Dynamical Modeling of Humanoid with Nonlinear Floor Friction

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Abstract: Biped locomotion created by a controller based on Zero-Moment Point [ZMP] known as reliable control method looks different from human's walking on the view point that ZMP-based walking does not include falling state, and it's like monkey walking because of knee-bended walking profiles. However, the walking control that does not depend on ZMP is vulnerable to turnover. Therefore, keeping the event-driven walking of dynamical motion stable is important issue for realization of human-like natural walking. In this research, a walking model of humanoid robot including slipping, bumping, surface-contacting and point-contacting of foot is discussed, and its dynamical equation is derived by the Extended NE method. And we considering the siliping of the foot of humanoid robot when the friction coefficient is small to create this walking model. In this paper we will introduce the new model which including the slipping supporting foot of humanoid robot and verify the model.

Keywords: Humanoid, siliping, friction, Bipedal, Dynamical, Extended Newton-Euler Method

1 INTRODUCTION

Human beings have acquired an ability of stable bipedal walking in evolving repetitions so far. From a view point of making a stable controller for the bipedal walking based on knowledge of control theory, it looks not easy because of the dynamics with high nonlinearity and coupled interactions between state variables with high dimensions. Therefore how to simplify the complicated walking dynamics to help construct stable walking controller has been studied intensively.

Avoiding complications in dealing directly with true dynamics without approximation, inverted pendulum has been used frequently for making a stable controller[1] [2] [3], simplifying the calculations to determine input torque. Further, linear approximation having the humanoid being represented by simple inverted pendulum enables researchers to realize stable gait through well-known control strategy[4] [5] [6].

Our research has begun from a view point of [7] [8] as aiming to describing gait's dynamics as correctly as possible, including point-contacting state of foot and toe, slipping of the foot and bumping. We discuss the dynamics of whole-body humanoid that contains head, waist and arms. And that what we think more important is that the dimension of dynamical equation will change depending on the walking gait's varieties, which has been discussed by [9] about concerning one legged hopping robot. In fact, this kind of dynamics with the dimension number of state variables varying by the result of its dynamical time transitions that are out of the arena of control theory that discusses how to control a system with fixed states' number. Further the tipping over motion has been called as non-holonomic dynamics that includes a joint without inputting torque, i.e., free joint.

Meanwhile, landing of the heel or the toe of lifting leg in

the air to the ground makes a geometrical contact. Based on [10]. We derive the dynamics of humanoid which is simulated as a serial-link manipulator including constraint motion and slipping motion by using the Extended Newton-Euler Method [11].

The conventional method of the NE could be applied to a robot having an open loop serial linkage structure, but the motion of hand was limited to motions without contacting external world. The NE method has not been formulated although it was very important for a robot that works under a premise that it must be contacted with the environment when the robot was doing some grinding work or assembling work. For this point, the extended NE method was proposed in [11] that is as same as the research of [12], in terms of that the constraints are strictly satisfied. Meanwhile, the constraint force which can be included in the iterative calculation of the NE method by calculating the constraint force by a substitution method [13].

In this research that based on [14], a walking model of humanoid robot including slipping, bumping, surfacecontacting and point-contacting of foot is discussed, and its dynamical equation is derived by the Extended NE method. And we considering the siliping of the foot of humanoid robot when the friction coefficient is small (on the ice, snow, or wet road) to create this walking model.

In this paper we will introduce the new model which including the slipping supporting foot of humanoid robot and verify the model.

2 DYNAMICAL WALKING MODEL

We discuss a biped robot whose definition is depicted in Fig. 1. Table 1 indicates length l_i [m], mass m_i [kg] of



Fig. 1. Definition of humanoid's link, joint and whole body

links and joints' coefficient of viscous friction d_i [N·m·s/rad], which are decided based on [15]. This model is simulated as a serial-link manipulator having ramifications and represents rigid whole body—feet including toe, torso, arms and so on—by 17 degree-of-freedom. Though motion of legs is restricted in sagittal plane, it generates varieties of walking gait sequences since the robot has flat-sole feet and kicking torque. In this paper, one foot including link-1 is defined as "supporting-foot" and another foot including link-7 is defined as "floating-foot" or "contacting-foot" according to the walking state.

