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Analysis of Bracing-Constraint Dynamics with Energy-efficient for Elbow-bracing Manipulator

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***Abstract*— The configuration of elbow-bracing is built by imitating human's handwriting behavior that human can do accurate task with less consumption energy by bracing the elbow or hand on the task. In this paper, first, the motion equation of the elbow-bracing manipulator under constrained condition has been derived. Second, as the consumption energy is calculated based on the voltage and current of the motor, the equation of motion of the motor has been proposed. Then, a control method based on the constraint dynamics of the elbow-bracing manipulator is proposed to simultaneously control constraint force and hand's trajectory and elbow-bracing position in work space. Moreover, we focus on the energy-efficient of the elbow-bracing manipulator, and analysis the factors which have a great effect on the consumption energy, i.e. elbow-bracing position, constraint force. Finally, a simulation experiment for 4-link elbow-bracing manipulator has been conducted to reveal the effectiveness of energy-efficient for the elbow-bracing manipulator and the influence of the above two factors to the energy-efficient.**

***Index Terms*— Elbow-bracing manipulator; Constraint dynamics; constraint force control ; Energy-efficient,**

I. Introduction

Humans can write characters accurately on a paper with less power by bracing and restricting the wrist, as shown in Fig.1. Moreover, this bracing strategy may overcome the hindrances of hyper-redundant manipulators being too heavy to spare the hand payload for desired tasks.

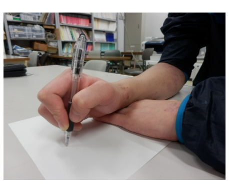

Fig. 1. Human's writing motion utilizing bracing wrist

Roy and Whitcomb [1] categorized motions and control methods of constrained robot as (a) model based control [2] [3] that assume undeformable robots and deformable environments, and (b) methods based on position/velocity control [4] that assume undeformable robots and also deformable environments. Park and Khatib [5] [6] proposed kinematics model of plural contact to control constraint motion in category (b). Finally, there is classification of (c) control method [7] that assume undeformable robots and undeformable environments. Yamane and Nakamura proposed walking of humanoid robot [8] and a concept of dynamics filter [9] in this category. Effectiveness and accuracy of hyper-redundant manipulators subject to constraint on environments have been discussed, West and Asada [10] proposed common contact mode of kinematics for designing position/force simultaneous controller of manipulator in constraint motion.

In this paper, considering the control method of undeformable robots and undeformable environments. Under these conditions, algebraic equation can be derived from constraint condition and equation of motion as (1).

$$\boldsymbol{A}\boldsymbol{f}_n = \boldsymbol{a} - \boldsymbol{B}\boldsymbol{\tau} \tag{1}$$

$\boldsymbol{f}_n$ is constraint force, $\boldsymbol{A}$, $\boldsymbol{a}$ and $\boldsymbol{B}$ are vector and matrices that will be defined in the next section, $\boldsymbol{\tau}$ is a vector of input torques. Eq.(1) shows an algebraic relation between input torques and constraint force when robot's hand is subject to constraint. The above equation has been derived by Hemami and Wyman [11] in discipline of biped walking, and applied by Peng and Adachi [12] in discipline of force/position control by robots at the beginning. Peng considered that $\boldsymbol{\tau}$ is input and $\boldsymbol{f}_n$ is output, and Eq.(1) was used as force sensor to detect $\boldsymbol{f}_n$. Despite the nature that the robot motions under a condition of (c) undeformable robot and undeformable environment be subject to the algebraic equation, Eq.(1), researches on robot force control in category (c) seems to be not based on the Eq.(1) except Peng as far as we know. In this paper, Eq.(1) is used for calculating input torque $\boldsymbol{\tau}$ to accomplish desired constraint force $\boldsymbol{f}_{nd}$ contrary to Peng's idea. Considering the hand writing motion, we know that too much pushing the wrist to table bears fatigue and meaningless, and also too less pushing makes us tired too. This suggests a hypothesis that appropriate supporting force exists, and also effective bracing position may exist.

