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Reconfiguration Manipulability Analyses for Redundant Robots in View of Strucuture and Shape

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***Abstract*—This paper is concerned with a concept of reconfiguration manipulability inspired from manipulability. The reconfiguration manipulability represents a shape-changeability of each intermediate link when a prior end-effector task is given. Taking reconfiguration manipulability into consideration, we analyze the redundant robots in view of its structure and shape by simulations.**

I. INTRODUCTION

There are many researches about configuration control of redundant manipulators discussing how to use the redundancy. Within the global methods, Ahuactzin and Gupta have proposed a global method (Kinematic Roadmap) [2] to find a series of reachable configurations (a feasible path) from a given initial configuration to goal position based on a concept of "reachability". Within the local methods, which controls robot's configuration with limited information about environments and so on, various approaches to obstacle avoidance for redundant manipulators have been presented [3] including real-time control methods to avoid singular configurations [4]. Above researches indicate that the focuses on research

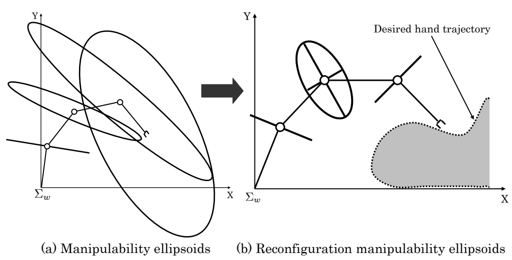


Fig. 1. Manipulability ellipsoids, reconfiguration manipulability ellipsoids

topics concerning redundant manipulators have been shifted from kinematical consideration into combined arguments of kinematics with dynamics. What we want to emphasize is that they were based on an implicit assumption that multiple reconfiguration motions could be realized. Please note that "reconfiguration" in this paper is used for shape-changing motion of the manipulator while the end-effector tracks a predetermined desired pose with designated dimension. Yet, whatever the choice for the secondary task, it may not necessarily lead to complete the desired internal motion, depending on the manipulator's configuration, even though the feasibility of the second or third reconfiguration subtasks is fully evaluated before execution. Though dozens of paper have been published on the subject, none of them has yet to analyze the feasibility of the reconfiguration subtasks.

On the other hand, the mobility of the end-effector can be evaluated by manipulability, e.g.[1] and it represents a kind of distance from singular configuration of manipulator. Contrarily to above end-effector's free motion, there has been no concept to describe reconfiguration manipulability for the secondary subtasks with prior end-effector task. We had presented a concept of the avoidance manipulability ellipsoid as an index evaluating shape-changeability of the intermediate links , while the end-effector tracks the desired trajectory as shown in Fig.1(b), which is inspired from the manipulability concept [1] as shown in Fig.1(a). The reconfiguration manipulability ellipsoids are depicted at the first and third links as partial reconfiguration manipulability ellipsoids in Fig.1(b), and at the second link as complete reconfiguration manipulability ellipsoid, which defines the reconfiguration space of intermediate links under the predefined end-effector task. What we want to discuss here is how to guarantee and maintain the expansion of the reconfiguration space to secure a dimension of the reconfiguration space as high as possible.

Through analyses of reconfiguration matrix, the reconfiguration ability has been closely examined, and in this paper we propose:

- Reconfiguration manipulability concept to analyze and measure shape-changeability of the intermediate links providing a prior end-effector task is given.
- Through analysis of reconfiguration matrices, whether multiple reconfiguration subtasks can be executed or not, and how many subtasks are realizable can be judged.
- The sufficient conditions have been presented, which can

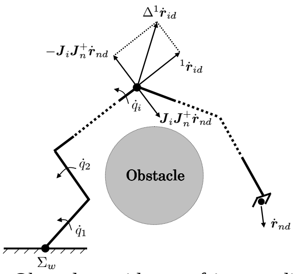


Fig. 2. Obstacle avoidance of intermediate links

mathematically guarantee the sustainability of the reconfiguration space of intermediate links.

II. Reconfiguration Manipulability

Here we assume that the desired end-effector's trajectory $\boldsymbol{r}_{nd}$ and the velocity $\dot{\boldsymbol{r}}_{nd}$ are given as primary task. Abbreviating $\dot{\boldsymbol{r}}_n(\boldsymbol{q}_n)$ to $\dot{\boldsymbol{r}}_n$, the desired $\dot{\boldsymbol{r}}_n$ is denoted by $\dot{\boldsymbol{r}}_{nd}$, then,

$$\dot{\boldsymbol{r}}_{nd} = \boldsymbol{J}_n \dot{\boldsymbol{q}}_n. \tag{1}$$

Solving $\dot{\boldsymbol{q}}_n$ in (1) as

$$\dot{\boldsymbol{q}}_n = \boldsymbol{J}_n^+ \dot{\boldsymbol{r}}_{nd} + (\boldsymbol{I}_n - \boldsymbol{J}_n^+ \boldsymbol{J}_n)\ {}^1\boldsymbol{l}. \tag{2}$$