In this paper, we derive the equation of motion following by NE formulation [?]-[?]. So we must consider the structure of the supporting-foot with two situations. When the supporting-foot is constituted by rotating joint. We first have to calculate relations of positions, velocities and accelerations between links as forward kinematics procedures from bottom link to top link. Serial link's angular velocity ${}^{i}\omega_{i}$, angular acceleration ${}^{i}\dot{\omega}_{i}$, acceleration of the origin ${}^{i}\ddot{p}_{i}$ and acceleration of the center of mass ${}^{i}\ddot{s}_{i}$ based on Σ_{i} fixed at *i*-th link are obtained as follows.

$${}^{i}\boldsymbol{\omega}_{i} = {}^{i-1}\boldsymbol{R}_{i}^{T} {}^{i-1}\boldsymbol{\omega}_{i-1} + \boldsymbol{e}_{z_{i}}\dot{q}_{i}$$

$$\tag{1}$$

$$\dot{\boldsymbol{\omega}}_{i} = {}^{i-1}\boldsymbol{R}_{i}^{T\ i-1}\dot{\boldsymbol{\omega}}_{i-1} + \boldsymbol{e}_{z}\ddot{q}_{i} + {}^{i}\boldsymbol{\omega}_{i} \times (\boldsymbol{e}_{z_{i}}\dot{q}_{i})$$
 (2)

$$\ddot{\boldsymbol{p}}_{i} = {}^{i-1}\boldsymbol{R}_{i}^{T} \Big\{ {}^{i-1}\ddot{\boldsymbol{p}}_{i-1} + {}^{i-1}\dot{\boldsymbol{\omega}}_{i-1} \times {}^{i-1}\hat{\boldsymbol{p}}_{i} \\ + {}^{i-1}\boldsymbol{\omega}_{i-1} \times ({}^{i-1}\boldsymbol{\omega}_{i-1} \times {}^{i-1}\hat{\boldsymbol{p}}_{i}) \Big\}$$
(3)

$${}^{i}\ddot{\boldsymbol{s}}_{i} = {}^{i}\ddot{\boldsymbol{p}}_{i} + {}^{i}\dot{\boldsymbol{\omega}}_{i} \times {}^{i}\hat{\boldsymbol{s}}_{i} + {}^{i}\boldsymbol{\omega}_{i} \times ({}^{i}\boldsymbol{\omega}_{i} \times {}^{i}\hat{\boldsymbol{s}}_{i})$$
(4)

Then if the supporting-foot is constituted by prismatic joint. We will switch the equations as the following.

Table 1. Physical parameters			
Link	l_i	m_i	d_i
Head	0.24	4.5	0.5
Upper body	0.41	21.5	10.0
Middle body	0.1	2.0	10.0
Lower body	0.1	2.0	10.0
Upper arm	0.31	2.3	0.03
Lower arm	0.24	1.4	1.0
Hand	0.18	0.4	2.0
Waist	0.27	2.0	10.0
Upper leg	0.38	7.3	10.0
Lower leg	0.40	3.4	10.0
Foot	0.07	1.3	10.0
Total	1.7	64.2	

$${}^{i}\boldsymbol{\omega}_{i} = {}^{i-1}\boldsymbol{R}_{i}^{\mathrm{T}\ i-1}\boldsymbol{\omega}_{i-1} \tag{5}$$
$${}^{i}\boldsymbol{\omega}_{i} = {}^{i-1}\boldsymbol{R}_{i}^{\mathrm{T}\ i-1}\boldsymbol{\omega}_{i-1} \tag{6}$$

