SICE Annual Conference 2012
August 20-23, 2012, Akita University, Akita, Japan

Position/Orientation Control of an Underactuated Flight Object Based on Two Degree-of-Freedom PID Control

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Abstract: This research explores position/orientation control of an underactuated flight object based on two degree-of-freedom PID control. Helicopter is applied in large field because of flight ability such as vertical ascent, vertical descent and hovering. However the helicopter, which is one of the underactuated flight objects, is complex and a nonlinear dynamics. In this research, controlled target is two inputs three outputs underactuated flight object. The control strategy for the experimental device is examined by two degree-of-freedom PID control. This method aims to adjust target-tracking and disturbance-reduction ability independently.

Keywords: UMA, Two degree-of-freedom, PID control, Underactuated system

1. INTRODUCTION

The helicopter is applied in large field because of flight ability such as vertical ascent, vertical descent and hovering. Especially manned helicopter is used for rescue, emergency activity and fire fighting at the time of disaster, and unmanned helicopter is precious sources of information in the danger spot where people cannot approach. But construction of helicopter is complex and sensitive to the influence of the wind. There are mainly two models in a helicopter. One is a single-rotor helicopter having main rotor and tail rotor. Tail rotor is to generate anti torque. Another one is a twin-rotor helicopter having two main rotors. Our laboratory has an experimental device of three degree-of-freedom underactuated flight object like twin-rotor helicopter. Twin-rotor helicopter has an advantage in the safety, because this model has a characteristic that operation stability against roll is high. This device controls the angles of vertical and rotation direction by thrust gained by two rotors. Controlling an underactuated flight object has attracted a lot of attention, due to the fact that flight object is an underactuate nonlinear system. It is considered that it is possible to contribute for reducing weight, lowering the cost, and the energy saving if the system can be controlled with the number of control inputs less than the number of the system outputs. The control strategy for the experimental device is examined by two degree-of-freedom PID control. This method aims to adjust target-tracking and disturbance-reduction ability independently by restraining the excessive control inputs.

This paper is organized as follows. Section 2 shows the underactuated flight object model of our experimental system, section 3 shows the system design of two degree-of-freedom PID control, section 4 and section 5 are simulation and experiment to compare PID control with two degree-of-freedom PID control.

2. MODELING

Controlled target is three degree-of-freedom underactuated flight object(Fig. 1). It is a two inputs and three outputs system which attaches motors for turning left and right rotor, and rotary encoders for detecting roll angle, pitch angle and yaw angle. To keep flight object from spinning by rotor drag torque, rotation of the right rotor is reverse rotation of the left rotor. The motion equation of three degree-of-freedom underactuated flight object is shown as follows.

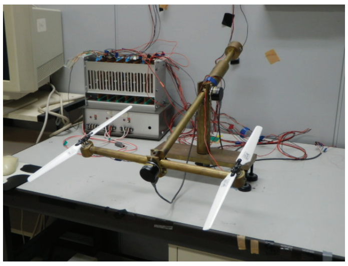

Fig. 1 Underactuated flight object

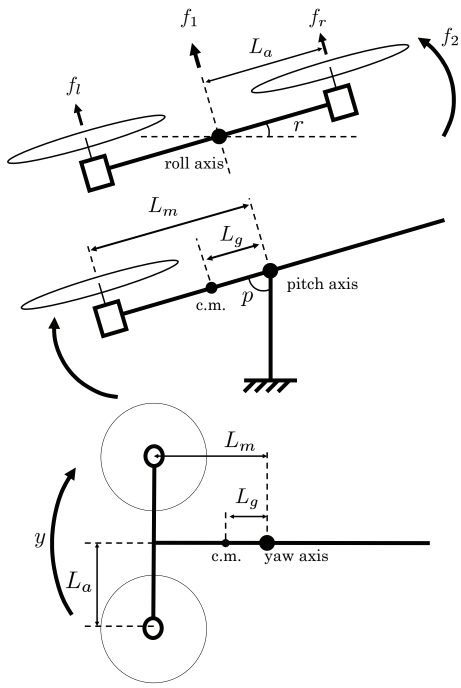

Fig. 2 Roll, pitch and yaw directions

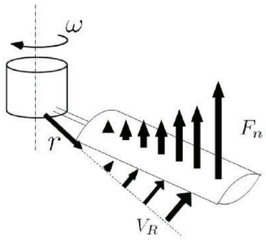

Fig. 3 Forces which act on the face of rotor

Direction of the roll angle:

$$I_r\ddot{r} + D_r\dot{r} = f_2 \tag{1}$$

Direction of the pitch angle:

$$I_p\ddot{p} + D_p\dot{p} + mgL_g\sin p = L_m f_1 \cos r \tag{2}$$

Direction of the yaw angle:

$$I_y\ddot{y} + D_y\dot{y} = L_m f_1 \sin r \tag{3}$$

Where r, p and y are angles of each direction, m is weight of the system, g is gravity acceleration, I_r, I_p and I_y are moments of inertia of each direction, D_r, D_p and D_y are coefficient of friction of each direction, L_m is distance from pitch axis to roll link and L_g is distance from pitch axis to center of mass.

f_1 is a resultant force of f_l and f_r, f_2 is a moment of roll direction.

$$f_1 = f_r + f_l \tag{4}$$
$$f_2 = L_a(f_l - f_r) \tag{5}$$

Where f_r and f_l are right rotor thrust and left one respectively. L_a is arm length from roll axis to the motor.

$$f_r = \omega_r^2 A = A(ku_r)^2 = Ak^2u_1 \tag{6}$$
$$f_l = \omega_l^2 A = A(ku_l)^2 = Ak^2u_2 \tag{7}$$

Where $\omega_{r,l}$ are rotor angular speeds, A is a coefficient based on shape of the rotor, $u_{r,l}$ are input voltages into each left and right motor, k is a coefficient between voltage and angular speed, where $\omega_r = ku_r$, $\omega_l = ku_l$.

2.1 Aerodynamical forces

The state of the flight object is shifted the desired state by aerodynamical forces which act on the rotors. The equation of aerodynamical forces is shown by using the rotor angular speed as follows. Aerodynamical force per microscopic area is shown as follows.

$$F_n = \frac{1}{2}\rho V_R^2 S C_z \tag{8}$$
$$V_R = \omega r \tag{9}$$

Where F_n is aerodynamical force per microarea, ρ is airdensity, V_R is airspeed, S is surface area of the rotor, C_z is a coefficient of aerodynamical forces and r is distance from shaft. F_n is a function of r as shown in Fig. 3. Airdensity ρ and airspeed V_R are variables. Surface area of the rotor S, shape of rotor and rotor area which affect C_z are constants. As a result, force of aerodynamical forces per microscopic area is the rotor thrust F_N,

$$\begin{aligned} F_N &= 2\int_0^R F_n dr \\ &= \int_0^R \rho(r\omega)^2 S C_z dr \\ &= \omega^2 S \int_0^R \rho r^2 C_z dr \\ &= \omega^2 A \end{aligned} \tag{10}$$

Coefficient based on shape of rotor A is

$$A = S\int_0^R \rho r^2 C_z dr \tag{11}$$

Where R is a radius of the rotor.

2.2 Input voltage

Input voltage sets a limit because of hardware's specification.

$$0[\mathrm{V}] \le u_r \le 7[\mathrm{V}] \tag{12}$$
$$0[\mathrm{V}] \le u_l \le 7[\mathrm{V}] \tag{13}$$

3. CONTROL SYSTEM DESIGN

3.1 PID control design

Based on equation of motion which shows the relationship between input voltage and rotor angular speed, equation of motion is given as follows.

$$\begin{aligned} I_r\ddot{r} + D_r\dot{r} &= L_aAk^2(u_2 - u_1) \\ I_p\ddot{p} + D_p\dot{p} + mgL_g\sin p &= L_mAk^2(u_1 + u_2)\cos r \\ I_y\ddot{y} + D_y\dot{y} &= L_mAk^2(u_1 + u_2)\sin r \end{aligned} \tag{14}$$

Where u_1 is the square of u_r, and u_2 is the square of u_l. Parameters of the equations are replaced and shown as follows.

$$\begin{aligned} a_1\ddot{r} + a_2\dot{r} &= u_2 - u_1 \\ b_1\ddot{p} + b_2\dot{p} + b_3\sin p &= (u_1 + u_2)\cos r \\ c_1\ddot{y} + c_2\dot{y} &= (u_1 + u_2)\sin r \end{aligned} \tag{15}$$