In (2), $\boldsymbol{J}_n^+$ is the pseudo-inverse of $\boldsymbol{J}_n$, $\boldsymbol{I}_n$ is $n \times n$ unit matrix, and ${}^1\boldsymbol{l}$ is an arbitrary vector satisfying ${}^1\boldsymbol{l} \in R^n$. The left superscript "1" of ${}^1\boldsymbol{l}$ means the first reconfiguration subtask. In the right side of (2), the first term denotes the solution making $\|\dot{\boldsymbol{q}}_n\|$ minimize in the null space of $\dot{\boldsymbol{q}}_n$ while realizing $\dot{\boldsymbol{r}}_{nd}$. The second term denotes the components of angular velocities at each joint, which can change the manipulator's shape regardless with the influence of $\dot{\boldsymbol{r}}_{nd}$ given arbitrarily as end-effector velocity for tracking the desired trajectory. Providing the first reconfiguration subtask, that is the first demanded velocity ${}^1\dot{\boldsymbol{r}}_{id}$, is given to the i-th link by geometric relation of manipulator and obstacles, shall we discuss realizability of ${}^1\dot{\boldsymbol{r}}_{id}$ in the following argument. In this research, ${}^1\dot{\boldsymbol{r}}_{id}$ is assumed to be commanded by an reconfiguration control system of higher level and ${}^1\dot{\boldsymbol{r}}_{id}$ can be used for general reconfiguration subtask. The relation of ${}^1\dot{\boldsymbol{r}}_{id}$ and $\dot{\boldsymbol{r}}_{nd}$ is denoted in (3) by substituting (2) into ${}^1\dot{\boldsymbol{r}}_{id} = \boldsymbol{J}_i \dot{\boldsymbol{q}}_n$.

$${}^1\dot{\boldsymbol{r}}_{id} = \boldsymbol{J}_i \boldsymbol{J}_n^+ \dot{\boldsymbol{r}}_{nd} + \boldsymbol{J}_i (\boldsymbol{I}_n - \boldsymbol{J}_n^+ \boldsymbol{J}_n)\ {}^1\boldsymbol{l}. \tag{3}$$

Here, we define two variables shown as

$$\Delta^1 \dot{\boldsymbol{r}}_{id} \stackrel{\triangle}{=} {}^1\dot{\boldsymbol{r}}_{id} - \boldsymbol{J}_i \boldsymbol{J}_n^+ \dot{\boldsymbol{r}}_{nd} \tag{4}$$

and

$${}^1\boldsymbol{M}_i \stackrel{\triangle}{=} \boldsymbol{J}_i (\boldsymbol{I}_n - \boldsymbol{J}_n^+ \boldsymbol{J}_n). \tag{5}$$

In (4), $\Delta^1 \dot{\boldsymbol{r}}_{id}$ is called by "the first reconfiguration velocity". In (5), ${}^1\boldsymbol{M}_i$ is a $m \times n$ matrix called by "the first reconfiguration matrix". Then, (4) can be rewritten as

$$\Delta^1 \dot{\boldsymbol{r}}_{id} = {}^1\boldsymbol{M}_i {}^1\boldsymbol{l}. \tag{6}$$

The relation between ${}^1\dot{\boldsymbol{r}}_{id}$ and $\Delta^1 \dot{\boldsymbol{r}}_{id}$ is shown in Fig.2.

Recipe:

Providing primarily given end-effector task $\dot{\boldsymbol{r}}_{nd}$ and the first reconfiguration subtask of the i-th link ${}^1\dot{\boldsymbol{r}}_{id}$, $\Delta^1 \dot{\boldsymbol{r}}_{id}$ is determined by (4). Then the realizability of ${}^1\dot{\boldsymbol{r}}_{id}$ depends on $rank({}^1\boldsymbol{M}_i)$, meaning whether $\Delta^1 \dot{\boldsymbol{r}}_{id}$ has a solution ${}^1\boldsymbol{l}$ through ${}^1\boldsymbol{M}_i$ in (6) relies on $rank({}^1\boldsymbol{M}_i)$.

A. Complete Reconfiguration Manipulability Ellipsoid

When ${}^1\dot{\boldsymbol{r}}_{id}$ is given as the desired reconfiguration velocity of the i-th link, according to (4), we can obtain $\Delta^1 \dot{\boldsymbol{r}}_{id}$. However, the problem is whether we can realize $\Delta^1 \dot{\boldsymbol{r}}_{id}$, that is, whether we can find ${}^1\boldsymbol{l}$ to realize $\Delta^1 \dot{\boldsymbol{r}}_{id}$. From (6), we can obtain ${}^1\boldsymbol{l}$ as

$${}^1\boldsymbol{l} = {}^1\boldsymbol{M}_i^+ \Delta^1 \dot{\boldsymbol{r}}_{id} + (\boldsymbol{I}_n - {}^1\boldsymbol{M}_i^{+1} \boldsymbol{M}_i)^2 \boldsymbol{l}. \tag{7}$$

In (7), ${}^2\boldsymbol{l}$ is an arbitrary vector satisfying ${}^2\boldsymbol{l} \in R^n$. From (7), we can obtain

$$\|{}^1\boldsymbol{l}\|^2 \geq \Delta^1 \dot{\boldsymbol{r}}_{id}^T ({}^1\boldsymbol{M}_i^+)^{T1} \boldsymbol{M}_i^+ \Delta^1 \dot{\boldsymbol{r}}_{id}, \tag{8}$$

