

Switching PID Control for an Underactuated Flying Object Through Model-Based Prediction

Akira Yanou, Kohei Yoshikawa, Syouya Mizokami, Mamoru Minami and Takayuki Matsuno
 Graduate School of Natural Science and Technology, Okayama University
 3-1-1, Tsushimanaka, Kitaku, Okayama JAPAN 700-8530
 Tel/Fax: +81-86-251-8924
 Email: yanou-a@cc.okayama-u.ac.jp

Abstract—This paper explores switching PID controller for an underactuated flying object through model-based prediction. Helicopter is applied in large field because of flight ability such as vertical ascent, vertical descent and hovering. However the helicopter, which is one of the underactuated flight objects, is complex and has nonlinear dynamics. In this research, controlled target is an underactuated flight object with two inputs and three outputs. The proposed method predicts the system outputs using the model of controlled target, and the control inputs are calculated by using their values. That is, PID gains are switched at each sampling time by the model-based controlled result with time passing virtually. A numerical example is shown to verify the validity of the proposed method.

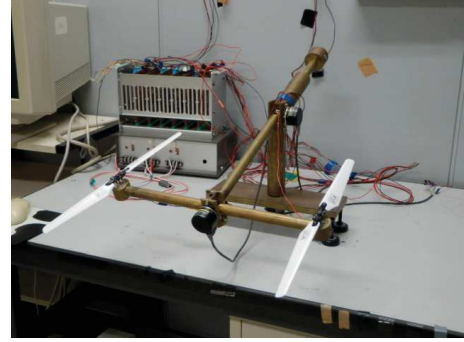


Fig. 1. Underactuated flying object

I. INTRODUCTION

Helicopter is applied in large field because of flight ability such as vertical ascent, vertical descent and hovering. Especially, manned helicopter is used for rescue, emergency activity and fire fighting at the time of disaster, and unmanned helicopter is precious sources of information in the danger spot where people cannot approach. But operation of helicopter is complex and sensitive to the influence of the wind. Our laboratory has an experimental device of three degree-of-freedom underactuated flying object. This device can control roll, pitch and yaw angles by thrust gained by two rotors. Controlling an underactuated flying object has attracted a lot of attention, due to the fact that flying object is an underactuated nonlinear system. That is, it may be possible to contribute for reducing weight, lowering the cost, and the energy saving if the system can be controlled with the number of control inputs less than the number of outputs. We have been controlling three outputs using interference of roll angle through PID control[1] and combined control[2], [3], in their researches the prediction of the flying object has not been considered for its control. PID with fixed control gains is simple and the tuning method such as Ziegler-Nichols' ultimate gain method is well known, but it seems to be insufficient for nonlinear system. Although combined control is seemed to be appropriate for the system, the derived controller is complicated. Therefore this paper explores simple switching PID control for underactuated flying object with model-based prediction and confirm the effectiveness of the prediction. The model-based prediction to settle PID control gains is executed virtually in the control interval time (one sampling time). Because the number of predictions in the virtual time, which affects computation time, is given by the prediction horizon and the virtual sampling time, the proposed method depends on computer performance

such as CPU frequency if long-range prediction horizon and virtual short sampling time are given.

This paper is organized as follows. Section 2 models the underactuated flying object of our experimental system. Section 3 shows the concept of switching PID control through model-based prediction. Section 4 gives a numerical simulation in order to check the validity of proposed method.

II. MODELING

Controlled target is three degree-of-freedom underactuated flying object(Fig. 1). The system has two inputs and three outputs, and attaches motors for rotating left and right rotor. Rotary encoders are installed for detecting roll, pitch and yaw angles. To avoid the controlled object from spinning by rotor drag torque, rotation of right rotor is the reverse rotation of left one. The equation of motion of three degree-of-freedom underactuated flying object is given as follows.