Where

$$a_1 = \frac{I_r}{L_aAk^2} \quad b_1 = \frac{I_p}{L_mAk^2}$$
$$a_2 = \frac{D_r}{L_aAk^2} \quad b_2 = \frac{D_p}{L_mAk^2}$$
$$c_1 = \frac{I_y}{L_mAk^2} \quad b_3 = \frac{mgL_g}{L_mAk^2}$$
$$c_2 = \frac{D_y}{L_mAk^2}$$

From (15), defining that;

$$\begin{aligned} F_r &= \frac{1}{a_1}\{-a_2\dot{r} + (u_2 - u_1)\} \\ F_p &= \frac{1}{b_1}\{-b_2\dot{p} - b_3\sin p + (u_1 + u_2)\cos r\} \\ F_y &= \frac{1}{c_1}\{-c_2\dot{y} + (u_1 + u_2)\sin r\} \end{aligned} \tag{16}$$

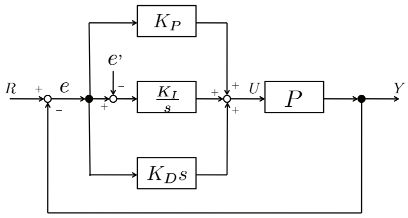

Fig. 4 Two degree-of-freedom PID control structure

Moreover, we design the desired F_p^*, F_y^* as follows.

$$F_p^* = \frac{1}{b_1}\left(-b_2\dot{p} - b_3 \sin p + z_p \cos r\right) \tag{17}$$

$$F_y^* = \frac{1}{c_1}\left(-c_2\dot{y} + z_y \sin r\right) \tag{18}$$

Where z_p and z_y are ideal input voltages replaced u_1 and u_2. From (17) and (18), we can obtain the desired roll angle as follows.

$$r^* = \tan^{-1}\left(\frac{c_1}{b_1}\frac{F_y^* + \frac{c_2}{c_1}\dot{y}}{F_p^* + \frac{b_2}{b_1}\dot{p} + \frac{b_3}{b_1}\sin p}\right) \tag{19}$$

And the desired input voltages F_p^*, F_y^* are given by the following PID control.

$$F_p^* = -K_{P2}(p - p_d) - K_{I2}\int (p - p_d) - K_{D2}\dot{p} \tag{20}$$

$$F_y^* = -K_{P3}(y - y_d) - K_{I3}\int (y - y_d) - K_{D3}\dot{y} \tag{21}$$

Desired input voltage F_r^* is also given as follows.

$$F_r^* = -K_{P1}(r - r^*) - K_{D1}\dot{r} \tag{22}$$

Where defining as;

$$u_2 - u_1 = a_1 F_r^* + a_2\dot{r} = z_1 \tag{23}$$

$$u_1 + u_2 = \frac{b_1 F_p^* + b_2\dot{p} + b_3 \sin p}{\cos r} = z_2 \tag{24}$$

From (23) and (24), u_1 and u_2 are obtained as follows.

$$u_1 = \frac{z_2 - z_1}{2} \tag{25}$$

$$u_2 = \frac{z_1 + z_2}{2} \tag{26}$$

Because $u_1 = u_r^2$ and $u_2 = u_l^2$, u_r and u_l are given as follows.

$$u_r = \sqrt{u_1} \tag{27}$$

$$u_l = \sqrt{u_2} \tag{28}$$

Since this device isn't carried out the reverse rotation, u_r and u_l are only positive signal. If u_r and u_l are negative signal, u_r and u_l are set to be zero.

3.2 Two degree-of-freedom PID control design

Two degree-of-freedom PID control law aims to adjust target-tracking and disturbance-reduction ability independently.

Two degree-of-freedom PID control structure is shown in Fig. 4. Where R is output such as a present angle, U is control input obtained by each gain, R is reference signal of each angle, $e^{'}$ is target-tracking error with PD control using the same gain as PID control. The difference between e and $e^{'}$ is applied to the integrator, that is, if there is neither disturbance nor modeling error, the integral compensation doesn't appear. Then the proposed controller is given as follows.

$$F_p^* = -K_{P2}(p - p_d) - K_{I2}\int \left((p - p_d) - e_p^{'}\right) - K_{D2}\dot{p}$$

$$F_y^* = -K_{P3}(y - y_d) - K_{I3}\int \left((y - y_d) - e_y^{'}\right) - K_{D3}\dot{y}$$

In this controller, the effect of integral compensation performs only when there is disturbance or modeling error.