Assuming that ${}^1\boldsymbol{l}$ is restricted as $\|{}^1\boldsymbol{l}\| \leq 1$, then we obtain next relation,

$$\Delta^1 \dot{\boldsymbol{r}}_{id}^T ({}^1\boldsymbol{M}_i^+)^T\ {}^1\boldsymbol{M}_i^+ \Delta^1 \dot{\boldsymbol{r}}_{id} \leq 1, \quad \Delta^1 \dot{\boldsymbol{r}}_{id} \in R({}^1\boldsymbol{M}_i). \tag{9}$$

If $rank({}^1\boldsymbol{M}_i) = m$, (9) represents an ellipsoid expanding in m-dimensional space, holding

$$\Delta^1 \dot{\boldsymbol{r}}_{id} = {}^1\boldsymbol{M}_i {}^1\boldsymbol{M}_i^+ \Delta^1 \dot{\boldsymbol{r}}_{id}, \quad \Delta^1 \dot{\boldsymbol{r}}_{id} \in R^m, \tag{10}$$

which indicates that $\Delta^1 \dot{\boldsymbol{r}}_{id}$ can be arbitrarily realized in m-dimensional space and (6) always has the solution ${}^1\boldsymbol{l}$ corresponding to all $\Delta^1 \dot{\boldsymbol{r}}_{id} \in R^m$. In this way, the ellipsoid represented by (9) when $rank({}^1\boldsymbol{M}_i) = m$ is named "the first complete reconfiguration manipulability ellipsoid", which is denoted by ${}^{1C}P_i$.

B. Partial Reconfiguration Manipulability Ellipsoid

If $rank({}^1\boldsymbol{M}_i) = p < m$, $\Delta^1 \dot{\boldsymbol{r}}_{id}$ does not value arbitrarily in R^m. In this case, reduced $\Delta^1 \dot{\boldsymbol{r}}_{id}$ is denoted as $\Delta^1 \dot{\boldsymbol{r}}_{id}^*$. Then (9) is written as

$$\begin{gathered} \Delta^1 \dot{\boldsymbol{r}}_{id}^{*\,T} ({}^1\boldsymbol{M}_i^+)^{T1} \boldsymbol{M}_i^+ \Delta^1 \dot{\boldsymbol{r}}_{id}^* \leq 1, \\ (\Delta^1 \dot{\boldsymbol{r}}_{id}^* = {}^1\boldsymbol{M}_i {}^1\boldsymbol{M}_i^+ \boldsymbol{s},\ \boldsymbol{s} \in R^m). \end{gathered} \tag{11}$$

(11) describes an ellipsoid expanded in p-dimensional space. This ellipsoid is named "the first partial reconfiguration manipulability ellipsoid", which is denoted by ${}^{1P}P_i$. Because $p < m$, the partial reconfiguration manipulability ellipsoid can be thought as regressed ellipsoid of the complete reconfiguration manipulability ellipsoid.

C. Reconfiguration Manipulability Measure

Representing the volume of the "reconfiguration manipulability ellipsoid" of the i-th link as S_{RMi}, "reconfiguration manipulability measure [8]" S_{RM} is defined as,

$$S_{RM} = \sum_{i=1}^{n-1} S_{RMi}. \tag{12}$$

III. Plural Reconfiguration Subtasks

This section discusses the multi-reconfiguration subtasks realization. If the first reconfiguration subtask, that is, the first reconfiguration velocity, $\Delta^1\dot{\boldsymbol{r}}_{id}$ or $\Delta^1\dot{\boldsymbol{r}}^*_{id}$ has been realized at a certain i-th link, we will consider the possibility to execute the second reconfiguration velocity except the i-th link. Substituting (7) into (2), we can obtain

$$\begin{aligned}\dot{\boldsymbol{q}}_n &= \boldsymbol{J}_n^+\dot{\boldsymbol{r}}_{nd} + (\boldsymbol{I}_n - \boldsymbol{J}_n^+\boldsymbol{J}_n)^1\boldsymbol{M}_i^+\Delta^1\dot{\boldsymbol{r}}_{id} \\ &\quad +(\boldsymbol{I}_n - \boldsymbol{J}_n^+\boldsymbol{J}_n)(\boldsymbol{I}_n - {}^1\boldsymbol{M}_i^{+1}\boldsymbol{M}_i)^2\boldsymbol{l}. \quad (13)\end{aligned}$$

Substituting (13) into ${}^2\dot{\boldsymbol{r}}_{jd} = \boldsymbol{J}_j\dot{\boldsymbol{q}}_n$, we can obtain

$$\begin{aligned}{}^2\dot{\boldsymbol{r}}_{jd} &= \boldsymbol{J}_j\boldsymbol{J}_n^+\dot{\boldsymbol{r}}_{nd} + \boldsymbol{J}_j(\boldsymbol{I}_n - \boldsymbol{J}_n^+\boldsymbol{J}_n)^1\boldsymbol{M}_i^+\Delta^1\dot{\boldsymbol{r}}_{id} \\ &\quad +\boldsymbol{J}_j(\boldsymbol{I}_n - \boldsymbol{J}_n^+\boldsymbol{J}_n)(\boldsymbol{I}_n - {}^1\boldsymbol{M}_i^{+1}\boldsymbol{M}_i)^2\boldsymbol{l}. \quad (14)\end{aligned}$$