In the past of this research, the control of constraint motion has been applied to many robots. The grinding robot that has been researching by Minami and Adachi [13], the hand of robot is constrainted to a changing surface to grinding a target object into desired shape with force-sensorless feed-forward control. And the bracing control also be used to a mobile robot with redundant manipulator and the bracing manipulator that have been researching by Washino and Minami [14], Kondo and Itoshima [15] to maintaining the balance of the mobile robot or conserve energy of the redundant manipulator. Xiang Li, et al.[16] derive the dynamical equation for humanoid robot using Extended Newton-Euler and discuss its walking models, such as slipping, bumping, surface-contacting and point-contacting of foot.

In this paper, we focus on the energy-efficient of the elbow-bracing manipulator through two factors, i.e. elbow-bracing position and constraint force. In section 2, the motion equation of elbow-bracing manipulator with constrained force and motor has been derived. In section 3, PD controller has been used to achieve the task for the elbow-bracing manipulator. In section 4, a simulation experiment for a 4-link elbow-bracing manipulator has been conducted. Finally, we give our conclusion.

II. Motion equation with constraint and motor for elbow-bracing manipulator

A. Constrained condition

As shown in Fig.2, the intermediate links of an n-link manipulator are contacted with the environment at p points.

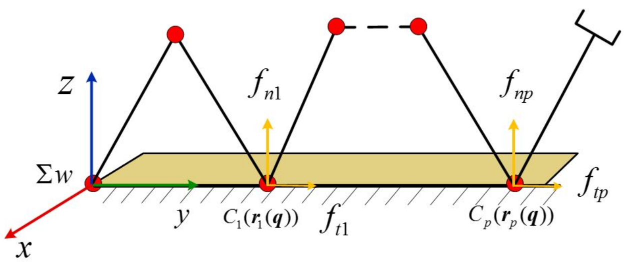

Fig. 2. Constrained model of the elbow-bracing manipulator

The constraint function is expressed as,

$$\begin{aligned} \boldsymbol{C}(\boldsymbol{r}(\boldsymbol{q})) &= [C_1(\boldsymbol{r}_1(\boldsymbol{q})), C_2(\boldsymbol{r}_2(\boldsymbol{q})), \cdots, C_p(\boldsymbol{r}_p(\boldsymbol{q}))]^{\mathrm{T}} \\ &= \boldsymbol{0} \end{aligned} \tag{2}$$

Here, $\boldsymbol{q} \in \boldsymbol{R}^n$ is joint angle vector with n joints, $\boldsymbol{r}_i \in \boldsymbol{R}^m (m < n)$ is i-th link position that is subject to constraint. The relation between $\boldsymbol{r}_i$ and $\boldsymbol{q}$ and the relation between $\dot{\boldsymbol{r}}_i$ and $\dot{\boldsymbol{q}}$ are expressed as,

$$\boldsymbol{r}_i = \boldsymbol{r}_i(\boldsymbol{q}) \tag{3}$$

$$\dot{\boldsymbol{r}}_i = \boldsymbol{J}_i(\boldsymbol{q})\dot{\boldsymbol{q}}, \quad \boldsymbol{J}_i(\boldsymbol{q}) = [\tilde{\boldsymbol{J}}_i(\boldsymbol{q}), \ \boldsymbol{0}] \ . \tag{4}$$

In (4), $\boldsymbol{J}_i$ is $m \times n$ matrix, $\tilde{\boldsymbol{J}}_i$ consists of $m \times i$ matrix and zero submatrix $\boldsymbol{0}$ with $m \times (n-i)$.

In the formulation of constraint motion of robot, we consider that a plural intermediate links are contacting with the environment. In Fig.2, a generalized surface can be defined with the position constraints along the tangents to this surface and force constraints along the normals. Then the unit vectors of normals, $\boldsymbol{j}_{cci}$, which represent direction of constraint forces, $\boldsymbol{f}_n = [f_{n1}, f_{n2} \dots f_{np}]^T$, and the unit vectors of tangents, $\boldsymbol{j}_{tti}$, which represent direction of friction forces,$\boldsymbol{f}_t = [f_{t1}, f_{t2} \dots f_{tp}]^T$, are expressed as,

$$\boldsymbol{j}_{cci} = \left(\frac{\partial \boldsymbol{C}_i}{\partial \boldsymbol{r}^{\mathrm{T}}}\right)^{\mathrm{T}} / \left\| \frac{\partial \boldsymbol{C}_i}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \tag{5}$$

$$\boldsymbol{j}_{tti} = \frac{\dot{\boldsymbol{r}}_i}{\|\dot{\boldsymbol{r}}_i\|} \tag{6}$$