4. SIMULATION

4.1 Simulation1

Simulation1 shows the case that the reference signal is step-type signal. Initial and reference signals for controlled target are given as follows. Initial signals of roll angle, pitch angle and yaw angle are 0.0[rad],1.23[rad] and 0.0[rad]. Reference signals of roll angle, pitch angle and yaw angle are r^*[rad], $\frac{\pi}{2}$[rad] and 0.0[rad]. Each parameter of controlled target and controller is given by Table 1 and Table 2 respectively.

Table 1 Parameters of controlled target

a_1	=	15.88022	b_1	=	43.69183
a_2	=	1.02117	b_2	=	1.02117
c_1	=	24.691833	b_3	=	36.11473
c_2	=	1.839416			

Table 2 Control parameters

	Roll($i = 1$)	Pitch($i = 2$)	Yaw($i = 3$)
K_{Pi}	1.4	1.5	0.375
K_{Ii}	0.0	0.1	0.075
K_{Di}	3.5	3.5	2.6

Simulation1 result is shown in Fig. 5 ～ Fig. 7. Pitch angle tracks the reference signal. Since roll angle and yaw angle are not used, the input voltages into each left and right motor hardly change. In PID control, transient response has overshoot. On the other hand, in two degree-of-freedom PID control, there is no overshoot. And the excessive control inputs are restrained with comparison to PID control.

4.2 Simulation2

Simulation2 shows the case that the reference signal is changed. In this simulation, the reference signal of yaw angle is changed at 15[s]. Initial and reference signals for controlled target are given as follows. Initial signals of roll angle, pitch angle and yaw angle are

0.0[rad],1.23[rad] and 0.0[rad]. Reference1 signals of roll angle, pitch angle and yaw angle are r^*[rad], $\frac{\pi}{2}$[rad] and 0.0[rad]. Reference2 signals of roll angle, pitch angle and yaw angle are r^*[rad], $\frac{\pi}{2}$[rad] and $\frac{\pi}{6}$[rad]. Each parameter of controlled target and control parameter are the same as Simulation1.

Simulation2 result is shown in Fig. 8 ∼ Fig. 11. Roll angle and yaw angle are not stabilizing to within a time. It is considered that this phenomenon is able to improve attenuation by raising a differential gain. In comparison with PID control and two degree-of-freedom PID control, the latter can be restrained from overshooting.

5. EXPERIMENT

5.1 Experiment1

Experiment1 shows the case that the reference signal is step-type signal. Initial and reference signals, and parameter of controlled target are considered to be the same as simulation1. And each control parameter is given by Table 3.

Table 3 Control parameters

	Roll($i = 1$)	Pitch($i = 2$)	Yaw($i = 3$)
K_{Pi}	2.0	4.1	2.2
K_{Ii}	0.0	0.01	0.005
K_{Di}	2.8	4.3	3.0

Experiment1 result is shown in Fig. 12 ∼ Fig. 16. In experiment1, pitch angle converges in the reference signal and be stabilized in steady state. In comparison with PID control and two degree-of-freedom PID control, as in the simulation1, the latter can be restrained from overshooting. Moreover, two degree-of-freedom PID control has an effect of energy saving since it can restrain input voltages. When pitch angle is stabilized, roll angle is converged in the vicinity of 0.0[rad]. Only required parts leans roll angle and track the reference signal of the yaw angle when a deviation arises on the yaw angle.

5.2 Experiment2

Experiment2 shows the case that the reference signal is changed. Initial and reference signals for controlled target are given as follows. Initial signals of roll angle, pitch angle and yaw angle are 0.0[rad],1.23[rad] and 0.0[rad]. Reference1 signals of roll angle, pitch angle and yaw angle are r^*[rad], $\frac{\pi}{2}$[rad] and 0.0[rad]. Reference2 signals of roll angle, pitch angle and yaw angle are r^*[rad], $\frac{\pi}{2}$[rad] and $-\frac{\pi}{12}$[rad].