By defining $\Delta^2\dot{\boldsymbol{r}}_{jd}$ and ${}^2\boldsymbol{M}_j$ as

$$\begin{aligned}\Delta^2\dot{\boldsymbol{r}}_{jd} &\triangleq {}^2\dot{\boldsymbol{r}}_{jd} - \boldsymbol{J}_j\boldsymbol{J}_n^+\dot{\boldsymbol{r}}_{nd} \\ &\quad -\boldsymbol{J}_j(\boldsymbol{I}_n - \boldsymbol{J}_n^+\boldsymbol{J}_n)^1\boldsymbol{M}_i^+\Delta^1\dot{\boldsymbol{r}}_{id} \quad (15)\end{aligned}$$

and

$${}^2\boldsymbol{M}_j \triangleq \boldsymbol{J}_j(\boldsymbol{I}_n - \boldsymbol{J}_n^+\boldsymbol{J}_n)(\boldsymbol{I}_n - {}^1\boldsymbol{M}_i^{+1}\boldsymbol{M}_i), \quad (16)$$

we can obtain

$$\Delta^2\dot{\boldsymbol{r}}_{jd} = {}^2\boldsymbol{M}_j{}^2\boldsymbol{l}. \quad (17)$$

The forms of (17) and (6) are similar. Therefore, the analysis method of the second reconfiguration manipulability ellipsoid ${}^2P_j$ $(j = 1,\cdots,n-1; \{j \neq i\})$ and the first reconfiguration manipulability ellipsoid ${}^1P_i$ are also similar. In other words, whether the second reconfiguration subtask can be realized or not depends on the rank value of second matrix ${}^2\boldsymbol{M}_j$ $(j = 1,\cdots,n-1; \{j \neq i\})$. If $rank({}^2\boldsymbol{M}_j) \neq 0$, the second subtask can be realized partially at least. If $rank({}^2\boldsymbol{M}_j) = 0$, the second reconfiguration subtask cannot be realized. Similarly, we can judge whether the third subtask can be realized or not by the third reconfiguration matrix ${}^3\boldsymbol{M}_k$ as

$$\begin{aligned}{}^3\boldsymbol{M}_k &\triangleq \boldsymbol{J}_k(\boldsymbol{I}_n - \boldsymbol{J}_n^+\boldsymbol{J}_n)(\boldsymbol{I}_n - {}^1\boldsymbol{M}_i^{+1}\boldsymbol{M}_i)(\boldsymbol{I}_n - {}^2\boldsymbol{M}_j^{+2}\boldsymbol{M}_j), \\ &(k = 1,\cdots,n-1;\ \{k \neq i\}\cap\{k \neq j\}\cap\{i \neq j\}). \quad (18)\end{aligned}$$

According to above analyses for ${}^1\boldsymbol{M}_i$, ${}^2\boldsymbol{M}_j$ and ${}^3\boldsymbol{M}_k$, the realizability of the fourth or more subtasks can be judged in a same manner.

Here, we show judgment sequence by a flow chart shown in Fig.3 when β reconfiguration subtasks are demanded. i denotes the number of link, $\alpha(\alpha = 1,2,\cdots,\beta)$ denotes the priority order of reconfiguration subtasks, ${}^\alpha\dot{\boldsymbol{r}}_{id}$ means the arbitrarily demanded reconfiguration velocity for the i-th link as the α-th reconfiguration subtask. According to Fig.3, whether the arbitrary ${}^\alpha\dot{\boldsymbol{r}}_{id}$ and the end-effector velocity $\dot{\boldsymbol{r}}_{nd}$ are both realized or not can be judged through $\Delta^\alpha\dot{\boldsymbol{r}}_{id}$ recurrently.