Direction of the roll angle:

$$I_r \ddot{r} + D_r \dot{r} = \tau \quad (1)$$

Direction of the pitch angle:

$$I_p \ddot{p} + D_p \dot{p} + mgL_g \sin p = L_m f_a \cos r \quad (2)$$

Direction of the yaw angle:

$$I_y \ddot{y} + D_y \dot{y} = L_m f_a \sin r \quad (3)$$

Where r , p and y are angles of each direction, m is the system weight, g is gravity acceleration, I_r , I_p and I_y are

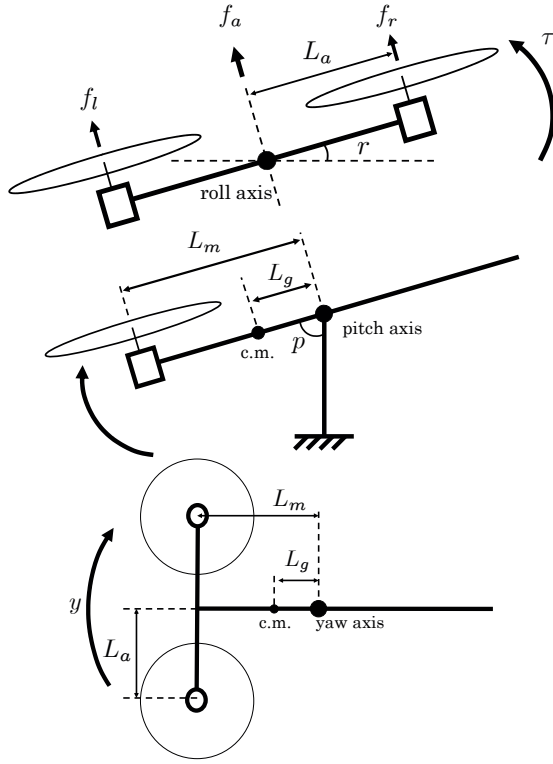


Fig. 2. Roll, pitch and yaw directions

moments of inertia of each direction, D_r , D_p and D_y are friction coefficients of each direction, L_m is distance from pitch axis to roll link and L_g is distance from pitch axis to center of mass.

Here f_a means a resultant force of f_l and f_r , τ is a moment of roll direction.

$$\begin{aligned} f_a &= f_r + f_l \\ \tau &= L_a(f_l - f_r) \end{aligned} \quad (4)$$

Where f_r and f_l are thrusts of right rotor and left one respectively. L_a is length from roll axis to the motor. The relation between rotor thrust and input voltage is expressed as follows.

$$\begin{aligned} f_r &= \omega_r^2 A = A(ku_r)^2 = Ak^2 u_1 \\ f_l &= \omega_l^2 A = A(ku_l)^2 = Ak^2 u_2 \end{aligned} \quad (5)$$

Where ω_r and ω_l are the angular velocities of right and left rotor, A is a coefficient based on the shape of rotor, u_r and u_l are the input voltages to right and left motor, k is a coefficient between voltage and angular velocity, where $\omega_r = ku_r$ and $\omega_l = ku_l$.

The equation of aerodynamical forces is shown by using the rotor angular velocity. And aerodynamical force in microscopic area is shown as follows.

$$F_n = \frac{1}{2} \rho V_R^2 S C_z \quad (6)$$

$$V_R = \omega r \quad (7)$$

Where F_n is aerodynamical force in microscopic area, ρ is air density, V_R is airspeed, S is surface area of the rotor, C_z is

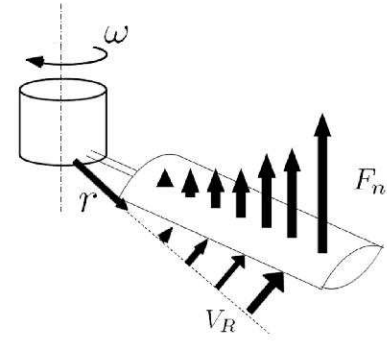


Fig. 3. Forces which act on the face of rotor

a coefficient of aerodynamical forces and r is distance from shaft. F_n is a function of r as shown in Fig. 3. Air density ρ and airspeed V_R are variables. Surface area of the rotor S , shape of rotor and rotor area which affect C_z are constants. As a result, total force of aerodynamical forces in microscopic area becomes the rotor thrust F_N ,

$$\begin{aligned} F_N &= 2 \int_0^R F_n dr \\ &= \int_0^R \rho (r\omega)^2 S C_z dr \\ &= \omega^2 S \int_0^R \rho r^2 C_z dr \\ &= \omega^2 A \end{aligned} \quad (8)$$

The coefficient A based on shape of rotor is

$$A = S \int_0^R \rho r^2 C_z dr \quad (9)$$

Where R is a radius of rotor.

