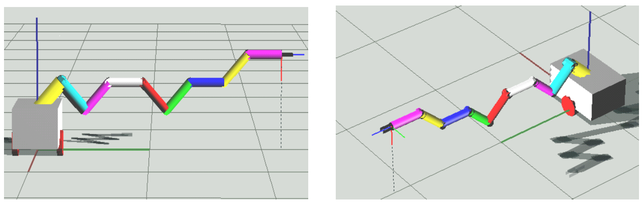


Fig. 3. The initial shape of simulation model

that the mobile robot will get a driving force caused by the movement of hyper-redundant manipulator, which will cause the mobile robot to move along x-axis. Fig.5 and Fig.6 show that the constraint force of two wheels of the mobile robot against the ground. Because the hyper-redundant manipulator is always moving at the left side of the mobile robot like Fig.3, and then the centrifugal force of each link exists caused by turning around the z-axis. Therefore the constraint force of left wheel is bigger than right wheel. Moveover if the constraint force of right wheel is negative, the right wheel maybe not contact with the ground and maybe the mobile robot will overturn toward the left side in some conditions. So this is the problem that we will consider next step.

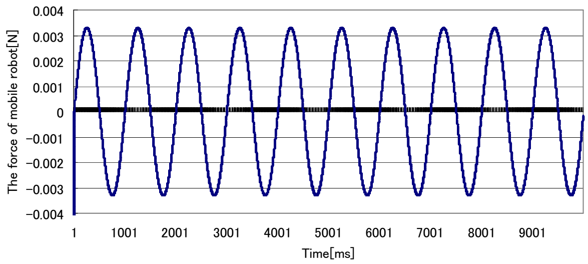


Fig. 4. The force of the mobile robot in x-axis direction

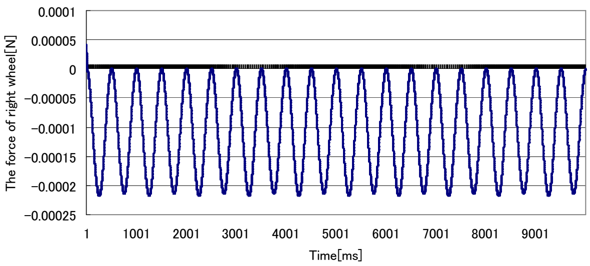


Fig. 5. The constraint force of right wheel against ground

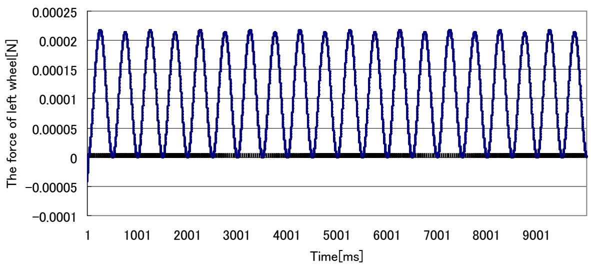


Fig. 6. The constraint force of left wheel against ground

V. CONCLUSION

We first propose a dynamical model of hyper-redundant mobile manipulator whose plural intermediate links are contacting with environment, second the motion of mobile robot without gravity under inputing a torque to manipulator is proved to be correct. Next we will consider the method how to input the suitable torque and force to control the mobile robot effectively and accurately without overturning with gravity when the hyper-redundant manipulator is given a task which should be completed with less-energy and high accuracy. Finally we think the combination of hyper-redundant manipulator with contacting and mobile robot is a new robot model with wide application in the future.

REFERENCES

[1] G.S.Chirikjian, J.W.Burdick:‘‘A Hyper-Redundant Manipulator”, IEEE Robotics & Automation Magazine, December 1994, pp.22-29 (1994)

[2] F.Fahimi, H.Ashrafiuon and C.Nataraj:‘‘Obstacle Avoidance for Spatial Hyper-Redundant Manipulators”, The First Asian Conference on Multibody Dynamics 2002,July 31-August 2, 2002, Iwaki, Fukushima, Japan.

