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Application of Self-tuning Generalized Predictive Control to Temperature Control Experimental Device of Aluminum Plate

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Abstract. This paper considers an application of self-tuning generalized predictive control (ST-GPC) to a temperature control experimental device of aluminum plate. In our research, although two DOF GPC can achieve to design the output response and the disturbance response independently, model parameters such as thermal conductivity had been fixed in the experiment. Therefore this paper applies ST-GPC to the experimental device and verifies the validity of the proposed controller. Moreover the control result, in the case of the controller constructed by the identified parameters though parameter identification law, is shown.
Keywords: GPC, Self-tuning, Temperature Control

1. Introduction

In general, the characteristics of the control object is already-known, when we design it, namely, mathematical model which describes the control object and parameters included in it are already-known because the control system must be designed to suit the characteristics of the controlled object. However, we consider the actual controlled object, the characteristic varies influence of the environment or operating conditions and often includes elements that cannot be described by mathematical model.
We derived the model as physical parameters such as the specific heat are already-known and confirmed the effectiveness of the proposed method by performing two degree-of-freedom configuration of the generalized predictive control system in the previous study[2][4].
However, in the real environment, the parameters of the model of the control object is changed due to temperature change. Therefor, we constructed Self-tuning Generalized Predictive Control system(STGPC system) which can sequentially update the control law according to add the sequential type of parameter estimation law. By this control law, if the parameters of the model of control object change we can update the control law by doing sequential estimation. Moreover, we report the result of simulation and experimental that applied the control law which was obtained to the aluminum plate temperature experimental device and that model.

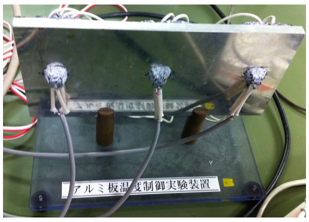

Fig. 1. Aluminum Plate Temperature Control Experimental Device

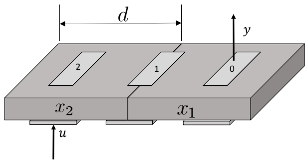

Fig. 2. Aluminum Plate Model

Table 1 Aluminum Plate Model Parameters

Density of aluminum	:	$2700[kg/m^3]$
Specific heat of aluminum	:	$917[J/kgK]$
Heat transfer coefficient	:	$25[W/m^2k]$
Thermal conductivity	:	$238[W/mK]$
Width of plate	:	$250[mm]$
Thickness of plate	:	$10[mm]$
Length of plate	:	$120[mm]$
Output range of heater	:	$40[W]$

2. Model

Firstly, the model of experimental device shown in Fig.1 is considered shown in Fig.2. The state quantity relating to the temperature of each part in Fig.2 (x_1, x_2) are defined as follows for the $i = 1, 2$.

$$x_i = T_i - T_0 \tag{1}$$

Where T_n is temperature of each part of the aluminum plate, T_o is ambient temperature. The parameters of aluminum plate model are given in Table I. Then, three laws are used in the derivation of the model. Fourier's law of heat conduction is given by,

$$q = -\lambda_f(d\theta/dn) \tag{2}$$

Where q $[W/m^2]$ is heat flow ratio, λ_f $[W/mK]$ is thermal conductivity, $d\theta/dn$ $[K/m]$ is temperature gradient. Newton's law of cooling is given by,

$$q = \alpha(\theta_s/\theta_f) \tag{3}$$

h $[W/m^2K]$ is heat transfer coefficient.
The law of heat conduction is given by,

$$dQ = mc \cdot d\theta \tag{4}$$

c $[J/kgK]$ is specific heat, m $[kg]$ is mass of each part. Then the following equations of the system are obtained from the state variables of Eq.(1) and the laws of Eq.(2),(3) and (4).

$$mc\frac{d(T_1 - T_0)}{dt} = -\left\{\alpha(T_1 - T_0)S_a + \lambda_f\frac{T_1 - T_2}{d}S_b\right\}$$
$$mc\frac{d(T_2 - T_0)}{dt} = -\left\{\alpha(T_2 - T_0)S_a + \lambda_f\frac{T_2 - T_1}{d}S_b\right\} + u$$

By using the state quantity defined above, when the input that is given to the model of the controlled object put u, state-space representation of the related formula of temperature change are given as follows.