$$\dot{\boldsymbol{\omega}}_{i} = {}^{i-1}\boldsymbol{R}_{i}^{T} {}^{i-1}\dot{\boldsymbol{\omega}}_{i-1}$$
(6)
$$\ddot{\boldsymbol{p}}_{i} = {}^{i-1}\boldsymbol{R}_{i}^{T} \Big\{ {}^{i-1}\ddot{\boldsymbol{p}}_{i-1} + {}^{i-1}\dot{\boldsymbol{\omega}}_{i-1} \times {}^{i-1}\hat{\boldsymbol{p}}_{i} \\ + {}^{i-1}\boldsymbol{\omega}_{i-1} \times ({}^{i-1}\boldsymbol{\omega}_{i-1} \times {}^{i-1}\hat{\boldsymbol{p}}_{i}) \Big\} \\ + 2({}^{i-1}\boldsymbol{R}_{i}^{Ti-1}\boldsymbol{\omega}_{i-1}) \times (\boldsymbol{e}_{z}\dot{q}_{i}) + \boldsymbol{e}_{z}\ddot{q}_{i}$$
(7)

$${}^{i}\ddot{s}_{i} = {}^{i}\ddot{p}_{i} + {}^{i}\dot{\omega}_{i} \times {}^{i}\hat{s}_{i} + {}^{i}\omega_{i} \times ({}^{i}\omega_{i} \times {}^{i}\hat{s}_{i})$$
(8)

Here, ${}^{i-1}\mathbf{R}_i$ means orientation matrix, ${}^{i-1}\hat{\mathbf{p}}_i$ represents position vector from the origin of (i - 1)-th link to the one of *i*-th, ${}^i\hat{s}_i$ is defined as gravity center position of *i*-th link and \mathbf{e}_{z_i} is unit vector that shows rotational axis of *i*-th link. However, velocity and acceleration of 4-th link transmit to 8-th link and ones of 10-th link transmit to 11-th, 14-th and 17-th link directly because of ramification mechanisms.

After the above forward kinematic calculation has been done, contrarily inverse dynamical calculation procedures is the next from top to base link. Newton equation and Euler equation of *i*-th link are represented by Eqs. 9, 10 when ${}^{i}I_{i}$ is defined as inertia tensor of *i*-th link.

$${}^{i}\boldsymbol{f}_{i} = {}^{i}\boldsymbol{R}_{i+1}{}^{i+1}\boldsymbol{f}_{i+1} + m_{i}{}^{i}\ddot{\boldsymbol{s}}_{i}$$

$${}^{i}\boldsymbol{n}_{i} = {}^{i}\boldsymbol{R}_{i+1}{}^{i+1}\boldsymbol{f}_{i+1} + {}^{i}\boldsymbol{I}_{i}{}^{i}\dot{\boldsymbol{\omega}}_{i} + {}^{i}\boldsymbol{\omega}_{i} \times ({}^{i}\boldsymbol{I}_{i}{}^{i}\boldsymbol{\omega}_{i})$$
(9)

$$+ {}^{i}\hat{s}_{i} \times (m_{i}{}^{i}\ddot{s}_{i}) + {}^{i}\hat{p}_{i+1} \times ({}^{i}R_{i+1}{}^{i+1}f_{i+1})$$
(10)

On the other hand, since force and torque of 5-th and 8-th

links are exerted on 4-th link, effects onto 4-th link as:

$${}^{4}\boldsymbol{f}_{4} = {}^{4}\boldsymbol{R}_{5}{}^{5}\boldsymbol{f}_{5} + {}^{4}\boldsymbol{R}_{8}{}^{8}\boldsymbol{f}_{8} + m_{4}{}^{4}\ddot{\boldsymbol{s}}_{4}, \qquad (11)$$

$${}^{4}\boldsymbol{n}_{4} = {}^{4}\boldsymbol{R}_{5}{}^{5}\boldsymbol{n}_{5} + {}^{4}\boldsymbol{R}_{8}{}^{8}\boldsymbol{n}_{8} + {}^{4}\boldsymbol{I}_{4}{}^{4}\dot{\boldsymbol{\omega}}_{4} + {}^{4}\boldsymbol{\omega}_{4} \times ({}^{4}\boldsymbol{I}_{4}{}^{4}\boldsymbol{\omega}_{4})$$