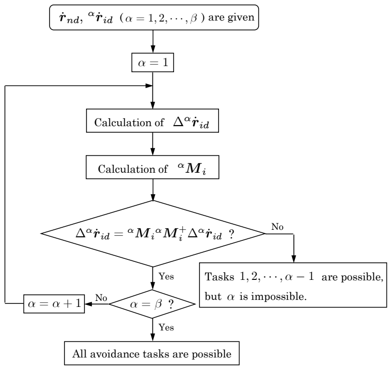


Fig. 3. Flow chart of judgment of reconfiguration possibility

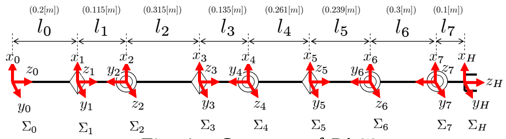


Fig. 4. Structure of PA11

IV. Analysis of $rank({}^1\boldsymbol{M}_i)$

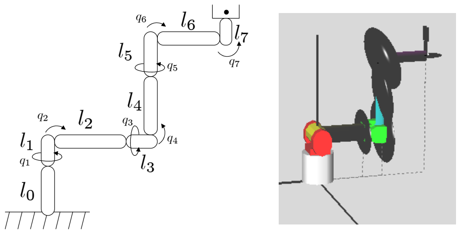
Maintaining $rank({}^1\boldsymbol{M}_i)$ of intermediate links to be as high as possible is the essential requirement for configuration control to optimize manipulator's shape in view of high reconfiguration manipulability. And it is the first step to design an on-line control system of a redundant manipulator with high shape-changeability based on reconfiguration manipulability. We want to stress here previous researches have not paid attention to how to guarantee $rank({}^1\boldsymbol{M}_i)$ to assure the required avoiding task to be realizable. In fact, a similar concept of ${}^1\boldsymbol{M}_i$ had initially been defined and used for controlling the redundant manipulator's configuration based on prioritized multiple tasks [5]. However, the proposed controller in reference [5] do not concern the possibility that the range space of ${}^1\boldsymbol{M}_i$ could be reduced by singular configuration and it cannot decouple the interacting motions of multiple tasks even though the redundant degree be much higher than the required motion degree of the multiple tasks. Even in our previous researches about avoidance manipulability optimization [6] and on-line control system [7], [8] of a redundant manipulator, we did not guarantee the sustainability of the range space of ${}^1\boldsymbol{M}_i$. In this section, we will propose two assumptions named as "General-Non-Singular Configuration Assumption" and "Desirable-Non-Singular Configuration Assumption", they can provide a configuration control criterion as primary control objective to keep the shape-changeability by avoiding singular configuration. The all proofs are shown in reference [9].

1) The General-Non-Singular Configuration Assumption:

Theorem a:

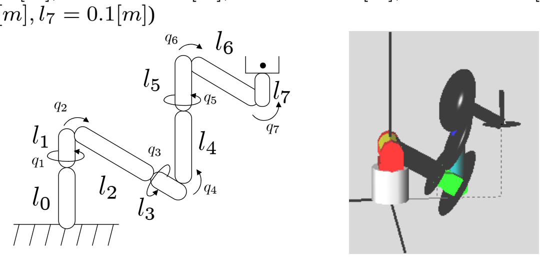
Giving the General-Non-Singular Configuration Assumption for any manipulator as

$$\begin{cases} (a).\ rank(\boldsymbol{J}_n{}^{n-m+1\to n}) = m \\ (b_i).\ rank(\boldsymbol{J}_i) = p_i, (i = 1,2,\cdots,n-1) \end{cases} \quad (19)$$



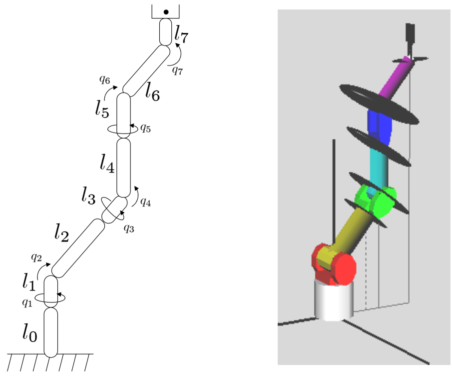
(a) Shape 1 (b) Reconfiguration manipulability ellipsoids

Fig. 5. Shape 1 of PA11 and reconfiguration manipulability ellipsoids ($q_1 = 0[deg], q_2 = -90[deg], q_3 = 0[deg], q_4 = 90[deg], q_5 = 0[deg], q_6 = -90[deg], q_7 = 90[deg]$; $l_0 = 0.2[m], l_1 = 0.115[m], l_2 = 0.315[m], l_3 = 0.135[m], l_4 = 0.261[m], l_5 = 0.239[m], l_6 = 0.3[m], l_7 = 0.1[m]$)



(a) Shape 2 (b) Reconfiguration manipulability ellipsoids

Fig. 6. Shape 2 of PA11 and reconfiguration manipulability ellipsoids ($q_1 = 0[deg], q_2 = -120[deg], q_3 = 0[deg], q_4 = 120[deg], q_5 = 0[deg], q_6 = -120[deg], q_7 = 120[deg]$)



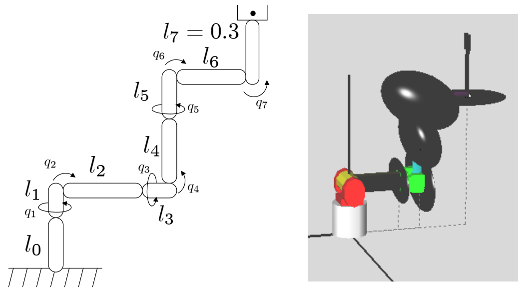
(a) Shape 3 (b) Reconfiguration manipulability ellipsoids

Fig. 7. Shape 3 of PA11 and reconfiguration manipulability ellipsoids ($q_1 = 0[deg], q_2 = -40[deg], q_3 = 0[deg], q_4 = 40[deg], q_5 = 0[deg], q_6 = -40[deg], q_7 = 40[deg]$)

with $p_i \in \{0, 1, \cdots, m\}$, *we have*

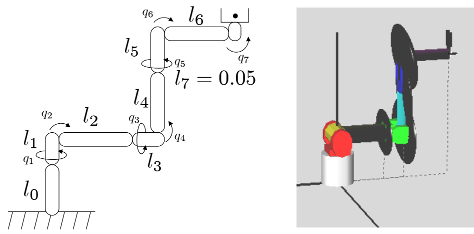
$$p_i + min\{i, n-m\} - i \leq rank({}^1\boldsymbol{M}_i) \leq min\{p_i, i, n-m\}. \quad (20)$$