Because of hardware specification, there is a limitation for the input voltages and rotors cannot rotate inverse as follows.

$$\begin{aligned} 0[V] &\leq u_r \leq 7[V] \\ 0[V] &\leq u_l \leq 7[V] \end{aligned} \quad (10)$$

III. CONTROL SYSTEM DESIGN

A. Structure of PID controller

In order to control the controlled object described in (1), (2) and (3), the control inputs for making pitch angle and yaw angle follow each reference signal are designed. The reference signal of roll angle, which means desired roll angle, is calculated so that pitch and yaw angle follows their reference signals respectively. Based on the equations of previous section, the relations between the input voltages and the output angles (r , p and y) are given as follows.

$$\begin{aligned} I_r \ddot{r} + D_r \dot{r} &= L_a A k^2 (u_2 - u_1) \\ I_p \ddot{p} + D_p \dot{p} + mg L_g \sin p &= L_m A k^2 (u_1 + u_2) \cos r \\ I_y \ddot{y} + D_y \dot{y} &= L_m A k^2 (u_1 + u_2) \sin r \end{aligned} \quad (11)$$

Where u_1 is the square of u_r , and u_2 is the square of u_l . The parameters in the equation (11) are replaced for simplicity and

shown as follows.

$$\begin{aligned} a_1 \ddot{r} + a_2 \dot{r} &= u_2 - u_1 \\ b_1 \ddot{p} + b_2 \dot{p} + b_3 \sin p &= (u_1 + u_2) \cos r \\ c_1 \ddot{y} + c_2 \dot{y} &= (u_1 + u_2) \sin r \end{aligned} \quad (12)$$

Each parameter is defined as follows.

$$\begin{aligned} a_1 &= \frac{I_r}{L_a A k^2} & b_1 &= \frac{I_p}{L_m A k^2} \\ a_2 &= \frac{D_r}{L_a A k^2} & b_2 &= \frac{D_p}{L_m A k^2} \\ c_1 &= \frac{I_y}{L_m A k^2} & b_3 &= \frac{mg L_g}{L_m A k^2} \\ c_2 &= \frac{D_y}{L_m A k^2} \end{aligned}$$

In (12), assuming that $F_r = \ddot{r}$, $F_p = \ddot{p}$ and $F_y = \ddot{y}$, the following equations are given.

$$\begin{aligned} F_r &= \frac{1}{a_1} \{-a_2 \dot{r} + (u_2 - u_1)\} \\ F_p &= \frac{1}{b_1} \{-b_2 \dot{p} - b_3 \sin p + (u_1 + u_2) \cos r\} \\ F_y &= \frac{1}{c_1} \{-c_2 \dot{y} + (u_1 + u_2) \sin r\} \end{aligned} \quad (13)$$

Considering that $z_r = u_2 - u_1$, $z_p = u_1 + u_2$ and $z_y = u_1 + u_2$ are given as ideal input voltages and F_p^* and F_y^* are given for z_p and z_y as ideal values, the following equations are obtained.

$$\begin{aligned} F_p^* &= \frac{1}{b_1} (-b_2 \dot{p} - b_3 \sin p + z_p \cos r) \\ F_y^* &= \frac{1}{c_1} (-c_2 \dot{y} + z_y \sin r) \end{aligned} \quad (14)$$

Because of $z_p = z_y$, the ideal roll angle r^* can be expressed from (14) as follows.

$$r^* = \tan^{-1} \left(\frac{c_1}{b_1} \frac{F_y^* + \frac{c_2}{c_1} \dot{y}}{F_p^* + \frac{b_2}{b_1} \dot{p} + \frac{b_3}{b_1} \sin p} \right) \quad (15)$$

And the ideal values F_p^* and F_y^* are generated by using the following PID controller through the reference signals p_d and y_d for pitch and yaw angle.