$$+ {}^{4}\hat{\boldsymbol{s}}_{4} \times (m_{4}{}^{4}\ddot{\boldsymbol{s}}_{4}) + {}^{4}\hat{\boldsymbol{p}}_{5} \times ({}^{4}\boldsymbol{R}_{5}{}^{5}\boldsymbol{f}_{5})$$

$$+ {}^{4}\hat{\boldsymbol{p}}_{8} \times ({}^{4}\boldsymbol{R}_{8}{}^{8}\boldsymbol{f}_{8}). \qquad (12)$$

Similarly, force and torque of 11-th, 14-th and 17-th links transmit to 10-th link directly. Then, rotational motion equation of *i*-th link is obtained as Eq. 13 by making inner product of induced torque onto the *i*-th link's unit vector e_{z_i} around rotational axis:

$$\tau_i = (\boldsymbol{e}_{z_i})^T \,^i \boldsymbol{n}_i + d_i \dot{q}_i. \tag{13}$$

However, when the supporting-foot (1-st link) is slipping (prismatic joint), the torque onto the 1-st link can be calculated by following equation.

$$\tau_1 = (\boldsymbol{e}_{z_1})^T \,{}^1\boldsymbol{f}_1 + k_{f1}\dot{y}_1. \tag{14}$$

Finally, we get motion equation with one leg standing as:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} = \boldsymbol{\tau}, \quad (15)$$

Here, τ is input torque, M(q) is inertia matrix, $h(q, \dot{q})$ and g(q) are vectors which indicate Coriolis force, centrifugal force and gravity. When the supporting-foot is slipping, the $D = diag[k_{f1}, d_2, \dots, d_{17}]$ is a matrix which means coefficients of joints and between foot and ground. And $q = [y_1, q_2, \dots, q_{17}]^T$ means the angle of joints and the relative position between foot and ground.

Making floating-foot contact with ground, contacting-foot appears with contacting-foot's position z_h or angle q_e to the ground being constrained. The constraints of foot's position and heel's rotation can be defined as C_1 and C_2 respectively, these constraints can be written as follow, where r(q) means the contacting-foot's heel or toe position in Σ_W .

$$\boldsymbol{C}(\boldsymbol{r}(\boldsymbol{q})) = \begin{bmatrix} C_1(\boldsymbol{r}(\boldsymbol{q})) \\ C_2(\boldsymbol{r}(\boldsymbol{q})) \end{bmatrix} = \boldsymbol{0}$$
(16)

Then, robot's equation of motion with external force f_n , friction force f_t and external torque τ_n corresponding to C_1 and C_2 can be derived as:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}}$$
$$= \boldsymbol{\tau} + \boldsymbol{j}_{c}^{T}f_{n} - \boldsymbol{j}_{t}^{T}f_{t} + \boldsymbol{j}_{r}^{T}\boldsymbol{\tau}_{n}, \qquad (17)$$

where j_c , j_t and j_r are defined as:

$$\boldsymbol{j}_{c}^{T} = \left(\frac{\partial C_{1}}{\partial \boldsymbol{q}^{T}}\right)^{T} \left(1 / \left\|\frac{\partial C_{1}}{\partial \boldsymbol{r}^{T}}\right\|\right), \quad \boldsymbol{j}_{t}^{T} = \left(\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^{T}}\right)^{T} \dot{\boldsymbol{r}}_{t}, \quad (18)$$

$$\boldsymbol{j}_{r}^{T} = \left(\frac{\partial C_{2}}{\partial \boldsymbol{q}^{T}}\right)^{T} \left(1 / \left\|\frac{\partial C_{2}}{\partial \boldsymbol{q}^{T}}\right\|\right).$$
(19)

It is common sense that (i) f_n and f_t are orthogonal, and (ii) value of f_t is decided by $f_t = K f_n$ ($0 < K \le 1$). The differentiating Eq. (16) by time for two times, we can derive the constraint condition of \ddot{q} .