The assumption (a) in (19) represents that the configuration from the $(n-m)$-th link to the n-th link is non-singular. The next assumption (b_i) is affected by many factors such as the structure of manipulator, variables choice of end-effector task and manipulator's configuration and so on, so $rank(\boldsymbol{J}_i)$ is given by an unspecified value p_i. For verifying the practicality of concept of reconfiguration manipulability, here we use our original robot named "PA11" to evaluate reconfiguration manipulability ellipsoid. "PA11" is a 7-link redundant manipulator ($n = 7$) and its end-effector can



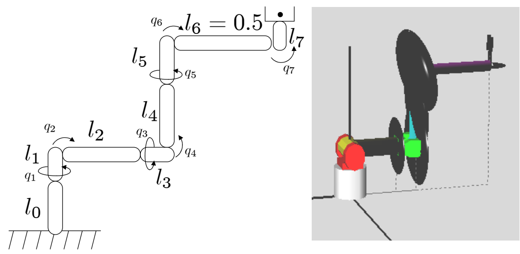
(a) Shape 4 (b) Reconfiguration manipulability ellipsoids

Fig. 8. Shape 4 and reconfiguration manipulability ellipsoids ($q_1 = 0[deg], q_2 = -90[deg], q_3 = 0[deg], q_4 = 90[deg], q_5 = 0[deg], q_6 = -90[deg], q_7 = 90[deg]$; The 7-th link of PA11 is increased as $l_7 = 0.3[m]$)



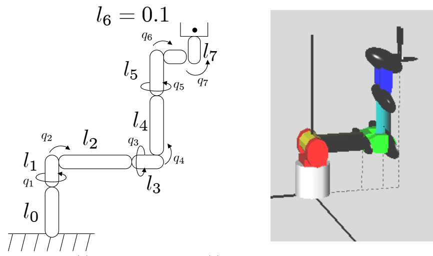
(a) Shape 5 (b) Reconfiguration manipulability ellipsoids

Fig. 9. Shape 5 and reconfiguration manipulability ellipsoids (The 7-th link of PA11 is decreased as $l_7 = 0.05[m]$)



(a) Shape 6 (b) Reconfiguration manipulability ellipsoids

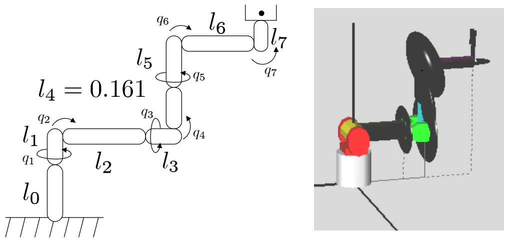
Fig. 10. Shape 6 and reconfiguration manipulability ellipsoids (The 6-th link of PA11 is increased as $l_6 = 0.5[m]$)



(a) Shape 7 (b) Reconfiguration manipulability ellipsoids

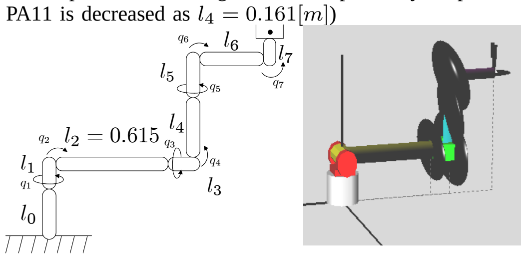
Fig. 11. Shape 7 and reconfiguration manipulability ellipsoids (The 6-th link of PA11 is decreased as $l_6 = 0.1[m]$)

execute the task in 3-dimensional position space ($m = 3$). The structure of "PA11" is shown in Fig.4. Where, all joints are rotational and their rotational directions are given by z-axis of each link coordinate. Considering the structure of "PA11", and assuming that the end-effector of "PA11" executes the task in 3-dimensional position space, that is $\boldsymbol{p}_i = [x, y, z]^T$. When "PA11" is set by $q_1 = 0[deg], q_2 = -90[deg], q_3 = 0[deg], q_4 = 90[deg], q_5 = 0[deg], q_6 = -90[deg], q_7 = 90[deg]$ shown in Fig.5(a), we can simply find that the



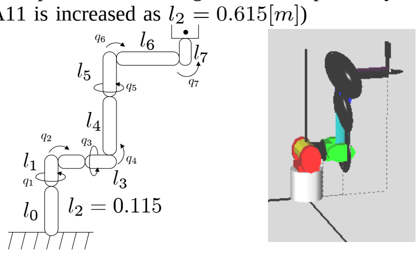
(a) Shape 9 (b) Reconfiguration manipulability ellipsoids

Fig. 12. Shape 9 and reconfiguration manipulability ellipsoids (The 4-th link of PA11 is decreased as $l_4 = 0.161[m]$)



(a) Shape 10 (b) Reconfiguration manipulability ellipsoids

Fig. 13. Shape 10 and reconfiguration manipulability ellipsoids (The 2-nd link of PA11 is increased as $l_2 = 0.615[m]$)



