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Availability of Multi-Preview Control of PA10 with Avoidance Manipulability Analyses

Yang Hou[1], Yusaku Nakamura[1], Maki Yamazaki[1], Tongxiao Zhang[1] and Mamoru Minami[2]

[1]Graduate school of Engineering, University of Fukui, Bunkyo3-9-1, Fukui, Japan
(Tel,Fax : +81-776-27-8527;Email: {houyang,yuusaku,yamazaki,zhangtongxiao}@rc.his.u-fukui.ac.jp)
[2]Faculty of Natural Science and Technology, University of Okayama, Tushima3-1-1, Okayama, Japan
(Tel,Fax: +81-86-251-8233; E-mail: minami@suri.sys.okayama-u.ac.jp)

Abstract: This paper proposes a new approach named Multi-Preview Control that is developed from Single-Preview Control to achieve an on-line control of trajectory tracking and obstacle avoidance for redundant manipulators effectively without predefined working object's shape. In the trajectory tracking process, the manipulator is required to keep a configuration with maximal avoidance manipulability in real-time. Multi-Preview Control uses several future optimal configurations to control current configuration to complete task of trajectory tracking and obstacle avoidance on-line with higher avoidance manipulability and reachability. In addition, we analyze the relationship between distribution of AMSI(Avoidance Manipulability Shape Index) and shape of manipulator. We verify the effectiveness and validity of Multi-Preview Control through simulations and experiments.

Keywords: Redundant manipulators, Single-Preview Control, Multi-Preview Control, AMSI, PA10.

1. INTRODUCTION

Over the past two decades, redundant manipulators were used for various tasks, for example, welding, sealing and grinding. These kinds of tasks require that the manipulator plan its hand onto a desired trajectory (trajectory tracking) and avoid its intermediate links, meaning all comprising links of robot except the top link with the end-effecter, from obstacles existing near the target object and also the target object itself (obstacle avoidance).

There are many researches on the motion of redundant manipulators discussing how to use the redundancy. The proposed solutions to this problem can be broadly categorized into two classes: Global Methods and Local Methods. Global Methods [1],[2] solve the collision avoidance problem by an entire path planning which is only suited for structured and static environment. Moreover, the computational cost of Global Methods is expensive and usually increases exponentially along with the number of manipulator's joints. On the other hand, Local Methods [3],[4] solve the collision avoidance problem in unstructured and dynamic environment. Local Method's system has the ability to be flexible even in surroundings with limited information. The information of the environment used in Local Method is naturally restricted to perform the tasks on-line in limited recognition time.

Our main concern in this research is whether we can connect the concepts of Local Method and Global Method by introducing a concept of Multi-Preview Control strategy. If the future information required for path planning can be available to use for Local Method, then it should be possible that the real-time configuration control in Local Method may approach the configuration behavior of Global Method. This is the research direction what we want to pursue and to make clear. Though this question has not been announced in this report, we can posit clue to approach it.

Our research pursues adaptive system using Local Method. The features of our system are shown in Fig.1 where the camera scene area symbolizes the restricted information of environment. In Fig.1, the camera and the manipulator's hand are supposed to move synchronously to achieve on-line operation depending on the real-time restricted information. When the camera detects a new obstacle appearing suddenly in the scene, the manipulator must change its configuration immediately to avoid it.

We have to measure the shape of working object before starting task to complete path-planning using Global Methods, so we have to predetermine the shape of manipulator that does not collide from the start to goal of the hand's task, and the manipulator traces the shape successively to complete the task. Single-Preview Control can refer to a shape of manipulator optimized by avoidance manipulability to induce the current manipulator's shape [5], and avoid collisions with the obstacles. However, because future information is limited, there are still existing possible situations that manipulator could not avoid collision. Then we considered that the manipulator can possibly avoid collision to complete tasks by increasing the number of future times and we named it as Multi-Preview Control. In our research, we propose Multi-Preview Control by improving Single-Preview Control, and from experiment and simulation results we have proved the effectiveness of Multi-Preview Control. Moreover we analyzed the relationship between AMSI distribution and shape of manipulator.