B. Motion equation with constraint

As we known, the jacobian transpose maps Cartesian forces into equivalent joint torques. Then, we define that,

$$\boldsymbol{j}_{ci}^{\mathrm{T}} = \boldsymbol{J}_i(\boldsymbol{q})^{\mathrm{T}} \boldsymbol{j}_{cci} = \left(\frac{\partial C_i}{\partial \boldsymbol{q}^{\mathrm{T}}}\right)^T / \left\| \frac{\partial C_i}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \tag{7}$$

$$\boldsymbol{j}_{ti}^{\mathrm{T}} = \boldsymbol{J}_i(\boldsymbol{q})^{\mathrm{T}} \boldsymbol{j}_{tti} = \left(\frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{q}^{\mathrm{T}}}\right)^{\mathrm{T}} \frac{\dot{\boldsymbol{r}}_i}{\|\dot{\boldsymbol{r}}_i\|} \tag{8}$$

$$\boldsymbol{J}_c^{\mathrm{T}} = [\boldsymbol{j}_{c1}^{\mathrm{T}}, \ \boldsymbol{j}_{c2}^{\mathrm{T}}, \ \cdots, \ \boldsymbol{j}_{cp}^{\mathrm{T}}] \tag{9}$$

$$\boldsymbol{J}_t^{\mathrm{T}} = [\boldsymbol{j}_{t1}^{\mathrm{T}}, \ \boldsymbol{j}_{t2}^{\mathrm{T}}, \ \cdots, \ \boldsymbol{j}_{tp}^{\mathrm{T}}] \tag{10}$$

$\boldsymbol{J}_c^{\mathrm{T}}$, $\boldsymbol{J}_t^{\mathrm{T}}$ are $n \times p$ matrices, and $\boldsymbol{f}_n$, $\boldsymbol{f}_t$ are $p \times 1$ vectors. Using above definitions, equation of motion of the manipulator subject to constraints at p points is expressed as

$$\begin{aligned} & \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ = & \ \boldsymbol{\tau} + \sum_{i=1}^{p} (\boldsymbol{j}_{ci}^{\mathrm{T}} f_{ni}) - \sum_{i=1}^{p} (\boldsymbol{j}_{ti}^{\mathrm{T}} f_{ti}) \\ = & \ \boldsymbol{\tau} + \boldsymbol{J}_c^{\mathrm{T}} \boldsymbol{f}_n - \boldsymbol{J}_t^{\mathrm{T}} \boldsymbol{f}_t \end{aligned} \tag{11}$$

Differentiating (2) with respect to time t twice, constraint condition of $\ddot{\boldsymbol{q}}$ is set up like

$$\dot{\boldsymbol{q}}^{\mathrm{T}} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} + \left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \ddot{\boldsymbol{q}} = \boldsymbol{0} \tag{12}$$

The solution of (11) that is $\dot{\boldsymbol{q}}$ and $\boldsymbol{q}$ must satisfy (12) disregarding time t that the manipulator be always subject to constraint. When the $\ddot{\boldsymbol{q}}$ satisfying (12) and the $\ddot{\boldsymbol{q}}$ in (11) is equal to each other, the solution $\boldsymbol{q}(t)$ in (11) satisfies (2) regardless of time.