$$\begin{aligned} F_p^* &= -K_{P2}(p - p_d) - K_{I2} \int (p - p_d) - K_{D2} \dot{p} \\ F_y^* &= -K_{P3}(y - y_d) - K_{I3} \int (y - y_d) - K_{D3} \dot{y} \end{aligned} \quad (16)$$

Where it is assumed that the reference signals p_d and y_d are constant. Moreover, in this paper the ideal value F_r^* to follow the ideal roll angle r^* is given by PD control.

$$F_r^* = -K_{P1}(r - r^*) - K_{D1}(\dot{r} - \dot{r}^*) \quad (17)$$

Replacing F_r and F_p in (13) to F_r^* in (17) and F_p^* in (16), the following relations of the input voltages are given.

$$\begin{aligned} u_2 - u_1 &= a_1 F_r^* + a_2 \dot{r} = z_r \\ u_1 + u_2 &= \frac{b_1 F_p^* + b_2 \dot{p} + b_3 \sin p}{\cos r} = z_p \end{aligned} \quad (18)$$

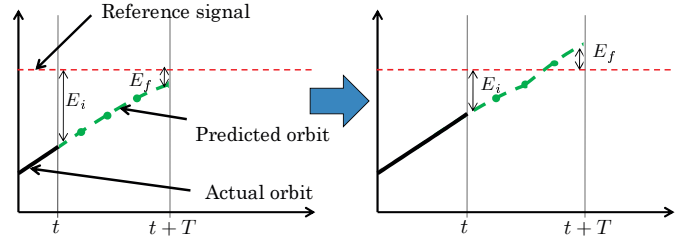


Fig. 4. Concept of model-based prediction for switching PID gains

From (18), u_1 and u_2 are obtained as follows.

$$u_1 = \frac{z_p - z_r}{2}, \quad u_2 = \frac{z_r + z_p}{2} \quad (19)$$

Because of $u_1 = u_r^2$ and $u_2 = u_l^2$, u_r and u_l are given as follows.

$$u_r = \sqrt{u_1}, \quad u_l = \sqrt{u_2} \quad (20)$$

Since the experimental device cannot carried out the reverse rotation, u_r and u_l are only positive signal. If u_r and u_l are negative signal, u_r and u_l are set to be zero.

B. Concept of switching PID gains and model-based prediction

In the previous research[1], PID gains have been fixed for controlling the experimental device. This paper aims at improving the control performance through switching PID gains and model-based prediction. Therefore the behavior of the controlled model described in (12) with the control input (20) is virtually calculated between each sampling time. The calculated behavior is given in the prediction horizon which is from the time t to $t + T$ as shown in Fig.4. Because the behavior of the model is calculated at each sampling period, the proposed controller checks the condition of whether PID gains should be switched or not at each sampling period. The procedure about switching condition of PID gains is shown below. In other words, a virtual error between the output and the reference signal is calculated by the model-based prediction executed virtually.

- 1) The initial error E_i between the output and the reference signal is calculated at the beginning of prediction horizon.
- 2) The final error E_f is calculated at the end of prediction horizon.
- 3) The following prediction error E_p is calculated.

$$E_p = |E_i| - |E_f| \quad (21)$$

- 4) PID gains are switched using E_p .
- 5) Control inputs are generated by the switched gains. The switched gains are used in the next prediction.

For simplicity, this paper explores switching the proportional gain of pitch angle only. In the proposed method, two cases for switching gain are designed. Supposing $E_p < 0$, it seems that the actual error will become larger. In such a case, the proposed method switches the proportional gain of pitch angle (reverses its sign) so as to make the actual error smaller.

$$K_{P2_i} = -1.0 * K_{P2_{i-1}} \quad (22)$$

TABLE I. PARAMETERS OF CONTROLLED OBJECT

a_1	=	15.9	b_1	=	43.7
a_2	=	1.02	b_2	=	1.02
c_1	=	24.7	b_3	=	36.1
c_2	=	1.84			

TABLE II. INITIAL PID GAINS

	Roll($i = 1$)	Pitch($i = 2$)	Yaw($i = 3$)
K_{Pi}	0.03	0.03	0.01
K_{Ii}	0.0	0.001	0.001
K_{Di}	0.3	0.3	0.3

