Posture Control of Inverted Pendulum Cart with Balancing Mechanism Using Full-Order State Observer

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Abstract—A lot of researches on inverted pendulum cart are conducted in recent years. And it attracts as a personal vehicle that realizes energy saving for many practical applications. However, because most conventional personal vehicles require a certain level of physical ability from the driver, it is not suitable for elderly and disabled people to drive.

Therefore, an inverted pendulum cart with a sliding mechanism for posture control that could be ridden by anyone has been developed. In this paper, we propose that inverted pendulum cart runs keeping posture perpendicularly. First, algorithm to estimate the lean angle, that is calculated based on the center of gravity of the cart, using the full-order state observer is proposed. Secondly, algorithm to keep the posture of cart perpendicular against the change of the center of gravity is proposed. Then, desired position of slider is calculated based on the estimated lean angle. Finally, the effectiveness of algorithm is confirmed through a simulation.

I. INTRODUCTION

An inverted pendulum is a mechanical system as same mechanism as playing with umbrella balanced on one's palm. Because of no actuators on the bottom of inverted pendulum, this is also a kind of a system with fewer actuators than degrees of movement, so-called under actuated system[1]. Inverted pendulum mechanism is adopted to inverted pendulum cart. And it is controlled by using acceleration caused by intentionally imbalance for movement. An inverted pendulum cart was proposed by Yamafuji et al.[2] as example of application of modern control theory. And the realization of function due to a difference in structure has been proposed by a number of researchers in conventional research. In addition, a lot of researches on inverted pendulum cart are conducted in recent years[3]-[6]. It is attracted as a personal vehicle which realizes energy saving for many practical applications, such as Segway[7], P.U.M.A.[8] and MOBIRO[9]. Especially, the Segway secured a no practical problem level of stability, with only two parallel wheels, it has demonstrated the reliability of control of wheel type inverted pendulum. The P.U.M.A is two sheeter personal vehicle that is developed by Segway and General Motors Company. These personal mobilities are small and light. Thus, energy efficiency for transportation per person[10] is large advantage of these personal mobilities. In addition, when the body assumes a small four-wheeled vehicle, various grounding states occur, and it is necessary to change control for the small four-wheeled vehicle. However, since most

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Fig. 1. Inverted pendulum cart

conventional personal vehicles require a certain level of physical ability because the driver controls posture in person, it is not suitable for elderly and disabled people. Therefore, it is demanded that anyone can ride it to realize the energysaving society.

In our research, an inverted pendulum cart with a sliding mechanism for posture control that could be ridden by anyone has been developed[11]. A slider part moves back and forth for change the center of gravity position. And the movement keeps a posture of inverted pendulum cart perpendicularly. Therefore, the driver does not need to make moved the center of gravity for oneself when it accelerates, decelerates and curves.

In the posture stabilization of the inverted pendulum cart, the center of gravity position is important. However, the center of gravity position of the inverted pendulum cart changes by a driver. Therefore, we propose algorithm where posture is kept perpendicular state even if the center of gravity position changes. First, algorithm to estimate the lean angle, that is calculated based on the center of gravity of the cart, using the full-order state observer is proposed in this paper. Secondly, algorithm to keep the posture of cart perpendicular against the change of the center of gravity is proposed. Then, desired position of slider is calculated based on the estimated lean angle in order to keep cart perpendicular. Finally, the effectiveness of algorithm is confirmed through simulations.



Fig. 2. Dynamics model of the inverted pendulum cart

PARAMETERS OF INVERTED PENDULUM CART		
SYMBOLS	MEANINGS	VALUES
$m_1[kg]$	Weight of cart	4.831
$m_2[kg]$	Weight of slider	3.456
$m_{\omega}[kg]$	Weight of wheel	0.419
$r_{\omega}[m]$	Radius of wheel	0.0825
$I_1[\text{kgm}^2]$	Inertia moment of cart	29.2×10 ⁻³
$I_2[\text{kgm}^2]$	Inertia moment of slider	50.0×10 ⁻³
$I_{\omega}[\text{kgm}^2]$	Inertia moment of wheel	1.43×10 ⁻³
$l_1[m]$	Height of COG of cart	0.0191
	from center of wheel	
$l_2[m]$	Height of COG of slider	0.0912
	from center of wheel	
$D_1[Nms]$	Coefficient of viscous friction	0.05
	for wheel axis	
$D_2[Ns/m]$	Coefficient of viscous friction	0.05

TABLE I

II. MODELING OF INVERTED PENDULUM CART

for velocity of slider

Coefficient of viscous friction

or angular velocity of wheel angle

Gravitational acceleration

0.05

9.8

A. Structure of Inverted Pendulum Cart

 $D_{\omega}[\text{Nms}]$

 $g[m/s^2]$

An appearance of proposed inverted pendulum cart is shown in Fig. 1. Board on the cart are defined as base plate, bottom plate, top plate, cover plate sequentially from the bottom. A rack gear, rails, and stoppers are mounted on base plate. A motor, motor driver, and encoder for slider are mounted on bottom plate. Inertial sensors, and microcomputer are mounted on top plate. Wheels, and motors for wheels are mounted on bale plate.