2. AVOIDANCE MANIPULABILITY SHAPE INDEX WITH POTENTIAL

We proposed Avoidance Manipulability Ellipsoid and Avoidance Manipulability Shape Index (AMSI) in [6], and Avoidance Manipulability Shape Index with Potential (AMSIP) in [5]. Avoidance Manipulability Ellipsoid

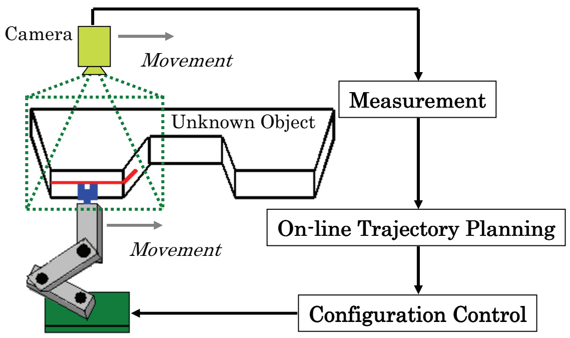

Fig. 1 Processing system for unknown object

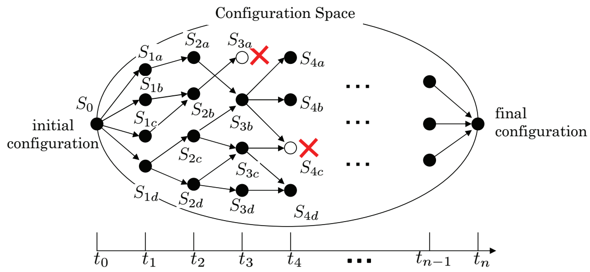

Fig. 2 Configuration space

is applied from Manipulability Ellipsoid proposed by Prof. Yoshikawa in [8]. We will elucidate them briefly in this section.

When the desired hand velocity $\dot{\boldsymbol{r}}_{nd}$ is given, $\dot{\boldsymbol{q}}_n$ is solved as

$$\dot{\boldsymbol{q}}_n = \boldsymbol{J}_n^+ \dot{\boldsymbol{r}}_{nd} + (\boldsymbol{I}_n - \boldsymbol{J}_n^+ \boldsymbol{J}_n)\ {}^1\boldsymbol{l}, \tag{1}$$

where $\boldsymbol{J}_n^+$ is the pseudo-inverse of Jacobean Matrix $\boldsymbol{J}_n$ and $\boldsymbol{I}_n$ is a $n \times n$ unit matrix. In addition, ${}^1\boldsymbol{l}$ is an arbitrary vector. Trajectory tracking of the hand and collision avoidance can executed simultaneously through this vector ${}^1\boldsymbol{l}$. Here, control variable ${}^1\boldsymbol{l}$ is determined so as to make actual manipulator's shape at current time $\boldsymbol{q}(t)$ close to future optimal shape by referring to the future optimal shapes of imaginary manipulators. The relation of the desired velocity of the i-th link ${}^1\dot{\boldsymbol{r}}_{id}$ and the desired hand velocity $\dot{\boldsymbol{r}}_{nd}$ is shown in Eq.(2).

$${}^1\dot{\boldsymbol{r}}_{id} = \boldsymbol{J}_i \boldsymbol{J}_n^+ \dot{\boldsymbol{r}}_{nd} + \boldsymbol{J}_i (\boldsymbol{I}_n - \boldsymbol{J}_n^+ \boldsymbol{J}_n)\ {}^1\boldsymbol{l} \tag{2}$$

Here we define two variables shown in Eq.(3) and Eq.(4).

$$\Delta {}^1\dot{\boldsymbol{r}}_{id} \triangleq {}^1\dot{\boldsymbol{r}}_{id} - \boldsymbol{J}_i \boldsymbol{J}_n^+ \dot{\boldsymbol{r}}_{nd}, \tag{3}$$

$${}^1\boldsymbol{M}_i \triangleq \boldsymbol{J}_i (\boldsymbol{I}_n - \boldsymbol{J}_n^+ \boldsymbol{J}_n). \tag{4}$$

According to Eq.(2),Eq.(3) and Eq.(4), $\Delta {}^1\dot{\boldsymbol{r}}_{id}$ can be rewritten as

$$\Delta {}^1\dot{\boldsymbol{r}}_{id} = {}^1\boldsymbol{M}_i\ {}^1\boldsymbol{l}. \tag{5}$$

In Eq.(5), $\Delta {}^1\dot{\boldsymbol{r}}_{id}$ is called the first avoidance velocity and ${}^1\boldsymbol{M}_i$ is a $m \times n$ matrix called the first avoidance matrix.