TABLE III. INITIAL VALUES OF OUTPTS AND THEIR REFERENCE SIGNALS

	Roll	Pitch	Yaw
Initial signal [rad]	0.0	1.23	0.0
Reference signal [rad]	r^*	$\frac{\pi}{2}$	0.1

Also, supposing $|E_f| > 0.08 * |p_d|$, it seems that it does not follow the reference signal sufficiently. In this case, the proportional gain is switched so that the absolute value of its gain gradually becomes larger in order to follow the reference signal.

$$K_{P2i} = K_{P2i-1} + \frac{K_{P2i-1}}{|K_{P2i-1}|} * 0.03 \quad (23)$$

IV. NUMERICAL EXAMPLES

This section shows a numerical example to verify the validity of proposed method. The parameters of controlled object described in (12) are given in Table I. In this paper, the proposed method is focused on controlling the pitch angle only because of basic exploration. The numerical example has two simulation results which are the cases of the proposed method and fixed PID gains. The initial PID gains for the proposed method and the fixed PID gains are the same and given in Table II. The initial values of outputs and their reference signals are given in Table III. r^* is calculated by (15) in order to make the pitch and the yaw angles follow their reference signals. In numerical simulation, the behavior of controlled object is solved by Runge-Kutta method on C programming language. The sampling time (step size) for the controlled object is assumed to be defined by 0.01 [s]. And the virtual sampling time for the model-based prediction is set to 0.6 [s] and the prediction horizon is 30 [s], that is, the model-based prediction is executed 50 times in 0.01 [s]. For the case of applying to the experimental device, it is easy to reduce the computation of the model-based prediction because the virtual sampling time and the prediction horizon can be designed.

Fig.5, Fig.6, Fig.7 and Fig.8 show the control results of yaw angles and roll angles respectively. For each angle the proposed method also uses fixed PID gain. Fig.9 and Fig.10 show the errors to switch the proportional gain for pitch angle. And Fig.11 shows the switched proportional gain for pitch angle. From Fig.12 and Fig.13, which are the control results of pitch angle, it finds that the proposed method can reduce the maximum value of overshoot comparing to the case of using