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \frac{1}{mc}\begin{bmatrix} -\left(\alpha S_a + \frac{\lambda_f S_b}{d}\right) & \frac{\lambda_f S_b}{d} \\ \frac{\lambda_f S_b}{d} & -\left(\alpha S_a + \frac{\lambda_f S_b}{d}\right) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{mc} \end{bmatrix} u \tag{5}$$

The sampling time put Δt and assume that it can be approximated as Eq.(6), Eq.(5) can be expressed as Eq.(7).

$$\frac{dx_i}{dt} \approx \frac{x_i(t+\Delta t) - x_i}{\Delta t} \tag{6}$$

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \quad y(t) = \mathbf{C}\mathbf{x}(t) \tag{7}$$

Where, $\mathbf{x}(t) = \left[x_1(t)\ x_2(t)\right]^T$. $y(t)$ is the value of the temperature change from ambient temperature of a portion x_1, and the output of the controlled object.

$$\mathbf{A} = \begin{bmatrix} 1 - \frac{\Delta t}{mc}\left(\alpha S_a + \frac{\lambda_f S_b}{d}\right) & \frac{\lambda_f S_b \Delta t}{d} \\ \frac{\lambda_f S_b \Delta t}{d} & 1 - \frac{\Delta t}{mc}\left(\alpha S_a + \frac{\lambda_f S_b}{d}\right) \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 0 \\ \frac{\Delta t}{mc} \end{bmatrix},\ \mathbf{C} = \left[1\ 0\right]$$

From the above, if you define waste time lag k_m, model of Fig.2 is represented by the transfer function of the following.

$$y(k) = \frac{z^{-km}B[z^{-1}]}{A[z^{-1}]}u(k) \tag{8}$$

Where, Substitution of $A[z^{-1}]$, $B[z^{-1}]$ and $a = \frac{\alpha S_a \Delta t}{mc} - 1$, $b = \frac{2S_b\Delta t}{mc}$, $c = \left(\frac{\Delta t}{mc}\right)^2 \cdot \frac{S_b}{d}$, it can be described as follows.

$$A[z^{-1}] = 1 + (b\lambda_f + 2a)z^{-1} + (ab\lambda_f + a^2)z^{-2} \quad B[z^{-1}] = c\lambda_f \quad km = 2$$

In the following it define $a_1 = b\lambda_f + 2a$, $a_2 = ab\lambda_f + a^2$ and $b_0 = c\lambda_f$.

3. Controller

First, we derive the prediction formula for the control target of the deviation system. If $y(k)$ is equal to the target value r in the steady state, the relationship between the steady-state value of input u_∞ and output y_∞ are given as follows.

$$A[z^{-1}]y_\infty = z^{-k_m}B[z^{-1}]u_\infty \tag{9}$$

The deviations from these steady-state value are defined as $\tilde{y}(k) = y(k) - y_\infty$ and $\tilde{u}(k) = u(k) - u_\infty$, and configure the following deviations system.

$$A[z^{-1}]\tilde{y}(k) = z^{-k_m}B[z^{-1}]\tilde{u}(k) \tag{10}$$

To the eq.(9), to derive a prediction formula $\hat{\tilde{y}}(k+j|t)$ by using the following Diophantine equation respect to $j = 1, 2, \cdots, N_2$.

$$1 = A[z^{-1}]E_j[z^{-1}] + z^{-j}F_j[z^{-1}] \tag{11}$$

$$E_j[z^{-1}]B[z^{-1}] = R_j[z^{-1}] + z^{-j}S_j[z^{-1}] \tag{12}$$

Where, $E_j[z^{-1}], F_j[z^{-1}], R_j[z^{-1}]$ and $S_j[z^{-1}]$ are given as follows, n and m are the degree of $A[z^{-1}]$ and $B[z^{-1}]$.

$$E_j = e_0 + e_1 z^{-1} + e_2 z^{-2} + \cdots + e_{j-1} z^{-j+1},\ F_j = f_0 + f_1 z^{-1} + f_2 z^{-2} + \cdots + f_n z^{-n}$$
$$R_j = r_0 + r_1 z^{-1} + r_2 z^{-2} + \cdots + r_{j-1} z^{-j+1},\ S_j = s_0 + s_1 z^{-1} + s_2 z^{-2} + \cdots + s_m z^{-m}$$

Where, R_j represent the following matrix.

$$\mathbf{R} = \begin{bmatrix} r_0 & 0 & \cdots\cdots & 0 \\ r_1 & r_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots\ \ddots & \vdots \\ r_{N_2-1} & & \ddots\ r_0 & 0 \\ r_{N_2-1} & r_{N_2-1} & \cdots\ r_1 & r_0 \end{bmatrix}$$