Here, the relation between constraint force $\boldsymbol{f}_n$ and friction force $\boldsymbol{f}_t$ is shown in the following equation with coefficients of sliding friction.

$$\begin{aligned} \boldsymbol{f}_t = \boldsymbol{K}\boldsymbol{f}_n, \quad \boldsymbol{K} = \mathrm{diag}[K_1, K_2, \cdots, K_p] \\ 0 < K_i < 1, (i = 1, 2, \cdots, p) \end{aligned} \tag{13}$$

Therefore, Eq.(11) can be translated into the following equation.

$$\begin{aligned} & \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ = \; & \boldsymbol{\tau} + (\boldsymbol{J}_c^{\mathrm{T}} - \boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K})\boldsymbol{f}_n \end{aligned} \tag{14}$$

The derivation of (1) is discussed here. First, eliminating $\ddot{\boldsymbol{q}}$ by using (11) and (12), and defining $(\partial\boldsymbol{C}/\partial\boldsymbol{q}^{\mathrm{T}})\boldsymbol{M}^{-1}(\partial\boldsymbol{C}/\partial\boldsymbol{q}^{\mathrm{T}})^{\mathrm{T}}$ as $\boldsymbol{M}_c$, the following equation is derived.

$$\begin{aligned} \boldsymbol{M}_c\boldsymbol{P}\boldsymbol{f}_n \; = \; & \left(\frac{\partial\boldsymbol{C}}{\partial\boldsymbol{q}^{\mathrm{T}}}\right)\boldsymbol{M}^{-1}(\boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K}\boldsymbol{f}_n + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g} - \boldsymbol{\tau}) \\ - \; & \dot{\boldsymbol{q}}^{\mathrm{T}}\left[\frac{\partial}{\partial\boldsymbol{q}}\left(\frac{\partial\boldsymbol{C}}{\partial\boldsymbol{q}^{\mathrm{T}}}\right)\right]\dot{\boldsymbol{q}} \end{aligned} \tag{15}$$

Where $\boldsymbol{P}$ =diag$[1/||\partial\boldsymbol{C}_1/\partial\boldsymbol{r}^{\mathrm{T}}||,\cdots,1/||\partial\boldsymbol{C}_p/\partial\boldsymbol{r}^{\mathrm{T}}||]$. Moreover, by using following definitions,

$$\boldsymbol{B} = \left(\frac{\partial\boldsymbol{C}}{\partial\boldsymbol{q}^{\mathrm{T}}}\right)\boldsymbol{M}^{-1}\;, \tag{16}$$

$$\boldsymbol{a} = \boldsymbol{B}\left\{\boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g}\right\} - \dot{\boldsymbol{q}}^{\mathrm{T}}\left[\frac{\partial}{\partial\boldsymbol{q}}\left(\frac{\partial\boldsymbol{C}}{\partial\boldsymbol{q}^{\mathrm{T}}}\right)\right]\dot{\boldsymbol{q}}\;, \tag{17}$$

(15) can be translated into,

$$\boldsymbol{M}_c\boldsymbol{P}\boldsymbol{f}_n = \boldsymbol{B}\boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K}\boldsymbol{f}_n - \boldsymbol{B}\boldsymbol{\tau} + \boldsymbol{a}\;. \tag{18}$$

By defining the following matrix $\boldsymbol{A}$,

$$\boldsymbol{A} = \boldsymbol{M}_c\boldsymbol{P} - \boldsymbol{B}\boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K}. \tag{19}$$

and by inputting (19) to (18), (1) has been derived. And we assume that $\boldsymbol{A}$ is positive definite matrix. Therefore, $\boldsymbol{f}_n$ can be expressed as,

$$\boldsymbol{f}_n = \boldsymbol{A}^{-1}(\boldsymbol{a} - \boldsymbol{B}\boldsymbol{\tau}) \tag{20}$$

C. Equation of Motor

To represent the motion of each motor, the following symbols are used.

v_i is the voltage of the motor, $\boldsymbol{v} = [v_1, v_2, \cdots, v_l]^T$;
i_i is the current of the motor, $\boldsymbol{i} = [i_1, i_2, \cdots, i_l]^T$;
v_{gi} is the back EMF of the motor;
R_i is the resistance of the motor, $\boldsymbol{R} = \mathrm{diag}[R_1, R_2, \cdots, R_l]$;
L_i is the inductance of the motor, $\boldsymbol{L} = \mathrm{diag}[L_1, L_2, \cdots, L_l]$;
θ_i is the rotational angle of the motor;
τ_{gi} is the electromagnetic torque of the motor;
τ_{Li} is the load torque of the motor;
I_{mi} is the inertia of the motor;
K_{Ei} is the coefficient of the back EMF;
K_{Ti} is the coefficient of the electromagnetic torque;
d_{mi} is the viscous friction coefficient of the reducer;
k_i is the reduction radio of the reducer;