B. Derivation of Systems Dynamics

A model of dynamics of inverted pendulum cart is shown in Fig. 2. Here, state variables for control are posture angle of inverted pendulum cart : θ_1 [rad], rotation angle of the tire wheel : θ_{ω} [rad], and displacement of the slider : λ_2 [m]. Control inputs are the torque of the motor wheel : τ_{ω} [Nm], and the seat movement driving force that the slider motor produces : $F_S[N]$.

Parameters of systems dynamics are listed in Table I. The kinetic energy T and the potential energy V are derivatived to obtain three equations of motion.

The kinetic energy T is defined as follows

$$T = \frac{1}{2} m_{\omega} (r_{\omega} \dot{\theta}_{\omega})^{2} + \frac{1}{2} I_{\omega} \dot{\theta}_{\omega}^{2} + \frac{1}{2} m_{1} (r_{\omega} \dot{\theta}_{\omega} + l_{1} \dot{\theta}_{1} \cos \theta_{1})^{2} + \frac{1}{2} m_{1} (-l_{1} \dot{\theta}_{1} \sin \theta_{1})^{2} + \frac{1}{2} m_{2} \{r_{\omega} \dot{\theta}_{\omega} + (l_{2} \cos \theta_{1} - \lambda_{2} \sin \theta_{1}) \dot{\theta}_{1} + \dot{\lambda}_{2} \cos \theta_{1}\}^{2} + \frac{1}{2} m_{2} \{(-l_{2} \sin \theta_{1} - \lambda_{2} \cos \theta_{1}) \dot{\theta}_{1} - \dot{\lambda}_{2} \sin \theta_{1}\}^{2} + \frac{1}{2} (I_{1} + I_{2}) \dot{\theta}_{1}^{2}.$$
(1)

The potential energy V is defined as follows

$$V = m_1 g l_1 \cos \theta_1 + m_2 g (l_2 \cos \theta_1 - \lambda_2 \sin \theta_1) . \qquad (2)$$

Therefore, the lagrangian L is defined as follows

$$L = T -V$$

$$= \frac{1}{2} I_{\omega}^{*}\dot{\theta}_{\omega}^{2}$$

$$+ \frac{1}{2}(m_{1}l_{1}^{2} + m_{2}l_{2}^{2} + m_{2}\lambda_{2}^{2} + I_{1} + I_{2})\dot{\theta}_{1}^{2}$$

$$+ \frac{1}{2}m_{2}\dot{\lambda}_{2}^{2}$$

$$+ \{m_{1}l_{1}\cos\theta_{1} + m_{2}(l_{2}\cos\theta_{1} - \lambda_{2}\sin\theta_{1})\}r_{\omega}\dot{\theta}_{\omega}\dot{\theta}_{1}$$

$$+ m_{2}r_{\omega}\cos\theta_{1}\dot{\theta}_{\omega}\dot{\lambda}_{2}$$

$$+ m_{2}l_{2}\dot{\theta}_{1}\dot{\lambda}_{2}$$

$$- m_{1}gl_{1}\cos\theta_{1} - m_{2}g(l_{2}\cos\theta_{1} - \lambda_{2}\sin\theta_{1}) . \quad (3)$$

Here, I_{ω}^* is constant value defined as

$$I_{\omega}^{*} = I_{\omega} + (m_{1} + m_{2} + m_{\omega})r_{\omega}^{2} .$$
 (4)

~ ` "

By lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_{\omega}}\right) - \frac{\partial L}{\partial \theta_{\omega}} + D_{\omega}\dot{\theta}_{\omega} + D_{1}(\dot{\theta}_{\omega} - \dot{\theta}_{1}) = \tau_{\omega} , \qquad (5)$$

the equation of motion is obtained as follows

$$I_{\omega}^{*} = \theta_{\omega} + r_{\omega} \{ (m_{1}l_{1} + m_{2}l_{2})\cos\theta_{1} - m_{2}\lambda_{2}\sin\theta_{1} \} \theta_{1}$$

$$+ m_{2}r_{\omega}\cos\theta_{1}\ddot{\lambda}_{2}$$

$$= \tau_{\omega} - (D_{\omega} + D_{1})\dot{\theta}_{\omega} + D_{1}\dot{\theta}_{1}$$

$$+ r_{\omega} \{ (m_{1}l_{1} + m_{2}l_{2})\sin\theta_{1}$$

$$+ m_{2}\lambda_{2}\cos\theta_{1} \} \dot{\theta}_{1}^{2} + 2m_{2}r_{\omega}\sin\theta_{1}\dot{\theta}_{1}\dot{\lambda}_{2} .$$
(6)