Next, we will represent the Avoidance Manipulability Ellipsoid. Providing that ${}^1\boldsymbol{l}$ is restricted as $\|{}^1\boldsymbol{l}\| \leq 1$, then the extent where $\Delta {}^1\dot{\boldsymbol{r}}_{id}$ can move is denoted as

$$\Delta {}^1\dot{\boldsymbol{r}}_{id}^T ({}^1\boldsymbol{M}_i^+)^T\ {}^1\boldsymbol{M}_i^+ \Delta {}^1\dot{\boldsymbol{r}}_{id} \leq 1. \tag{6}$$

If $rank({}^1\boldsymbol{M}_i) = m$, the ellipsoid represented by Eq.(6) is named as the first complete avoidance manipulability ellipsoid. If $rank({}^1\boldsymbol{M}_i) = p < m$, the ellipsoid is named as the first partial avoidance manipulability ellipsoid.

The volume of each Avoidance Manipulability Ellipsoid indicates mobility of each link (shape-changeability). The larger total volume indicates the higher whole avoidance manipulability. We evaluated total volume as Avoidance Manipulability Shape Index (AMSI). Then we proposed Avoidance Manipulability Shape Index with Potential (AMSIP) which considers AMSI and the distance between the manipulator and target object. And we verified the superiority of AMSIP through the simulation in [5].

3. PREVIEW CONTROL

In this chapter, we will use Fig.2 to show the concept of preview control. The ellipsoid in the figure shows the configuration space of manipulator and the various shapes are lined up in chronological order. The black circles $\bullet$ indicate the most suitable shape locally at each time, the white circles $\circ$ indicate the collision shape at the time, while connecting the arrows among them indicate the transition of the shape.

When we use path planning (Global Methods), this system refers to all possible shapes at all time from start to goal and can avoid collision with the working object by selecting reachable path like $S_0 \rightarrow S_{1d} \rightarrow S_{2c} \rightarrow S_{3b} \rightarrow S_{4b} \cdots$. On the other hand, Single-Preview Control refers to a manipulator at one future time (we named it as imaginary manipulator) to control the shape of the current manipulator.

When current time $t = t_0$, the reference future time is expressed by t_1. After evaluating the shape of this time t_1, suppose that S_{1b} is selected as a best configuration, and the manipulator changes its shape to S_{1b}. When current time $t = t_1$, the reference future time is t_2. In this time, reachable shape is only S_{2b}. Then the manipulator transits S_{2b} when time $t = t_2$. When current time $t = t_2$, the reference future time is t_3. In this time, reachable shape is only S_{3a} but S_{3a} destined to collide with the obstacle, then the control system will be obliged to halt the process. In this way, the manipulator cannot accomplish the task using Sing-Preview Control.

On the other hand, Multi-Preview Control controls current manipulator's shape by referring several imaginary manipulator's shape at several future times. We assume three imaginary manipulators are used for referring so as to make it easy to explain the effectiveness clearly. When