As the DC motor is considered in this paper, the coefficients, K_{Ei} and K_{Ti} are equal. And we assume that

$$K_{Ti} = K_{Ei} = K_i \tag{21}$$

The relationship between the parameters of the motor can be expressed as the following equations

$$v_i(t) \; = \; L_i\dot{i}_i + R_i i_i(t) + v_{gi}(t) \tag{22}$$
$$v_{gi}(t) \; = \; K_{Ei}\dot{\theta}_i(t) \tag{23}$$
$$I_{mi}\ddot{\theta} \; = \; \tau_{gi}(t) - \tau_{Li}(t) - d_{mi}\dot{\theta}_i \tag{24}$$
$$\tau_{gi}(t) \; = \; K_{Ti}i_i(t) \tag{25}$$

As the reduction radio of the reducer is k_i, the following equations can be obtained.

$$\theta_i \; = \; k_i q_i \tag{26}$$
$$\tau_{Li} \; = \; \frac{\tau_i}{k_i} \tag{27}$$

Based on the above equations, we can obtain the equation of motor.

$$L_i\frac{di_i}{dt} \; = \; v_i - R_i i_i - K_i k_i\dot{q}_i \tag{28}$$
$$\tau_i \; = \; -I_{mi}k_i^2\ddot{q}_i + K_i k_i i_i - d_{mi}k_i^2\dot{q}_i \tag{29}$$

The equation of motor can be rewritten in the form of vector, as shown in the following equations.

$$\boldsymbol{L}\frac{d\boldsymbol{i}}{dt} \; = \; \boldsymbol{v} - \boldsymbol{R}_i - \boldsymbol{K}_m\dot{\boldsymbol{q}} \tag{30}$$
$$\boldsymbol{\tau} \; = \; -\boldsymbol{J}_m\ddot{\boldsymbol{q}} + \boldsymbol{K}_m\boldsymbol{i} - \boldsymbol{D}_m\dot{\boldsymbol{q}} \tag{31}$$

Where

$$\begin{aligned} &\boldsymbol{K}_m = \mathrm{diag}[K_{m1}, K_{m2}, \cdots, K_{ml}], K_{mi} = K_i k_i \\ &\boldsymbol{J}_m = \mathrm{diag}[J_{m1}, J_{m2}, \cdots, J_{ml}], J_{mi} = I_{mi}k_i^2 \\ &\boldsymbol{D}_m = \mathrm{diag}[D_{m1}, D_{m2}, \cdots, D_{ml}], D_{mi} = d_{mi}k_i^2 \end{aligned}$$

The consumption energy can be expressed as the following equation.

$$E_i(T) = \int_0^T v_i(t)i_i(t)dt \tag{32}$$

D. Motion equation of manipulator including motor under constraint condition

By combining (12),(14) and (30), and substituting (31) into (14), we can obtain the motion equation of manipulator including motor under constraint condition, as shown in the following equation.

$$\begin{bmatrix} \boldsymbol{M} + \boldsymbol{J}_m & -(\boldsymbol{J}_c^{\mathrm{T}} - \boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K}) & \boldsymbol{0} \\ \dfrac{\partial\boldsymbol{C}}{\partial\boldsymbol{q}^{\mathrm{T}}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{f}_n \\ d\boldsymbol{i}/dt \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_m\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D} + \boldsymbol{D}_m)\dot{\boldsymbol{q}} \\ -\dot{\boldsymbol{q}}^{\mathrm{T}}\left[\frac{\partial}{\partial\boldsymbol{q}}\left(\frac{\partial\boldsymbol{C}}{\partial\boldsymbol{q}^{\mathrm{T}}}\right)\right]\dot{\boldsymbol{q}} \\ \boldsymbol{v} - \boldsymbol{R}\boldsymbol{i} - \boldsymbol{K}_m\dot{\boldsymbol{q}} \end{bmatrix} \tag{33}$$