(a) Shape 11 (b) Reconfiguration manipulability ellipsoids

Fig. 14. Shape 11 and reconfiguration manipulability ellipsoids (The 2-nd link of PA11 is decreased as $l_2 = 0.115[m]$)

conditions in (19) given as

$$\begin{cases} rank(\boldsymbol{J}_7{}^{5\to7}) = 3 \\ rank(\boldsymbol{J}_1) = 0 \\ rank(\boldsymbol{J}_2) = rank(\boldsymbol{J}_3) = 2 \\ rank(\boldsymbol{J}_4) = rank(\boldsymbol{J}_5) = rank(\boldsymbol{J}_6) = 3 \end{cases} \tag{21}$$

with

$$\boldsymbol{J}_7{}^{5\to7} = \begin{bmatrix} 0 & 0.1 & 0.1 \\ -0.3 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}. \tag{22}$$

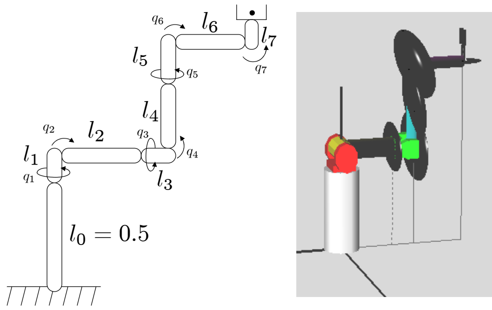
Substituting (21) into (20), we can obtain

$$\begin{cases} rank({}^1\boldsymbol{M}_1) = 0 \\ rank({}^1\boldsymbol{M}_2) = rank({}^1\boldsymbol{M}_3) = 2 \\ rank({}^1\boldsymbol{M}_4) = 3 \\ 2 \le rank({}^1\boldsymbol{M}_5) \le 3, \ 1 \le rank({}^1\boldsymbol{M}_6) \le 3 \end{cases}. \tag{23}$$

On the other hand, from (5) we can calculate

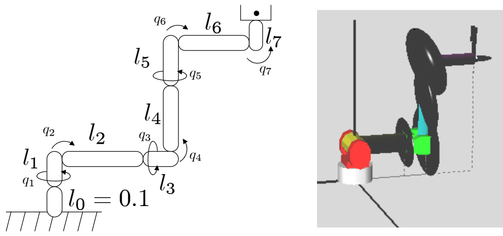
$$\begin{cases} rank({}^1\boldsymbol{M}_1) = 0 \\ rank({}^1\boldsymbol{M}_2) = rank({}^1\boldsymbol{M}_3) = 2 \\ rank({}^1\boldsymbol{M}_4) = 3 \\ rank({}^1\boldsymbol{M}_5) = 3, \ rank({}^1\boldsymbol{M}_6) = 2 \end{cases}. \tag{24}$$

In (24), $rank({}^1\boldsymbol{M}_1)$, $rank({}^1\boldsymbol{M}_2)$, $rank({}^1\boldsymbol{M}_3)$ and $rank({}^1\boldsymbol{M}_4)$ are completely coincide with (23), $rank({}^1\boldsymbol{M}_5)$ and $rank({}^1\boldsymbol{M}_6)$ are in the extent of (23).



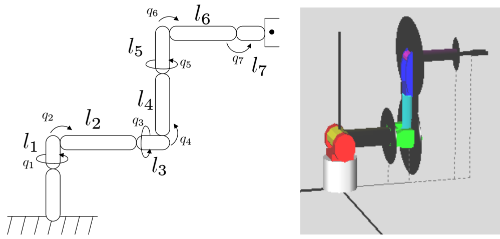
(a) Shape 12 (b) Reconfiguration manipulability ellipsoids

Fig. 15. Shape 12 and reconfiguration manipulability ellipsoids (The base of PA11 is increased as $l_0 = 0.5[m]$)



(a) Shape 13 (b) Reconfiguration manipulability ellipsoids

Fig. 16. Shape 13 and reconfiguration manipulability ellipsoids (The base of PA11 is decreased as $l_0 = 0.1[m]$)



(a) Shape 14 (b) Reconfiguration manipulability ellipsoids

Fig. 17. Shape 14 of PA11 and reconfiguration manipulability ellipsoids ($q_1 = 0[deg], q_2 = -90[deg], q_3 = 0[deg], q_4 = 90[deg], q_5 = 0[deg], q_6 = -90[deg], q_7 = 0[deg]$)

The reconfiguration manipulability ellipsoids given by (9) or (11) are shown in Fig.5(b). Where, the 1-st link does not possess the reconfiguration manipulability since $rank({}^1\boldsymbol{M}_1) = 0$ by our calculations in (24). The 2-nd and 3-rd links possess the reconfiguration manipulability in 2-dimensional position space since $rank({}^1\boldsymbol{M}_3) = rank({}^1\boldsymbol{M}_4) = 2$ in (24), the ellipsoids are vertical with the principal axes of the 2-nd link and 3-rd link respectively, here please note that the ellipsoid of the 3-rd link is somewhat larger than the ellipsoid of the 2-nd link because of the length of the 3-rd link, that is l_3. The 4-th and 5-th links possess the reconfiguration manipulability in 3-dimensional position space since $rank({}^1\boldsymbol{M}_4) = 3$ and $rank({}^1\boldsymbol{M}_5) = 3$ in (24), the 6-th link possesses the reconfiguration manipulability in 2-dimensional position space since $rank({}^1\boldsymbol{M}_6) = 2$ in (24), which is vertical with the 7-th link. These results prove the consistency between "**Theorem a**" and practice. The follows are similar.