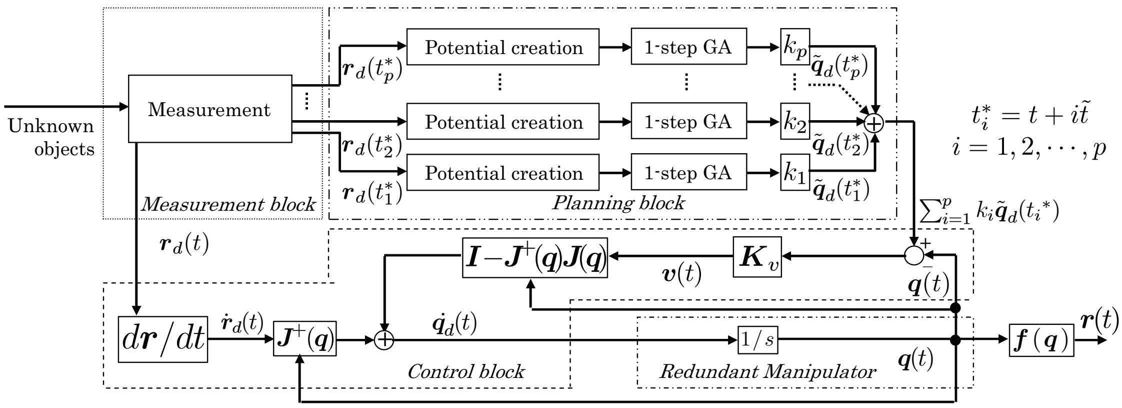

Fig. 3 Multi-Preview control system

current time $t = t_0$, the reference times are t_1,t_2,t_3. There are many paths like $S_0 \rightarrow S_{1b} \rightarrow S_{2b} \rightarrow S_{3a}$, but this path includes the collision shape then this path will not be selected. Assuming this system selects path $S_0 \rightarrow S_{1d} \rightarrow S_{2c} \rightarrow S_{3c}$ through evaluating possible future configuration by, e.g., AMSIP[7], then the manipulator transit S_{1d} when time $t = t_1$. When the current time $t = t_1$, the reference times are t_2,t_3,t_4. In the same way, the path $S_{1d} \rightarrow S_{2c} \rightarrow S_{3c} \rightarrow S_{4c}$ is not selected because of the same reason. For example, if this system selects path $S_{1d} \rightarrow S_{2c} \rightarrow S_{3b} \rightarrow S_{4b}$, then the manipulator transit S_{2c} when time $t = t_2$. In this way, the configuration control system can avoid collision with the working object by using Multi-Preview Control and we think Multi-Preview Control is superior to Single-Preview Control.

3.1 Multi-Preview Control

Multi-Preview Control System is shown in Fig.3 which is a configuration control method to change current manipulator's shape satisfying non-collision requirement by referring to the future configurations based on an on-line measurement.

It consists of an on-line measurement block, a path planning block, a redundancy control block and redundant manipulator. On the assumption that current time is represented by t, and the future times are defined as $t_i^* = t + i\tilde{t}$, $(i \in [1, p])$ where $\tilde{t}$ denotes preview time and i is the number of future times. A measurement block detects a desirable hand position $\boldsymbol{r}_d(t_i^*)$ on the surface of the target object at time t_i^*, which is reasonably assumed to be possible to detect the future information only in the detected camera image in Fig.2. Firstly, potential space based on the detected shape of the target object is created around it at the path planning block. Then the path planning block outputs the optimal shape $\tilde{\boldsymbol{q}}_d(t_i^*)$ corresponding to the maximum ${}^1S$ presented in [5] at the future time t_i^* (imaginary manipulator) by 1-Step GA. The control block outputs desired joint angular velocity $\dot{\boldsymbol{q}}_d(t)$ that makes actual manipulator's shape at current time $\boldsymbol{q}(t)$ close to the optimal shape in the future by referring to $\sum_{i=1}^{p} \tilde{\boldsymbol{q}}_d(t_i^*)$.

An equation which realizes this control system is named as Preview Control Equation and expressed as follows

$$\begin{aligned} \dot{\boldsymbol{q}}_d &= \boldsymbol{J}_n^+ \dot{\boldsymbol{r}}_{nd} \\ &+ (\boldsymbol{I}_n - \boldsymbol{J}_n^+ \boldsymbol{J}_n) \boldsymbol{K}_v \Big(\sum_{i=1}^{p} \tilde{\boldsymbol{q}}_d(t_i^*) - \boldsymbol{q}(t)\Big). \end{aligned} \tag{7}$$

where $n \times 1$ matrix $\sum_{i=1}^{p} \tilde{\boldsymbol{q}}_d(t_i^*) - \boldsymbol{q}(t)$ is defined as

$$\sum_{i=1}^{p} \tilde{\boldsymbol{q}}_d(t_i^*) - \boldsymbol{q}(t) = \begin{bmatrix} \sum_{i=1}^{p} \tilde{q}_{1d}(t_i^*) - q_1(t) \\ \vdots \\ \sum_{i=1}^{p} \tilde{q}_{jd}(t_i^*) - q_j(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{8}$$

when redundant degrees j remains and the redundancy is used for the joints from 1 to j.