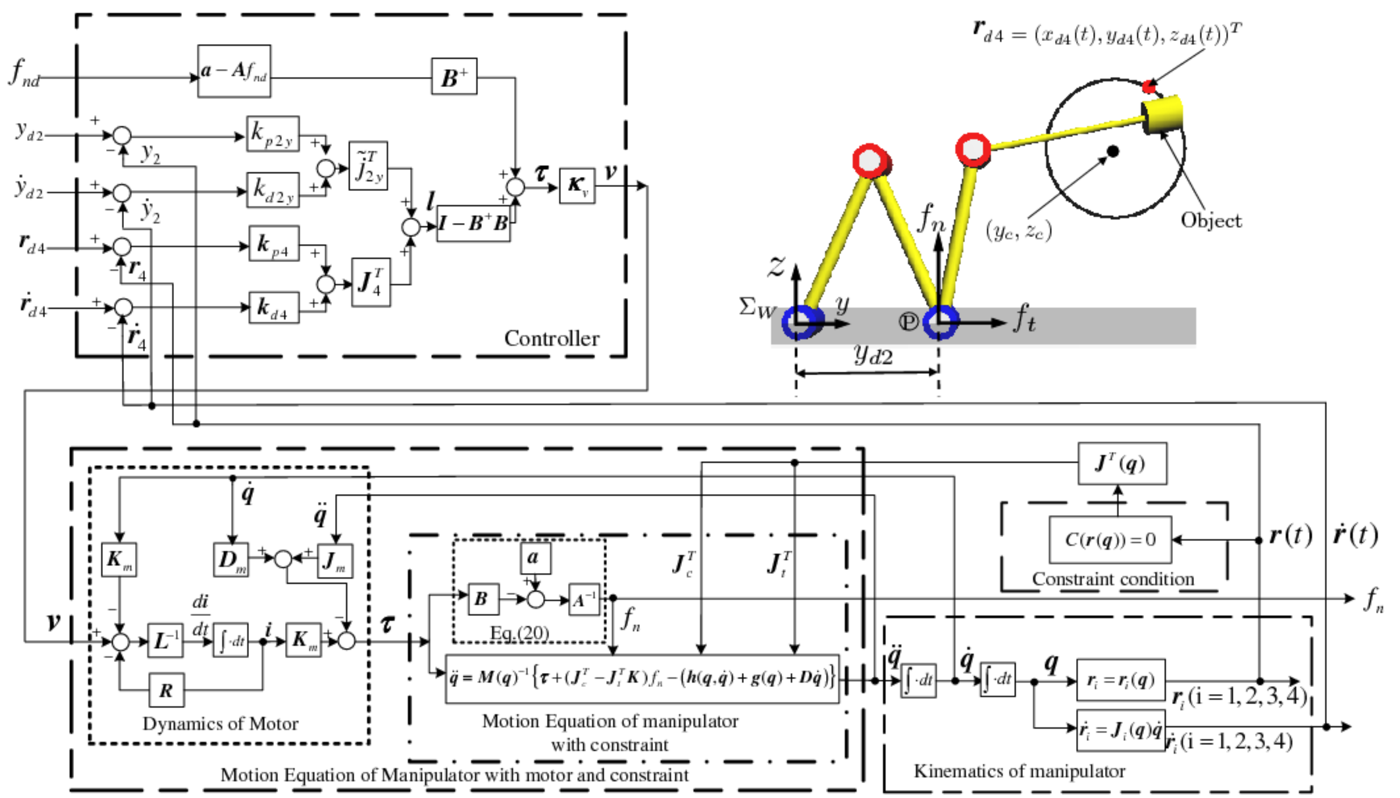

Fig. 3. Block diagram of control for 4-link elbow-bracing manipulator

III. Control method

Basede on (1), the control law of the torque $\boldsymbol{\tau}$ for the 4-link elbow-bracing manipulator is shown in the following equation.

$$\boldsymbol{\tau} = \boldsymbol{B}^{+}(\boldsymbol{a} - \boldsymbol{A}\boldsymbol{f}_{nd}) \tag{34}$$

Where, $\boldsymbol{B}^{+}$ is pseudo inverse matrix.