Also by lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_{1}}\right) - \frac{\partial L}{\partial \theta_{1}} + D_{1}\left(\dot{\theta}_{1} - \dot{\theta}_{\omega}\right) = -\tau_{\omega} , \qquad (7)$$

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the equation of motion is obtained as follows

$$r_{\omega} = \{(m_{1}l_{1} + m_{2}l_{2})\cos\theta_{1} - m_{2}\lambda_{2}\sin\theta_{1}\}\ddot{\theta}_{\omega} + (m_{1}l_{1}^{2} + m_{2}l_{2}^{2} + m_{2}\lambda_{2}^{2} + I_{1} + I_{2})\ddot{\theta}_{1} + m_{2}l_{2}\ddot{\lambda}_{2} = - \tau_{\omega} - D_{1}\dot{\theta}_{1} + D_{1}\dot{\theta}_{\omega} - 2m_{2}\lambda_{2}\dot{\theta}_{1}\dot{\lambda}_{2} + (m_{1}l_{1} + m_{2}l_{2})g\sin\theta_{1} + m_{2}g\lambda_{2}\cos\theta_{1}.$$
(8)

Also by lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\lambda}_2}\right) - \frac{\partial L}{\partial \lambda_2} + D_2 \dot{\lambda}_2 = F_S , \qquad (9)$$

the 3rd equation of motion is obtained as follows

$$m_2 \quad r_{\omega}\cos\theta_1\ddot{\theta}_{\omega} + m_2l_2\ddot{\theta}_1 + m_2\ddot{\lambda}_2$$

= $F_S \quad -D_2\dot{\lambda}_2 + m_2\lambda_2\dot{\theta}_1^2 + m_2g\sin\theta_1$. (10)

C. Linearization

A state equation can be calculated by linearing motion equations of Eq. (6), Eq. (8), and Eq. (10). First, assuming that state variables θ_{ω} , θ_1 , λ_2 to be slightly small amount, $\cos \theta_1 = 1$, $\sin \theta_1 = \theta_1$ are put into equations. Eq. (6), Eq. (8), and Eq. (10) are linearized as

$$I_{\omega}^{*} \quad \ddot{\theta}_{\omega} + r_{\omega} \{ (m_{1}l_{1} + m_{2}l_{2}) - m_{2}\lambda_{2}\theta_{1} \} \ddot{\theta}_{1} + m_{2}r_{\omega}\ddot{\lambda}_{2}$$

$$= \tau_{\omega} \quad -(D_{\omega} + D_{1})\dot{\theta}_{\omega} + D_{1}\dot{\theta}_{1} + r_{\omega} \{ (m_{1}l_{1} + m_{2}l_{2})\theta_{1}$$

$$+ m_{2}\lambda_{2} \} \dot{\theta}_{1}^{2} + 2m_{2}r_{\omega}\theta_{1}\dot{\theta}_{1}\dot{\lambda}_{2} , \qquad (11)$$

$$r_{\omega} = \{(m_{1}l_{1} + m_{2}l_{2}) - m_{2}\lambda_{2}\theta_{1}\}\dot{\theta}_{\omega} + (m_{1}l_{1}^{2} + m_{2}l_{2}^{2} + m_{2}\lambda_{2}^{2} + I_{1} + I_{2})\ddot{\theta}_{1} + m_{2}l_{2}\ddot{\lambda}_{2} \\ = -\tau_{\omega} - D_{1}\dot{\theta}_{1} + D_{1}\dot{\theta}_{\omega} - 2m_{2}\lambda_{2}\dot{\theta}_{1}\dot{\lambda}_{2} + (m_{1}l_{1} + m_{2}l_{2})g\theta_{1} + m_{2}g\lambda_{2}, \qquad (12)$$

and

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$$m_{2} \quad r_{\omega}\ddot{\theta}_{\omega} + m_{2}l_{2}\ddot{\theta}_{1} + m_{2}\ddot{\lambda}_{2} = F_{S} \quad -D_{2}\dot{\lambda}_{2} + m_{2}\lambda_{2}\dot{\theta}_{1}^{2} + m_{2}g\theta_{1} .$$
(13)

. ...