In this case, when defining the $\hat{\tilde{y}}(k+j|t)$ is the predicted value of the output deviation system(eq.(12)) at the time of k, it is possible to give the following prediction equation respect to $j = 1, 2, \cdots, N_2$.

$$\hat{\tilde{\mathbf{Y}}} = \mathbf{R}\tilde{\mathbf{U}} + \mathbf{H} \tag{13}$$

But, in this case, $\hat{\tilde{\mathbf{Y}}} = [\hat{\tilde{y}}(k+1|k), \hat{\tilde{y}}(k+2|k), \cdots, \hat{\tilde{y}}(k+N_2|k)]^T$,$\tilde{\mathbf{U}} = [\tilde{u}(k), \cdots, \tilde{u}(k+N_2-1)]^T$,$\mathbf{H} = [h_1(k), \cdots, h_{N_2}(k)]^T$ and $h_j(k) = F_j[z^{-1}]\tilde{y}(k) + z^{-km}S_j[z^{-1}]\tilde{u}(k)$. Also, consider the following evaluation function respect to eq.(10)) under the conditions of $\tilde{y}(k+j) = \hat{\tilde{y}}(k+j|t)$.

$$J = \sum_{j=N_1}^{N_2} \tilde{y}^2(k+j) + \sum_{j=1}^{N_u} \lambda \tilde{u}^2(k+j-1) = \left(\mathbf{R}\tilde{\mathbf{U}} + \mathbf{H}\right)^T \left(\mathbf{R}\tilde{\mathbf{U}} + \mathbf{H}\right) + \lambda \tilde{\mathbf{U}}^T \tilde{\mathbf{U}}$$

Where $[N_1, N_2]$ is the prediction horizon, $[1, N_u]$ is controlled horizon, λ is the weight factor of the control input. Also, in this paper, we defined as $N_u = N_2$.
To obtain the following formula to partial differential the J with respect to $\tilde{U}$.

$$\tilde{\mathbf{U}} = -(\mathbf{R}^T\mathbf{R} + \lambda\mathbf{I})^{-1}\mathbf{R}^T\mathbf{H} \tag{14}$$

From eq.(14), control law for eq.(8) is given by the following equation.

$$u(k) = H_0[z^{-1}]r(k) - F_0[z^{-1}]y(k) \tag{15}$$

Where,

$$H_0[z^{-1}] = \frac{F_p[z^{-1}] + (1 + z^{-k_m}S_p[z^{-1}])K}{1 + z^{-k_m}S_p[z^{-1}]},\ F_0[z^{-1}] = \frac{F_p[z^{-1}]}{1 + z^{-k_m}S_p[z^{-1}]},\ K = \frac{A[1]}{B[1]}$$

$$F_p[z^{-1}] = \sum_{j=N_1}^{N_2} p_j F_j[z^{-1}],\ S_p[z^{-1}] = \sum_{j=N_1}^{N_2} p_j S_j[z^{-1}],\ [p_{N_1}, \cdots, p_{N_2}] = [1, 0, \cdots, 0](\mathbf{R}^T\mathbf{R} + \lambda\mathbf{I})^{-1}\mathbf{R}^T$$

4. Algorithm

It is considered as control law obtained by eq.(15) that the parameters of the controlled object are known. So, it configure the self-tuning controller by adding the following parameter estimation law.