The transition of AMSIP and the manipulator's shape using Multi-Preview Control System is shown in Fig.4. According to Fig.4, we can find that the manipulator can always keep higher AMSIP value by using Multi-Preview Control. And the AMSIP value obtained by this system moves from one higher peak to another higher peak as time in multi peak AMSIP distributions. This verifies the validity of Multi-Preview Control in 2-dimension by 4-link planar manipulator.

4. SIMULATION

In order to compare the Single-Preview Control with Multi-Preview Control, we use a 7-link manipulator for simulations, which is produced by Mitsubishi Heavy Industries named PA10 and the structure of PA10 is shown in Fig.5. Hand tracking trajectory and given manipulator's shape are depicted in Fig.6, target hand trajectory is predefined. In addition, the kinematics and the dynamics of PA10 are implemented in the simulator. The solid line in Fig.6 expresses a target trajectory set to be followed.

As for the hand's desired position trajectory $\boldsymbol{r}_d$, when $0[s] \leq t < 12[s]$

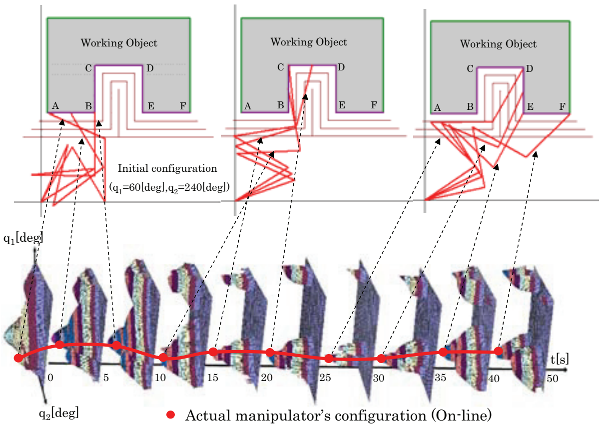

Fig. 4 Actual manipulator's configurations in whole tracking process based on multi-preview control

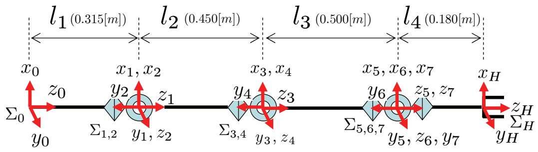

Fig. 5 Structure of PA10

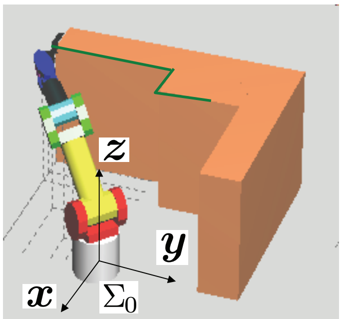

Fig. 6 Outside appearance of simulation

$$\boldsymbol{r}_d = \begin{bmatrix} r_{dx} \\ r_{dy} \\ r_{dz} \end{bmatrix} = \begin{bmatrix} -0.8[m] \\ -0.5 + 0.05t[m] \\ 0.6[m] \end{bmatrix}, \tag{9}$$

when $12[s] \leq t < 16[s]$

$$\boldsymbol{r}_d = \begin{bmatrix} r_{dx} \\ r_{dy} \\ r_{dz} \end{bmatrix} = \begin{bmatrix} -0.8 + 0.05(t-12)[m] \\ 0.1[m] \\ 0.6[m] \end{bmatrix}, \tag{10}$$

and when $16[s] \leq t < 22[s]$

$$\boldsymbol{r}_d = \begin{bmatrix} r_{dx} \\ r_{dy} \\ r_{dz} \end{bmatrix} = \begin{bmatrix} -0.6[m] \\ 0.1 + 0.05(t-16)[m] \\ 0.6[m] \end{bmatrix}. \tag{11}$$