And, when thought state variables θ_{ω} , θ_1 , λ_2 to be slightly small amount, the product of state variable is ignored. Eq. (11), Eq. (12), and Eq. (13) can be changed as follows

$$I_{\omega}^{*} \quad \ddot{\theta}_{\omega} + r_{\omega}(m_{1}l_{1} + m_{2}l_{2})\ddot{\theta}_{1} + m_{2}r_{\omega}\lambda_{2}$$

= $\tau_{\omega} - (D_{\omega} + D_{1})\dot{\theta}_{\omega} + D_{1}\dot{\theta}_{1}$, (14)

$$r_{\omega} \quad (m_{1}l_{1} + m_{2}l_{2})\ddot{\theta}_{\omega} + (m_{1}l_{1}^{2} + m_{2}l_{2}^{2} + I_{1} + I_{2})\ddot{\theta}_{1} \\ + m_{2}l_{2}\ddot{\lambda}_{2} \\ = - \tau_{\omega} - D_{1}\dot{\theta}_{1} + D_{1}\dot{\theta}_{\omega} + (m_{1}l_{1} + m_{2}l_{2})g\theta_{1} \\ + m_{2}g\lambda_{2} , \qquad (15)$$

and

$$m_2 \quad r_{\omega}\dot{\theta}_{\omega} + m_2 l_2 \dot{\theta}_1 + m_2 \lambda_2$$

= $F_S \quad -D_2 \dot{\lambda}_2 + m_2 g \theta_1$. (16)

To put Eq. (14), Eq. (15), and Eq. (16) together,

$$Z\dot{x} = Ax + Bu \tag{17}$$

is obtained.

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Fig. 3. Difference of the center of gravity position

Here,

$$A = \begin{bmatrix} -(D_{\omega} + D_{1}) & D_{1} & 0 & 0 & 0 & 0 & 0 \\ D_{1} & -D_{1} & 0 & 0 & (m_{1}l_{1} + m_{2}l_{2})g & m_{2}g \\ 0 & 0 & -D_{2} & 0 & m_{2}g & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$Z = \begin{bmatrix} I_{\omega}^{*} & r_{\omega}(m_{1}l_{1} + m_{2}l_{2}) & m_{2}r_{\omega} & 0 & 0 & 0 \\ r_{\omega}(m_{1}l_{1} + m_{2}l_{2}) & m_{1}l_{1}^{2} + m_{2}l_{2}^{2} + l_{1} + l_{2} & m_{2}l_{2} & 0 & 0 & 0 \\ m_{2}r_{\omega} & m_{2}l_{2} & m_{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$x = \begin{bmatrix} \dot{\theta}_{\omega} & \dot{\theta}_{1} & \dot{\lambda}_{2} & \theta_{\omega} & \theta_{1} & \lambda_{2} \end{bmatrix}^{T},$$
and
$$u = \begin{bmatrix} \tau_{\omega} & F_{S} \end{bmatrix}^{T}.$$

By rewritten above Eq. (17)

$$\dot{x} = Z^{-1}Ax + Z^{-1}Bu \tag{18}$$

which is state equation is abtained.

III. ALGORITHM

As indicated in the Fig. 3(a), a general inverted pendulum has a simple shape. Therefore, the center of gravity position exists on the plumb of the fulcrum when posture of the pendulum is perpendicular. However as indicated in the Fig. 3(b), a pendulum part of the inverted pendulum cart has complicated shape. Therefore, the center of gravity position may not exist on the plumb of a wheel when posture of cart is perpendicular. And, the center of gravity position of the inverted pendulum cart changes by a driver. Therefore, algorithm to estimate the lean angle, that is calculated based on the center of gravity of the cart, using the full-order





(a) Initial

state

state observer is proposed. Secondly, algorithm to keep the posture of cart perpendicular against the change of the center of gravity is proposed. Then, desired position of slider is calculated based on the estimated lean angle. Control based on full-order state observer simulation is compared with feedback control simulation in order confirm the effectiveness of proposed algorithm. The block diagram of feedback is shown in Fig. 4.

A. Control Low

The block diagram is shown in Fig. 5. In proposal method, control inputs are decided by estimated state values from observer. And simulated posture control simulation. Desired position of slider is calculated based on the estimated lean angle. And, a vertical state is maintained for the change of the center of gravity.