Experiment2 result is shown in Fig. 17 ∼ Fig. 22. Roll angle tends to be leaned because the reference signal of the yaw angle changed, yaw angle is going to track the reference signal. However, it turns out that yaw angle is changing so that it may be pulled by something when it approaches near the reference signal. This is seemed to be a problem of the experiment system. Since it is thought that it will be pulled by the tension of wiring of the motor for turning a rotor and wiring of the encoder which reads the roll angle, control of the yaw angle is affected.

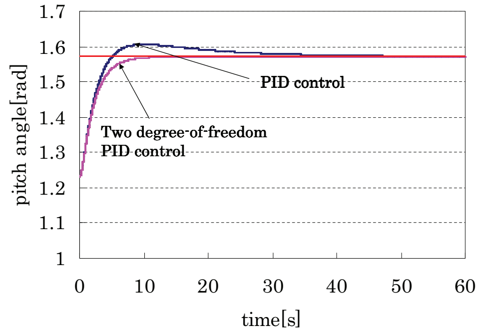

Fig. 5 Comparison of pitch angle

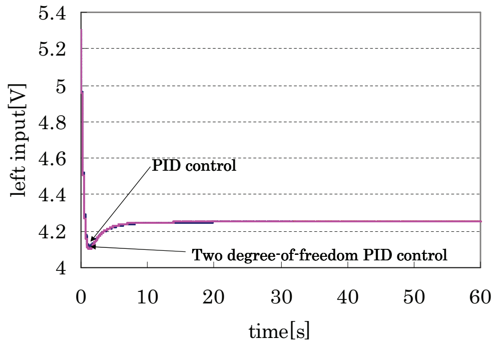

Fig. 6 Comparison of left input voltages

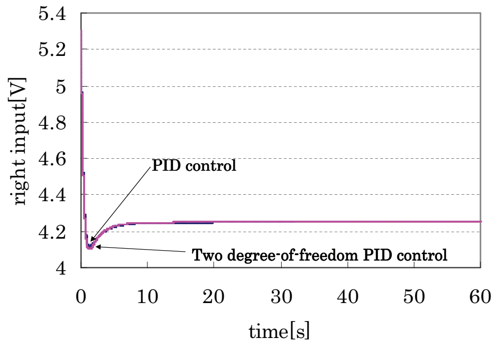

Fig. 7 Comparison of right input voltages

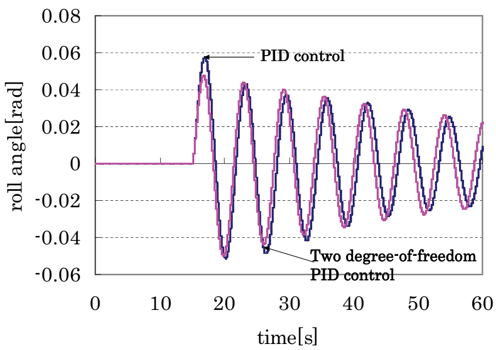

Fig. 8 Comparison of roll angle

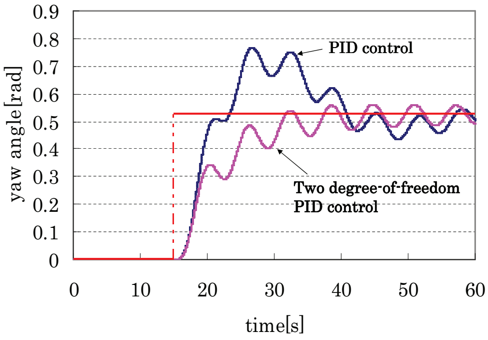

Fig. 9 Comparison of yaw angle

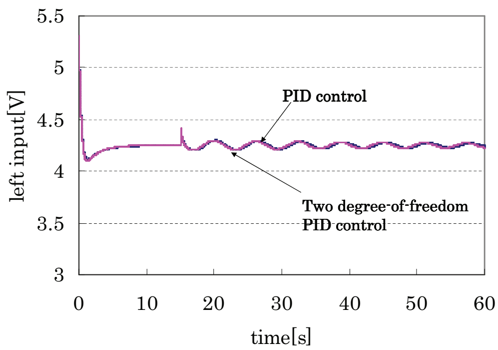

Fig. 10 Comparison of left input voltages

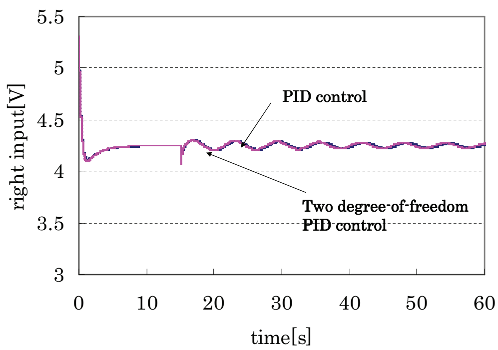