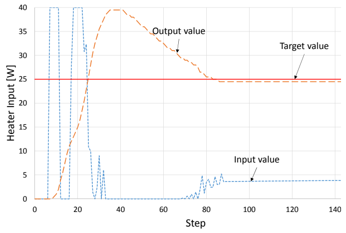

Fig. 3. STGPC Experiment Resu

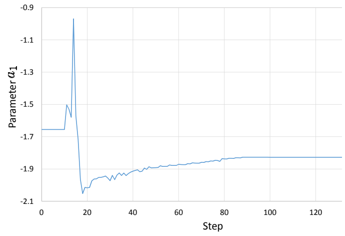

Fig. 4. Identified Parameter a_1

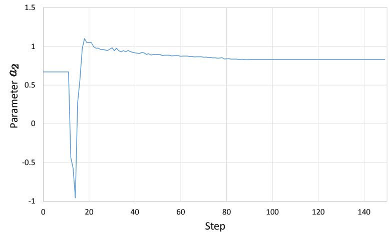

Fig. 5. Identified Parameter a_2

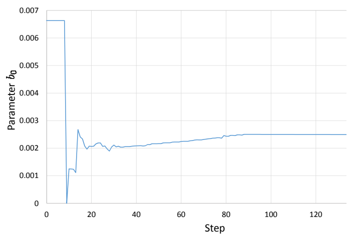

Fig. 6. Identified Parameter b_0

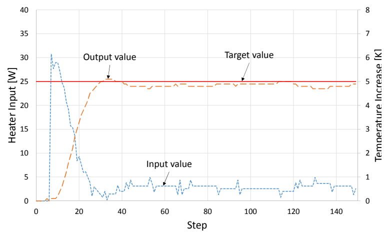

Fig. 7. GPC Experiment Result for Tuned Parameters

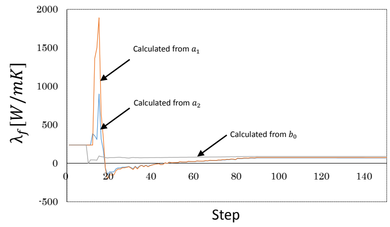

Fig. 8. Calculated Result from Each of dates

$$\Gamma_k = \Gamma_{k-1} - \frac{\Gamma_{k-1}\varphi_k\varphi_k^T\Gamma_{k-1}}{1+\varphi_k^T\Gamma_{k-1}\varphi_k} \qquad \hat{\theta}_k = \hat{\theta}_{k-1} - \frac{\Gamma_{k-1}\varphi_k}{(1+\varphi_k^T\Gamma_{k-1}\varphi_k)}(y_k - \hat{\theta}_{k-1})$$

$$\varphi_k = \left[-y_{k-1}\cdots -y_{k-n}\ u_{k-k_m}\cdots u_{k-k_m-m}\right]^T \qquad \hat{\theta}_k = \left[\hat{a_1}(k)\cdots\hat{a_n}(k)\ \hat{b_0}(k)\cdots\hat{b_m}(k)\right]^T$$

$$\Gamma_0 = \alpha I, \alpha > 0$$

5. Experiment

From the simulation results, it can be confirmed that the parameters by the controller which is designed converge to true value while the target value follow-up is achieved. Also, we chose the design parameters of the controller as $N_1 = 1, N_2 = N_u = 5, \lambda = 0.005$ at that time. The parameters which is the model of the controlled object are $a_1 = -1.655, a_2 = 0.6696$ and $b_0 = 0.00663$ as true values and when performing the self-tuning gave 0.8 times the true value. $\Gamma(0)$ for parameter estimation law we chose $100I$. Based on the results of the simulation, it shows the experimental results. The result configure the controller when the parameters of the controlled object are as $a_1 = -1.655, a_2 = 0.6696$ and $b_0 = 0.00663$ is shown in Fig.8.

From this result, it see that the steady-state deviation occur between the target value and the output value. It is considered that it is deviation due by modeling error. Next, the experimental results when it have configured the self-tuning controller in Fig.9-Fig.12. In this case, it give the nominal value ($a_1 = -1.655, a_2 = 0.6696, b_0 = 0.00663$) of the controlled object, and the other conditions are the same as in the simulation.

From Fig.9, it can be seen that although the overshoot is observed in transient state , as the the estimated parameter converges to a constant value, to achieve the target value follow-up. The reason is considered

that the modeling error is reduced. So it shows the experimental results in the case of configuring the controller using the values ($a_1 = -1.828, a_2 = 0.829, b_0 = 0.00249$) of the parameter estimation results in Fig.10, 11 and 12 in Fig.13. This result indicates that it has achieved the target value follow-up. In other words, it see that the parameters of the experimental device as control object is properly determined. Furthermore, by using the data of the obtained parameters, there were calculated thermal conductivity from the $a_1 = b\lambda_f + 2a$, $a_2 = ab\lambda_f + a^2$ and $b_0 = c\lambda_f$.
It shows the result of the above calculation in Fig.14 λ_f which is calculated from a_1 is 70.4, λ_f which is calculated from a_2 is 74.9, λ_f which is calculated from b_0 is 89.5. λ_f as the theoretical value is 238. So, large difference occurs. The reason is considered that it does not take into account the entry and exit of the heat by radiation in the model that is constructed in this time.

6. conclusion

In this paper, we estimate the parameters of the controlled object by sequential estimation algorithm, and construct self-tuning generalized predictive control system using the value. In addition, it experimented and simulate by using aluminum plate temperature control experimental device, it was confirmed the effectiveness of this method. As future works, there are consideration of the estimation method of physical parameters based on the input and output data[3][5] and application of this approach to the two-dimensional mode of the aluminum plate[4].

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