Desired position of slider is calculated as

$$\lambda_2^* = P \int \hat{\theta}_1 dt \ . \tag{19}$$

Here,

$$\hat{x} = \begin{bmatrix} \dot{\theta}_{\omega} & \dot{\theta}_1 & \dot{\lambda}_2 & \dot{\theta}_{\omega} & \dot{\theta}_1 & \dot{\lambda}_2 \end{bmatrix}^T .$$
(20)

IV. SIMULATION

A. Simulation Environment

Scilab[12] is used for simulation. The desired movement of cart on simulation is shown in Fig. 6. Inverted pendulum cart must keep perpendicular state, while inverted pendulum cart runs. Inverted pendulum cart is desired acceleration from a stop state, and ran, and stop. At 1 second, desired angular velocity 3.03[rad/s] is given. At 14 seconds, weight of 10 % of slider part weight m_2 is added to the slider part to simulate the change of the center of gravity position by the movement of the driver. At 20 seconds, desired angular velocity 0[rad/s] is given to stop. The posture control simulations are carried out. P in Ep. (19) is decided heuristically in this simulation. P is

$$P = -3.0 \left[\frac{\mathrm{m}}{\mathrm{rad} \cdot \mathrm{s}}\right] \,. \tag{21}$$

3.03

[rad/s]

(C)

One

second later

(b)

14

seconds later

 $= m_2 + 0.1m_2$

Here.

$$F = \begin{bmatrix} -0.97 & -3.53 & -17.23 & -0.29 & -25.99 & -101.24 \\ 0.41 & 0.54 & 7.44 & 0.13 & 6.66 & 30.59 \end{bmatrix}, (22)$$

and
$$K = \begin{bmatrix} 8.03 & -4.76 & -1.50 & 7.39 & -9.01 & -1.18 \\ -4.76 & 16.57 & 0.24 & -2.88 & 10.15 & 3.57 \\ -1.50 & 0.24 & 0.67 & -1.17 & 1.41 & 0.24 \\ 0.74 & -0.29 & -0.12 & 3.02 & -0.46 & -0.12 \\ -0.90 & 1.01 & 0.14 & -0.46 & 1.57 & 0.12 \\ -0.12 & 0.36 & 0.02 & -0.12 & 0.16 \end{bmatrix}. (23)$$

B. Simulation Results

Full-order state observer control simulation is compared with feedback control simulation. Simulation results of feedback control are shown in Fig. 7. The wheel angle θ_{ω} is shown in Fig. 7(a). The lean angle θ_1 is shown in Fig. 7(b). The slider position λ_2 is shown in Fig. 7(c). Also, simulation results of the full-order state observer control are shown in Fig. 8. The wheel angle θ_{ω} is shown in Fig. 8(a). The lean angle θ_1 is shown in Fig. 8(b). The slider position λ_2 is shown in Fig. 8(c).



Fig. 7. Simulation results of state feedback control

Fig. 8. Simulation results of control based on full-order state observer

C. Discussion

As shown in Fig. 7(a) and Fig. 8(a), acceleration and deceleration are realized in both simulation without unstable condition. As shown in Fig. 7(b), the lean angle was stable

in 3.5×10^{-3} [rad] from 15.7sec to 20sec in the state that declined by the posture control using the state feedback. On the other hand, as shown in Fig. 8(b), by the posture control using the feedback by full-order state observer, the lean angle was stable in 4.3×10^{-5} [rad] from 15.7sec to 20sec and stood

with approximately perpendicular state. As shown in Fig. 7(c) and Fig. 8(c), when acceleration and deceleration, the change of a sudden slider position is seen in both simulation. However, error of lean angle is smaller than posture control using the feedback that posture control using the feedback that posture control using the feedback by full-order state observer. According to the above results, algorithmic effectiveness to estimate the lean angle, that is calculated based on the center of gravity of the cart, using the full-order state observer is confirmed. And, effectiveness of algorithm to keep the posture is confirmed. In proposed algorithm, a slider moves back and forth according to information of estimated lean angle in order to keep cart perpendicular against change of the center of gravity.

V. CONCLUSION

In this paper, algorithmic effectiveness to estimate the lean angle, that is calculated based on center of gravity of the cart, using the full-order state observer. And algorithmic effectiveness to keep the posture of cart perpendicular against the change of the center of gravity by desired position of slider is calculated based on the estimated lean angle. Two algorithmic effectiveness of the above are confirmed using simulation.

The posture control of inverted pendulum cart using fullorder state observer with the actual machine and stabilization of the posture by the movement of a slow slider are future problem. At the time of gain decision, change the weighing matrix and aim at the movement of a slow slider. We work these assignments and aim to realize acceleration and deceleration with keeping the angle of the cart perpendicular in the future.

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