$$\left(\frac{\partial C_i}{\partial \boldsymbol{q}^T}\right)\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T \left\{\frac{\partial}{\partial \boldsymbol{q}}\left(\frac{\partial C_i}{\partial \boldsymbol{q}^T}\right)\dot{\boldsymbol{q}}\right\} = 0 \quad (i = 1, \ 2) \quad (20)$$

Making the \ddot{q} in Eqs. (17) and (20) be identical, we can obtain the equation of contacting motion as follow.

$$\begin{bmatrix} \mathbf{M}(\mathbf{q}) & -(\mathbf{j}_{c}^{T} - \mathbf{j}_{t}^{T} \mathbf{K}) & -\mathbf{j}_{r}^{T} \\ \frac{\partial C_{1}}{\partial \mathbf{q}^{T}} & 0 & 0 \\ \frac{\partial C_{2}}{\partial \mathbf{q}^{T}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ f_{n} \\ \tau_{n} \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{\tau} - \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{g}(\mathbf{q}) - \mathbf{D}\dot{\mathbf{q}} \\ -\dot{\mathbf{q}}^{T} \left\{ \frac{\partial}{\partial \mathbf{q}} \left(\frac{\partial C_{1}}{\partial \mathbf{q}^{T}} \right) \right\} \dot{\mathbf{q}} \\ -\dot{\mathbf{q}}^{T} \left\{ \frac{\partial}{\partial \mathbf{q}} \left(\frac{\partial C_{2}}{\partial \mathbf{q}^{T}} \right) \right\} \dot{\mathbf{q}} \end{bmatrix}$$
(21)

Here, since motion of the foot is constrained only vertical direction, walking direction has a degree of motion. That is, contacting-foot may slip forward or backward depending on the foot's velocity in traveling direction.

3 VALIDATION OF MODEL

3.1 Verification by Mechanical Energy

To verify this complex model, we used the mechanical energy conservation law. Because to verify the conservation of mechanical energy, the equation of motion must be correct. We make the model to do a free fall with the input torque $\tau_i = 0$ and the viscous friction $D_i = 0$. In this case, there is no friction. So, it will has no loss of energy during free fall. During the motion the mechanical energy will be saved at the initial potential energy. To derive the mechanical energy, it is necessary to calculate all of the potential energy, rotational energy and translational energy.

3.1.1 Calculation of Mechanical Energy

It is necessary to calculate the height of the center of gravity of each link before the calculation of the potential energy. We use the homogeneous transformation matrix to calculate it as following equation.

$${}^{W}z_{Gi} = {}^{W}z_i + \frac{{}^{W}z_{i+1} - {}^{W}z_i}{2}$$
(22)

Here, ${}^{W}z_{Gi}$ is the height of the center of gravity of *i*-th link which seen from the world coordinate. ${}^{W}z_i$ is the height of the joint which seen from the world coordinate. So, we can calculate the potential energy as following equation.

$$E_p = \sum_{i=1}^{17} m_i^W z_i g$$
 (23)

Here, E_p is the potential energy of the model. m_i is the mass of each link. g is the gravitational acceleration.

Then, we can calculate the rotational energy as following equation.

$$E_k = \sum_{i=1}^{17} \frac{1}{2}^W \boldsymbol{\omega}_i^{TW} \boldsymbol{I}_i^W \boldsymbol{\omega}_i$$
(24)

Here, E_k is the rotational energy of the model. I_i is the moment of inertia of each link.

Then, we can also calculate the translational energy as following equation.

$$E_{v} = \sum_{i=1}^{17} \frac{1}{2} m_{i}^{W} \dot{\boldsymbol{r}}_{gi}^{TW} \dot{\boldsymbol{r}}_{gi}$$
(25)

Here, E_v is the translational energy of the model, r_{gi} is the translational velocity of the center of gravity of *i*-th link.