Fig. 11 Comparison of right input voltages

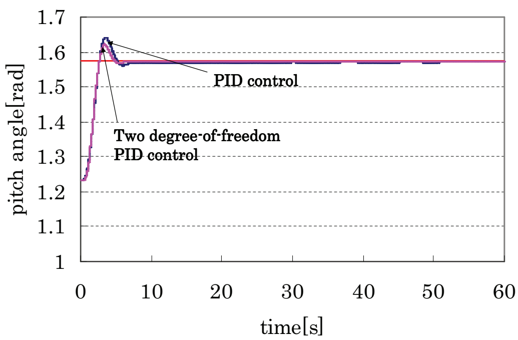

Fig. 12 Comparison of pitch angle

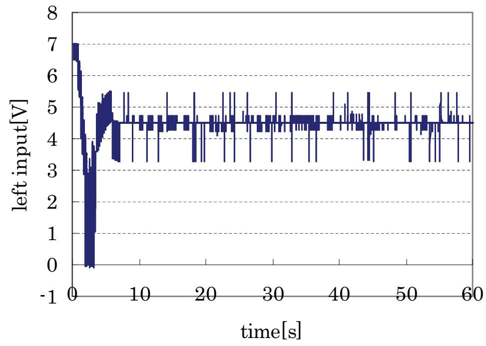

Fig. 13 Left input voltage when PID control applies

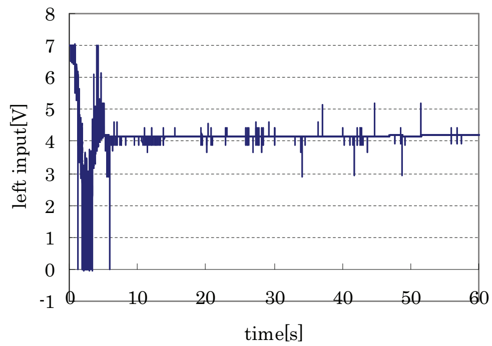

Fig. 14 Left input voltage when two degree-of-freedom PID control applies

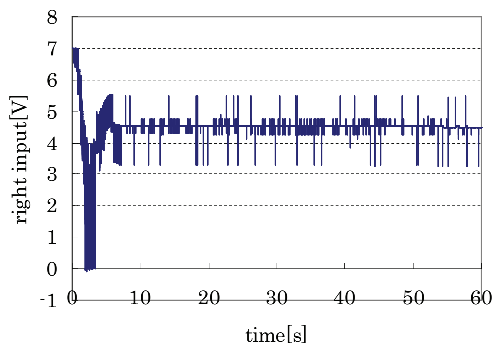

Fig. 15 Right input voltage when PID control applies

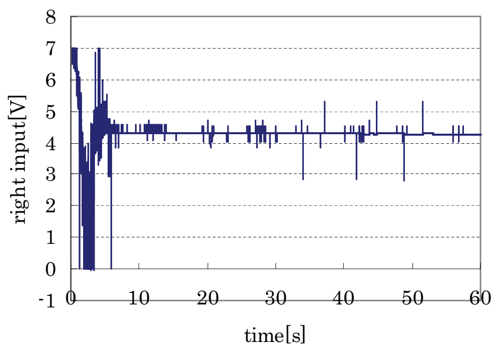

Fig. 16 Right input voltage when two degree-of-freedom PID control applies

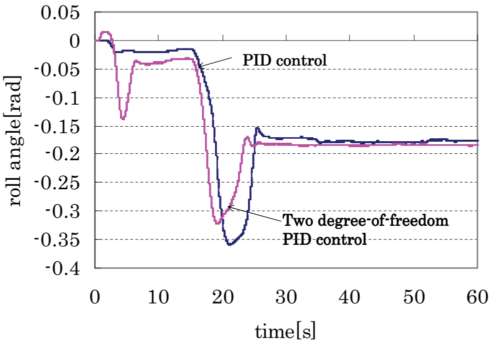

Fig. 17 Comparison of roll angle

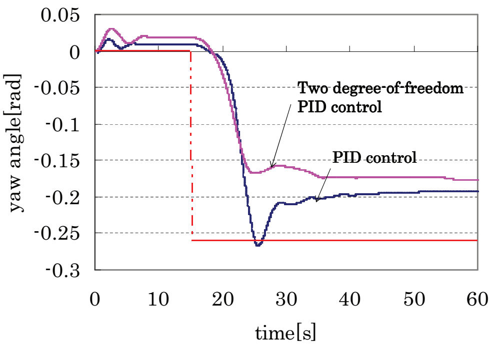

Fig. 18 Comparison of yaw angle

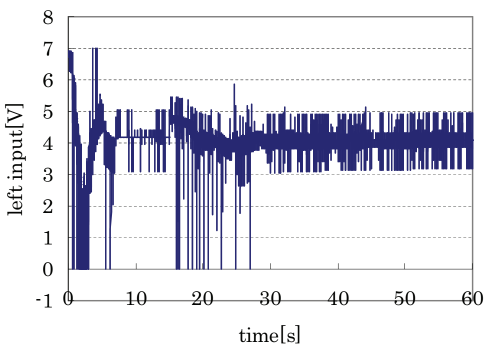

Fig. 19 Left input voltage when PID control applies

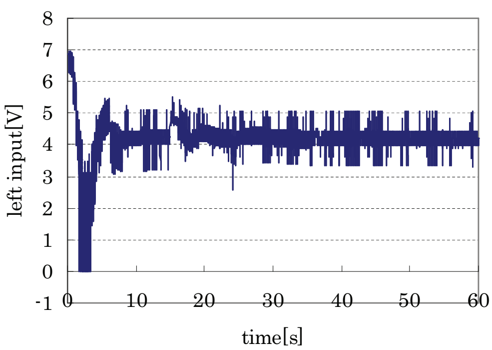

Fig. 20 Left input voltage when two degree-of-freedom PID control applies

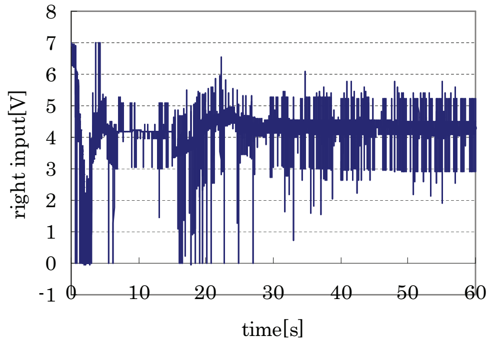

Fig. 21 Right input voltage when PID control applies

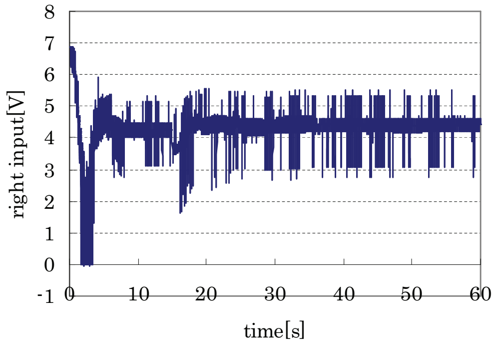

Fig. 22 Right input voltage when two degree-of-freedom PID control applies

6. CONCLUSION

This paper explored modeling to carry out the position/orientation control of two inputs three outputs underactuated flight object based on two degree-of-freedom PID control, design of the PID control system and expansion to two degree-of-freedom PID control. Comparing two degree-of-freedom PID control to PID control, in PID control, overshoot arose both simulations and experiments, but in two degree-of-freedom PID control, it restrained and tracked the reference signal. Moreover, input voltages of the left and right motor brought similar result in the simulation, but in the experiment, input voltages of two degree-of-freedom PID control are less than that of PID control. Therefore it concludes that two degree-of-freedom PID control can contribute to energy saving. As a future work, it is necessary to improve the experiment system since it is difficult to control the direction of the yaw angle on account of the problem of wiring.

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