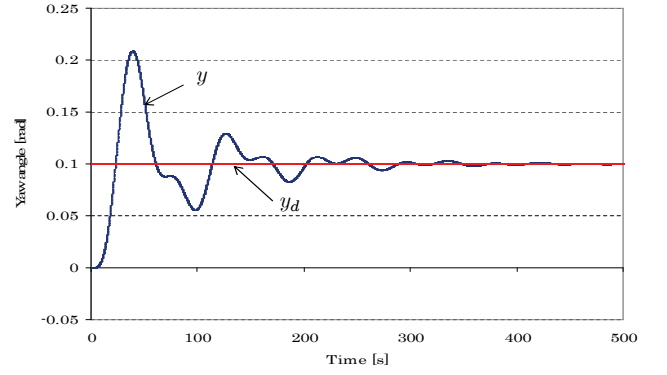


Fig. 5. Yaw angle by the proposed method

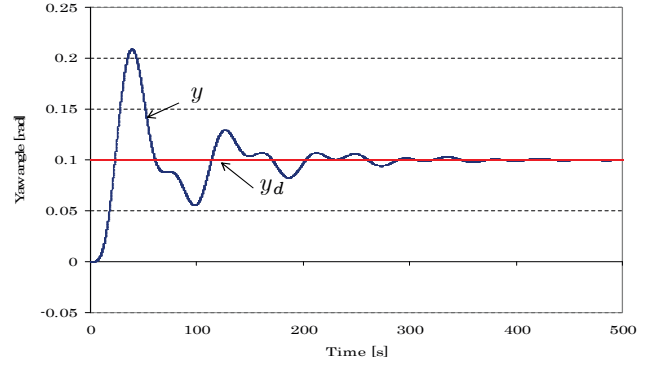


Fig. 6. Yaw angle by fixed PID controller

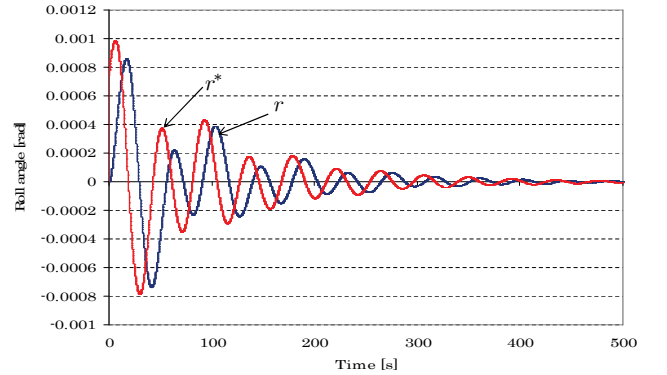


Fig. 7. Roll angle by the proposed method

fixed PID gains. That is, it can find that the proposed method is superior to the fixed PID controller because of using switching mechanism through the model-based prediction.

V. CONCLUSION

In our previous research, the fixed PID controller and the combined controller have been applied to the underactuated flying object. In order to overcome a limit of control performance and aim at a simplification of controller, this paper explored a switching PID control method through model-based prediction. With comparison to the previous methods, a simple controller was obtained and it showed a possibility of better control performance.

As future works, there are an application of the proposed method to the experimental device, and investigation into

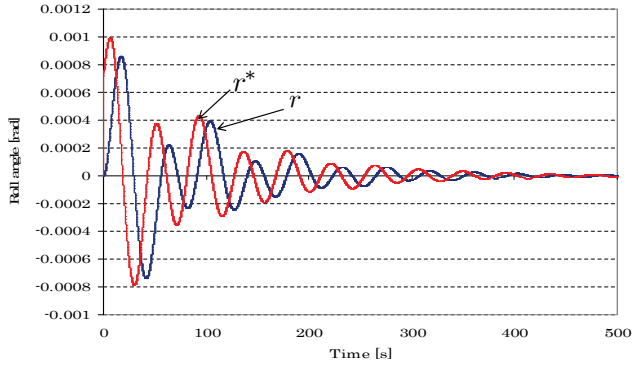


Fig. 8. Roll angle by fixed PID controller

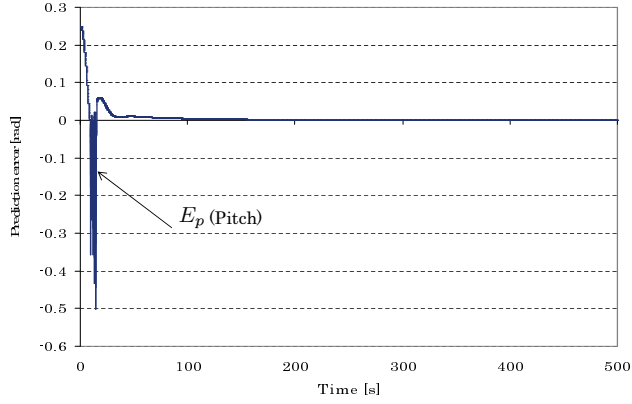


Fig. 9. Prediction error E_p (one of the switching conditions)

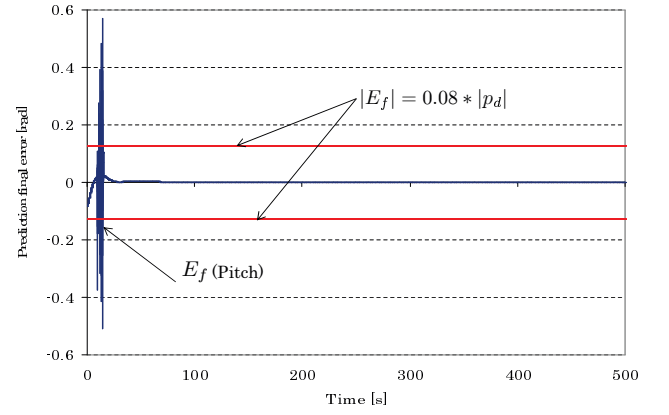


Fig. 10. Final error E_f (one of the switching conditions)