For desired hand posture, when $0[s] \leq t < 10[s]$ and $18[s] \leq t < 22[s]$

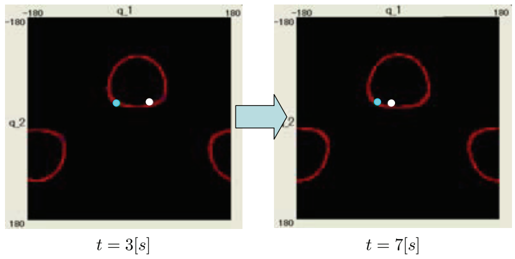

Fig. 7 Convergence of $\boldsymbol{q}(t)$

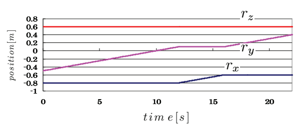

Fig. 8 Transition of hand position $\boldsymbol{r}(t)$ using Single-Preview

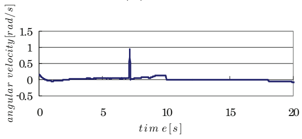

Fig. 9 Transition of hand posture $\omega_x(t)$ using Single-Preview

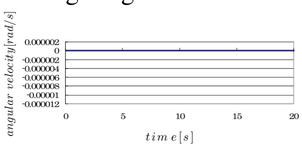

Fig. 10 Transition of hand posture $\omega_y(t)$ using Single-Preview

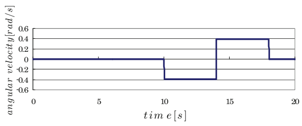

Fig. 11 Transition of hand posture $\omega_z(t)$ using Single-Preview

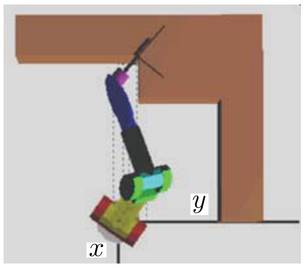

Fig. 12 Configuration of PA10 when $t = 12[s]$ using Single-Preview

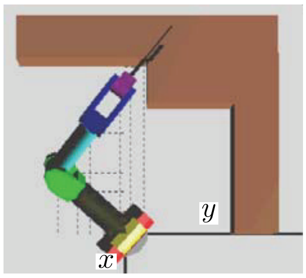

Fig. 13 Configuration of PA10 when $t = 12[s]$ using Multi-Preview

$$\boldsymbol{\omega}_d = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} free[rad/s] \\ 0.0[rad/s] \\ 0.0[rad/s] \end{bmatrix}, \tag{12}$$

when $10[s] \leq t < 14[s]$

$$\boldsymbol{\omega}_d = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0.0[rad/s] \\ 0.0[rad/s] \\ -\frac{\pi}{8}[rad/s] \end{bmatrix}, \tag{13}$$

and when $14[s] \leq t < 18[s]$

$$\boldsymbol{\omega}_d = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0.0[rad/s] \\ 0.0[rad/s] \\ \frac{\pi}{8}[rad/s] \end{bmatrix}. \tag{14}$$

When evaluating Single-Preview Control, we let t be current time and $t^* = t + 3[s]$ is future time for imaginary manipulator. The simulation results are shown in from Fig.7 to Fig.12. Fig.7 indicates AMSIP distribution at time $t = 3[s]$ and $t = 7[s]$. The white circle is the shape of current manipulator $\boldsymbol{q}(t)$, the other blue circles mean the shape of imaginary manipulator $\tilde{\boldsymbol{q}}_d(t^*)$. According to these figures, current manipulator changes its shape to make the configuration close to the imaginary manipulator's shape. From Fig.8 to Fig.11 indicate the transition of position and posture of current manipulator's hand. According to these figures, current manipulator is tracking the given target trajectory correctly. Moreover, Fig.12 indicates top view of the shape of the current manipulator when time $t = 12[s]$. According to this figure, we see the fourth joint exists in a vicinity of working object because the Single-Preview Control refers to only one imaginary manipulator. From the above results we know that when we use Single-Preview Control, manipulator is able to track the target trajectory correctly, but the shape may drop dangerously into being close to the working object.