When we change the shape of "PA11". For example, the shape is changed into $q_1 = 0[deg], q_2 = -120[deg], q_3 = 0[deg], q_4 = 120[deg], q_5 = 0[deg], q_6 = -120[deg], q_7 =$

$120[deg]$ and $q_1 = 0[deg], q_2 = -40[deg], q_3 = 0[deg], q_4 = 40[deg], q_5 = 0[deg], q_6 = -40[deg], q_7 = 40[deg]$, we can find the area or volume of all ellipsoids decrease, which are shown in Fig.6 and Fig.7.

When we change the structure of "PA11" in length of links under the fixed shape of $q_1 = 0[deg], q_2 = -90[deg], q_3 = 0[deg], q_4 = 90[deg], q_5 = 0[deg], q_6 = -90[deg], q_7 = 90[deg]$. For example, l_7 is increased into $0.3[m]$ from $0.1[m]$, or l_6 is increased into $0.5[m]$ from $0.3[m]$, or l_4 is increased into $0.561[m]$ from $0.261[m]$, or l_2 is increased into $0.615[m]$ from $0.315[m]$, the area and volume of the all ellipsoids will become large, which are shown in Figs.8, 10, **??** and 13 respectively. On the contrary, when l_7 is decreased into $0.05[m]$ from $0.1[m]$, or l_6 is decreased into $0.1[m]$ from $0.3[m]$, or l_4 is decreased into $0.161[m]$ from $0.261[m]$, or l_2 is decreased into $0.115[m]$ from $0.315[m]$, the area and volume of the all ellipsoids will become small, which are shown in Figs.9, 11, 12 and 14 respectively. In addition, from Figs.15 and 16, we can find the change of l_0 will not affect the ellipsoids. Figs.5 to 16 represent how do the length of links affect the area and volume of ellipsoids. Here, please note that the changes of shape and structure shown from Figs.5 to 16 are under the constraint of (21). By calculations, their ellipsoids are completely coincide with (24).

However, in the case of $q_1 = 0[deg], q_2 = -90[deg], q_3 = 0[deg], q_4 = 90[deg], q_5 = 0[deg], q_6 = -90[deg], q_7 = 0[deg]$ shown in Fig.17, the assumption (a) in (19) is not restrained to manipulator such as $rank(\boldsymbol{J}_7{}^{5\to7}) = 2$ with

$$\boldsymbol{J}_7{}^{5\to7} = \begin{bmatrix} 0 & 0 & 0 \\ -0.4 & 0 & 0 \\ 0 & 0.4 & 0.1 \end{bmatrix}, \quad (25)$$

we can find the 4-th link only possesses the reconfiguration manipulability in 2-dimensional position space, that is $rank({}^1\boldsymbol{M}_4) = 2$. This case shows the necessity of the assumption (a).

2) The Desirable-Non-Singular Configuration Assumption:

Theorem b:

Given the Desirable-Non-Singular Configuration Assumption as

$$rank(\boldsymbol{J}_i{}^{\nu\to\nu+m-1}) = min\{i, m\},$$
$$(all\ i\ satisfying\ 1{\le}i{\le}n;\ \nu = max\{i-m+1, 1\}), \quad (26)$$

Then, if $n \geq 2m$,

$$rank({}^1\boldsymbol{M}_i) = \begin{cases} i\ (1 \leq i < m) \\ m\ (m \leq i \leq n-m) \\ n-i{\sim}m\ (n-m < i \leq n-1) \end{cases}. \quad (27)$$

If $m < n < 2m$,

$$rank({}^1\boldsymbol{M}_i) = \begin{cases} i\ (1 \leq i < n-m) \\ n-m\ (n-m \leq i \leq m) \\ n-i{\sim}n-m\ (m < i \leq n-1) \end{cases}. \quad (28)$$

This means all possible partial configuration constituted by successive m links should be non-singular. If we compare "General-Non-Singular Configuration Assumption" with "Desirable-Non-Singular Configuration Assumption" from robotic viewpoint, on the one hand, the former is more suitable for general redundant robots than the latter in the consideration of restriction degree of assumptions themselves because of (b_i) in (19). That is to say, the former is wider than the latter in the consideration of their availability. However, on the other hand, given multiple reconfiguration subtasks, the configuration complying Desirable-Non-Singular Configuration Assumption can keep higher reconfiguration manipulability for multiple reconfiguration subtasks since General-Non-Singular Configuration Assumption allows singular confuguration in each intermediate link, which reduces reconfiguration ability for further subtasks.

V. CONCLUSIONS

This work was supported by Grant-in-Aid for Scientific Research (C) 19560254. In this paper, we proposed reconfiguration manipulability concept to measure shape-changeability of the intermediate links providing a prior end-effector task is given. Through analyses of multiple reconfiguration matrices, whether multiple reconfiguration subtasks can be executed or not, and how many subtasks are realizable can be judged online. Furthermore the sufficient conditions have been shown that they can mathematically guarantee the sustainability of the reconfiguration space of intermediate links.

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