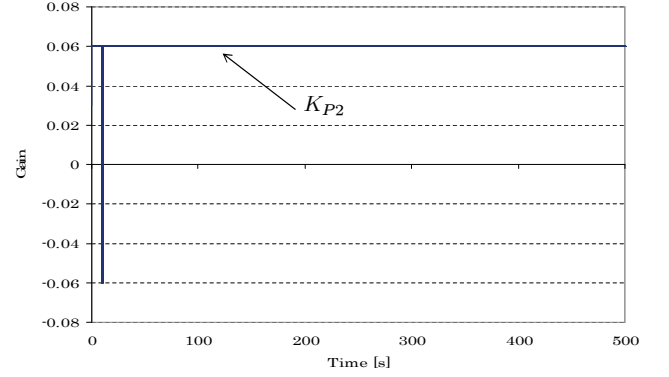


Fig. 11. Switched proportional gain for pitch angle

the switching condition for various reference signals and the relation between the virtual sampling time and the prediction horizon for model-based prediction. Moreover the proposed method should be improved aiming at the quick output responses and convergence to reference signal.

REFERENCES

- [1] K. Yoshikawa, A. Yanou and M. minami, *Position/Orientation Control of an Underactuated Flight Object Based on Two Degree-of-Freedom PID Control*, Proc. of SICE Annual Conference, pp.1204-1209, 2012.
- [2] M. Deng, A. Inoue and T. Shimizu, *Combined Attitude Control of an Underactuated Helicopter Experimental System*, Proc. of UKACC International Conference on Control 2008, 2008.
- [3] M. Deng, A. Inoue, H. Yu, T. Shimizu, *Combined attitude control application of an underactuated helicopter experimental system*, Int. J. Advanced Mechatronic Systems, Vol.1, No.4, pp.280-287, 2009.
- [4] M. Deng, A. Inoue, T. Kishida, and N. Ueki, *Modeling and control of an underactuated helicopter experimental system*, Proc. of the 26th Chinese Control Conference, pp.648-651, 2007.
- [5] A. Inoue, M. Deng, T. Shimizu and T. Harima, *Experimental study on attitude control system design of a helicopter experimental system*, Proc. of International Control Conference (ICC2006), 2006.
- [6] A. Kojima, T. Ohtsuka, *An Introduction to Model Predictive Control*, Journal of the Society of Instrument and Control Engineers, Vol.42, No.4, pp310-312, 2003.

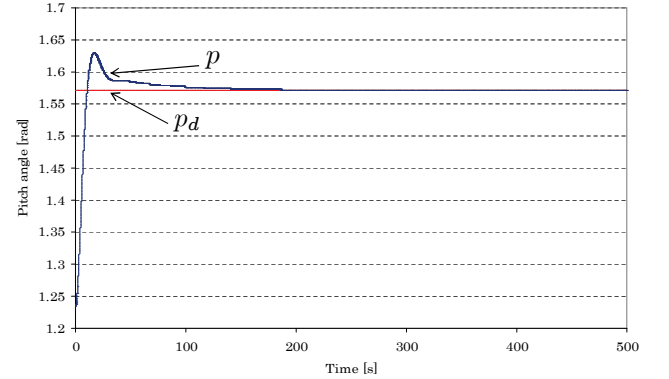


Fig. 12. Pitch angle by the proposed method

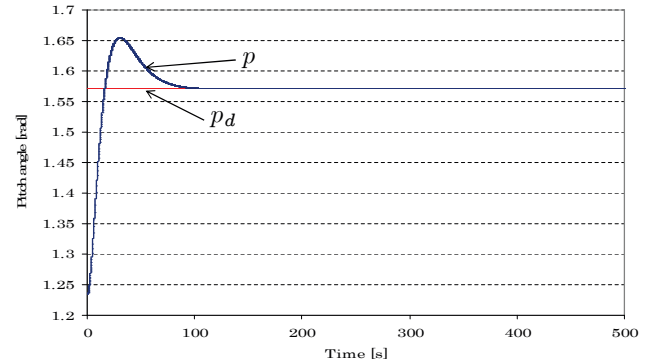


Fig. 13. Pitch angle by fixed PID controller