When we choose Multi-Preview Control, we set three imaginary manipulators at future time $t_i^* = t+3i[s], (i = 1, 2, 3)$. The result of simulation shown in Fig.13 represents that the current manipulator's hand position and posture have rather safer space against the wall of target trajectory. Fig.13 indicates top view of the current manipulator when time $t = 12[s]$. According to this figure, we see the second joint gets some distance form the working objects comparing with Single-Preview Control result shown in Fig.12. Multi-Preview Control can refer to several imaginary manipulator's shape, so the manipulator can change the shape and get away from working object.

From the results of simulation, although manipulator avoids collision with the target object no matter what method of Single-Preview Control Method or Multi-Preview Control Method to be used, but the shape of current manipulator get away from working object earlier and faster when we use the Multi-Preview Control. Therefore, we showed a validity of Multi-Preview Control through simulation.

5. EXPERIMENT

We used PA10 which is produced by Mitsubishi Heavy Industries to complete the experiment and show the validity of Multi-Preview Control. We set the same experimental environment as the one in simulation. The results using Single-Preview Control are showed in from Fig.14 to Fig.18. Fig.14 shows transition of manipulator's hand position, and from Fig.15 to Fig.17 show transition of manipulator's hand posture. According to these figures, although we find r_z includes position error from desired trajectory, we think that PA10 tracked desired trajectory and also desired posture. Fig.18 shows the shape of PA10 when time $t = 12[s]$.

On the other hand, the result using Multi-Preview Control is showed in Fig.19. We didn't show transition of manipulator's hand position or posture, however PA10 tracks desired trajectory or desired posture. Fig.19 shows the shape of PA10 when time $t = 12[s]$.

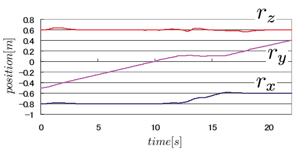

Fig. 14 Transition of hand position $\boldsymbol{r}(t)$ using Single-Preview

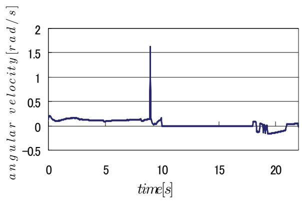

Fig. 15 Transition of hand posture $\omega_x(t)$ using Single-Preview

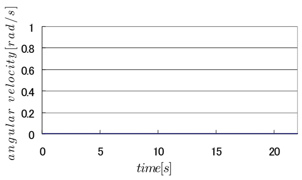

Fig. 16 Transition of hand posture $\omega_y(t)$ using Single-Preview

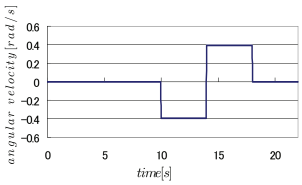

Fig. 17 Transition of hand posture $\omega_z(t)$ using Single-Preview

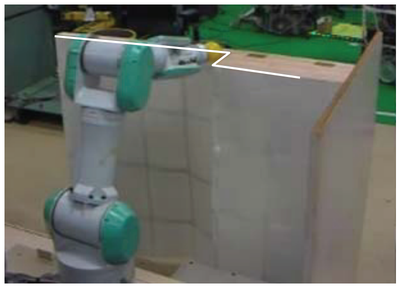

Fig. 18 Configuration of PA10 when $t = 12[s]$ using Single-Preview

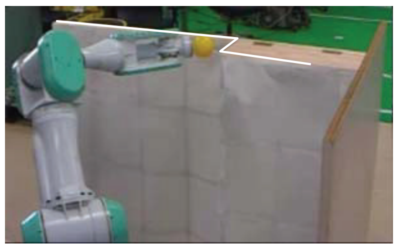

Fig. 19 Configuration of PA10 when $t = 12[s]$ using Multi-Preview

Compare Fig.18 and Fig.19, PA10's shape when using Multi-Preview Control keeps away from the working object. We consider it has more roots in a number of reference future times, a number of imaginary manipulators can be used for control current manipulator.