Finally, the mechanical energy can be derived as following equation.

$$E_Q = E_p + E_k + E_v \tag{26}$$



Fig. 2. Ground state of the supporting-foot

3.1.2 Verification Simulation Experiment

First, the simulation which about the stopped state of the supporting-foot with surface contact and line contact are shown in Fig.2(a), (b). As shown in Fig.3, the humanoid model is doing a free fall with the condition that without any viscous friction ($D_i=0$) and input torque ($\tau_{input} = 0$). The Fig.4 shows the result of the mechanical energy when the supporting-foot is in the stopped and surface contact state. And Fig.5 is about the mechanical energy during the supporting-foot is in the stopped and line contact state. From Fig.4, 5, the total of mechanical energy is saved. So, it can be seen that support-foot stopped model is correct.

Similarly, the simulation which about the slip state of the supporting-foot with surface contact and line contact are shown in Fig.2(c), (d). As well as Fig.3, the humanoid model is also doing a free fall without any viscous friction and input torque. The Fig.6 shows the result of the mechanical energy when the supporting-foot is in the slip and surface contact state. And Fig.7 is about the mechanical energy during the



Fig. 3. Free-fall of humanoid model with the supporting-foot in the stopped state



Fig. 4. Mechanical energy conservation during the supporting-foot in the stopped and surface contact state



Fig. 5. Mechanical energy conservation during the supporting-foot in the stopped and line contact state

supporting-foot is in the slip and line contact state. From Fig.6, 7, the total of mechanical energy is saved. So, it can be seen that support-foot slip model is correct.

3.2 Verification by The Position of The Overall Center of Gravity

In the simulation of free fall, if the supporting-foot can slip on the ground (without any friction), the y-position of the overall center of gravity should not change. In order to confirm that, the y-position of the overall center of gravity can be calculated as following equation.

$$P_{GAy} = \frac{m_1 P_{G1y} + m_2 P_{G2y} + \dots + m_{17} P_{G17y}}{m_1 + m_2 + \dots + m_{17}}$$
(27)

Here, P_{GAy} is the y-position of the overall center of gravity, $m_1 \sim m_{17}$ is the mass of each link, $P_{G1y} \sim P_{G17y}$ is the y-position of center of gravity of each link.



Fig. 6. Mechanical energy conservation during the supporting-foot in the stopped and surface contact state



Fig. 7. Mechanical energy conservation during the supporting-foot in the stopped and line contact state

The y-position of the overall center of gravity of free fall simulation that supporting-foot in slip surface contact state and slip line contact state are shown in Fig. 8, 9. From Fig. 8, 9, the y-position of the overall center of gravity is not changed. So, it also can be seen that support-foot slip model is correct.

4 TRANSITION OF GAITS

In this research, humanoid walking with the transition of gaits state as shown in Fig. 9. And the route of state transition is determined by the walking motion of humanoid. In other words, it depends on the solution of the dynamics in Eq. (21). The dynamics and the state variable which depending on the gaits has been selected. And it will transit to the next state when the branching condition is satisfied.



Fig. 8. The y-position of the overall center of gravity during the supporting-foot in the slip and surface contact state



Fig. 9. The y-position of the overall center of gravity during the supporting-foot in the slip and line contact state

5 BIPEDAL WALKING SIMULATION

By using the gait transition, the authors realized the bipedal walking of humanoid robot that just like Fig. 11. The humanoid was walking 1000 steps on the ground with friction coefficient $K_f = 0.7$. The walking distance is 424.13[m], the walking time is required to 710.35[s]. By the calculation, the average speed is 2.15[km/h], the length of stride is 0.42[m], and the period of walking is 1.42[s].



Fig. 11. The bipedal walking of humanoid robot

6 CONCLUSION

In this paper, we showed a walking model of humanoid including slipping, bumping, surface-contacting and pointcontacting of foot, which dynamical equation is derived by Newton-Euler method with constraint condition, named as the Extended Newton-Euler Method. For the future, we plan to extend the calculation method to the case of more than two constraint conditions, and evaluate it by simulations.



Fig. 10. States and gait transition including slip motion

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