[3] Shugen Ma, Watanabe.M, Kondo.H:‘‘Dynamic control of curve-constrained hyper-redundant manipulators”, Computational Intelligence in Robotics and Automation, 2001. Proceedings 2001 IEEE International Symposium, 29 July-1 Aug. 2001 pp,83-88.

[4] Oda.N, Murakami.T, Ohnishi.K,:‘‘A force based motion control strategy for hyper-redundant manipulator”, Industrial Electronics, Control and Instrumentation, IECON 97. 23rd International Conference, Volume 3, 9-14 Nov. 1997 pp,1385-1390 vol.3.

[5] Hirose.S, Chu.R:‘‘Development of a light weight torque limiting M-Drive actuator for hyper-redundant manipulator Float Arm ”, Robotics and Automation, 1999. Proceedings. 1999 IEEE International Conference , Volume 4, 10-15 May 1999 pp,2831-2836 vol.4 .

[6] Glass K., Colbaugh R., Lim D. and Seraji H, “Real-time collision avoidance for redundant manipulators”, *IEEE Transactions on Robotics and Automation*, Vol.11, pp.448-457, 1995.

[7] Homayoun Seraji and Bruce Bon, “Real-Time Collision Avoidance for Position-Controlled Manipulators”, *IEEE Transactions on Robotics and Automation*, Vol.15, No.4, pp.670-677, 1999.

[8] Leon Zlajpah and Bojan Nemec, “Kinematic Control Algorithms for On-line Obstacle Avoidance for Redundant Manipulator”, *International Conference on Intelligent Robots and Systems*, Lausanne, Vol.2, pp.1898-1903, 2002.

[9] Kwang-Kyu Lee and Martin Buss, “Obstacle Avoidance for Redundant Robots Using Jacobian Transpose Method”, *International Conference on Intelligent Robots and Systems*, San Diego, pp.3509-3514, 2007.

[10] Ryo Kurazume and Tsutomu Hasegawa, “Impedance Matching for Free Flying Robots”, (in Japanese) *Proc. 20th Annu. Conf. Robot. Soc. Jpn.*, p. 3J16, 2002.

[11] Ryo Kurazume and Tsutomu Hasegawa, “A New Index of Serial-Link Manipulator Performance Combining Dynamic Manipulability and Manipulating Force Ellipsoid”, *IEEE Transactions on Robotics*, Vol.22, No.5, pp.1022-1028, 2006.

[12] West,H.,and Asada,H.,‘‘A Method for the Design of Hybrid Position/Force Controllers for Manipulators Constrained by Contact with the Environment”, Proc. of 1985 IEEE International Conference on Robotics and Automation, pp.251-260

[13] Jaeheung Park and Oussama Khatib ,‘‘Multi-Link Multi-Contact Force Control for Manipulators”, Proc. of 2005 IEEE International Conference on Robotics and Automation, April 2005, pp.3624-3629

[14] Anna Petrovskaya, Jaeheung Park and Oussama Khatib ‘‘Probabilistic Estimation of Whole Body Contacts for Multi-Contact Robot Control”, Proc. of 2007 IEEE International Conference on Robotics and Automation, April 2007, pp.568-573

[15] Takeshi Ikeda and Mamoru Minami:‘‘Research of Grinding Robot without Force Sensor by Using Algebraic Equation”(in Japanese), Transactions of the Japan Society of Mechanical Engineers(C), Vol.71, No.702, pp.270-277, 2005.

[16] M.W.Walker and D.E.orin:‘‘Efficient Dynamic Computer Simulation of Robotic Mechanisms”, ASME J.of DSMC,104, pp.205-211, 1982

[17] L.R.Hunt, R.Su and G.Meyer:‘‘Global Transformation of Non Linear system,” IEEE Trans. AC., 28-1, pp.24-31, 1983