According to the above discussion, we show the validity of Multi-Preview Control through experiment.

6. AMSI DISTRIBUTION

We proposed index named AMSI (Avoidance Manipulator Shape Index)[1] E. In this section, we will analyze relation between AMSI distribution and shape of manipulator. In our research, we use 7-link manipulator showed in Fig.20. We set the hand's position and posture (Euler angle) arbitrarily and analyzed the relationship of 7-link

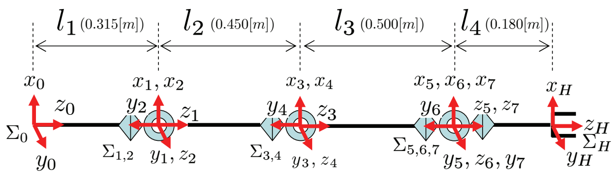

Fig. 20 Coordinate System of PA10

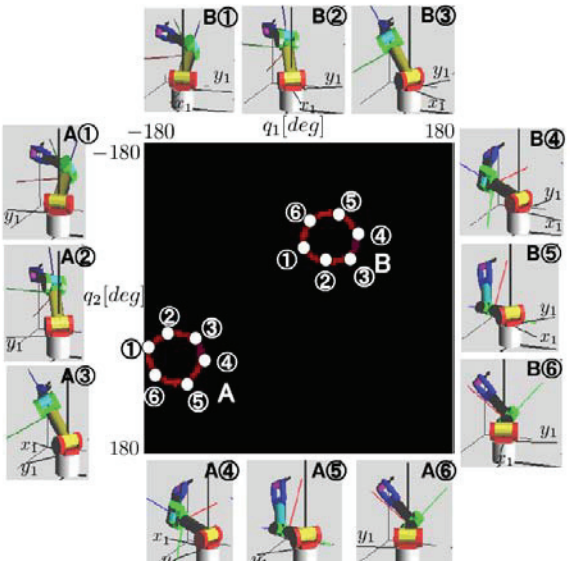

Fig. 21 AMSI Distribution and Shape of Manipulator

manipulator, if we set three variables of hand position and two variables of hand posture then we obtain two redundant degrees of freedom and take them into joint angles q_1, q_2.

When we set hand position and posture (Euler angles) $\boldsymbol{r} = [r_x, r_y, r_z, \phi, \theta, \psi]^T = [-0.8[m], -0.5[m], 0.6[m], 0.0, -\frac{\pi}{2}, free]^T$, the AMSI distribution is shown in Fig.21.

The shapes which can realize $\boldsymbol{r}$ is area of $0 < {}^1E$ and it is showed by two circles A, B in Fig.21, which expresses the possible range of redundant degree of freedom while realizing desired hand pose $\boldsymbol{r}$. To analyze the relationship, we set the angle of point ① ∼ ⑥ in circle A and the angle of point ① ∼ ⑥ in circle B expressed q_1, q_2 respectively, and show the shape in Fig.21.

When we observe shapes given by points ① ∼ ⑥ in order, we find that the shapes of A and B are same. In the A and B, q_1 get away from $180[deg]$ and q_2 is opposite sign, then q_2's direction in each axes in coordinate $\Sigma_{1,2}$ are not same. And q_3 gets away from $180[deg]$, the direction of each axes in coordinate Σ_3 is same. We find that A and B are relationship of front posture and rear posture.

7. CONCLUSION

In this paper, we propose Multi-Preview Control that is improved and modified from Single-Preview Control. When we use Multi-Preview Control, the manipulator can widen the space against the working object and the manipulator's shape change early to avoid collision with working object, and complete desired hand task in more secure situation. From experiment and simulation results we have proved the effectiveness of Multi preview control. However, when we use the real machine, the actual hand's position deviates from the target location. The error of position r_z which in the z axis direction is particularly swelled, so our future task and research direction is to consider seriously and to find the reason.

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