Noticed that the direction in which the position vector are constrained are complementary to the direction in which the constrained force is constrained. By dividing the woke space into two orthogonal domains, a position domains and a force domain, which are complementary to the directions of the corresponding constraints, in each of the two domains, position or force can be controlled independently and arbitrarily. Therefore, in order to simultaneously control the constraint force and position, the following equations can be obtained.

$$\boldsymbol{\tau} = \boldsymbol{B}^{+}(\boldsymbol{a} - \boldsymbol{A}\boldsymbol{f}_{nd}) + (\boldsymbol{I} - \boldsymbol{B}^{+}\boldsymbol{B})\boldsymbol{l}\ . \tag{35}$$

Where, $\mathrm{rank}(\boldsymbol{I}-\boldsymbol{B}^{+}\boldsymbol{B})$ equals $n-p$. Because $\boldsymbol{I}-\boldsymbol{B}^{+}\boldsymbol{B}$ is non-dimensional matrix, $\boldsymbol{l}$ has dimensions of torque. Considering $\boldsymbol{l}$ to be new input, $\boldsymbol{l}$ can be used to track target trajectory of hand $\boldsymbol{r}_{d4}$ and control bracing position through null-space $\boldsymbol{I} - \boldsymbol{B}^{+}\boldsymbol{B}$ of $\boldsymbol{B}^{+}$. By the nature of pseudo inverse matrix, adding any value to $\boldsymbol{l}$ has no influence on achieving $\boldsymbol{f}_{nd}$. So, the task of tracking trajectory and the task of achieving $\boldsymbol{f}_{nd}$ can be achieved in decoupled nature.

Here, a method to determine $\boldsymbol{l}$ is discussed. In the simulation to utilize four-link manipulator in this paper, one degree of freedom is used for force control of elbow, one degree is for contacting position control of elbow and two degrees are for two-dimensional position control of hand.

$$\begin{aligned} \boldsymbol{l} \;&=\; \tilde{\boldsymbol{j}}_{2y}^{\mathrm{T}}[K_{p2y}(y_{d2} - y_2) + K_{d2y}(\dot{y}_{d2} - \dot{y}_2)] \\ &+\; \boldsymbol{J}_4^{\mathrm{T}}[\boldsymbol{K}_{p4}(\boldsymbol{r}_{d4} - \boldsymbol{r}_4) + \boldsymbol{K}_{d4}(\dot{\boldsymbol{r}}_{d4} - \dot{\boldsymbol{r}}_4)] \end{aligned} \tag{36}$$

Here, $\tilde{\boldsymbol{j}}_{2y}^{\mathrm{T}}$ is the first column vector that comprises $\tilde{\boldsymbol{J}}_2^{\mathrm{T}}$ defined in Eq.(4). K_{p2y} and K_{d2y} are control gains of position and velocity in y axis direction of 2nd link that is shown in Fig.4, $\boldsymbol{J}_4$ is Jacobian matrix defined by Eq.(4) when $i = 4$, and $\boldsymbol{K}_{p4}$ and $\boldsymbol{K}_{d4}$ are control gain matrices of position and velocity of fourth link.

Equation(35) can be realizable in the case that robots are driven by DD motors, but the input of usual DC motor is driven by voltage input. In this paper, the following equation that gives input voltage $\boldsymbol{v}$ to the DC motors is used instead of the controller of (35), where $\boldsymbol{K}_v$ is coefficient matrix to convert torque into voltage.

$$\boldsymbol{v} = \boldsymbol{K}_v\left[\boldsymbol{B}^{+}(\boldsymbol{a} - \boldsymbol{A}\boldsymbol{f}_{nd}) + (\boldsymbol{I} - \boldsymbol{B}^{+}\boldsymbol{B})\boldsymbol{l}\right] \tag{37}$$

The block diagram of the control method for 4-link elbow-bracing manipulator is shown in Fig.3. The output of the controller $\boldsymbol{v}$ is considered as the input of the motor which also includes joint angular velocity $\dot{\boldsymbol{q}}$ and angular acceleration $\ddot{\boldsymbol{q}}$ that are the outputs of the manipulator. And the output of the motor is $\boldsymbol{\tau}$ which is the input of the 4-link elbow-bracing manipulator. As the second link of the manipulator is subject to the constraint surface, the constraint force in normal direction of the surface, which is made up of $\boldsymbol{J}_c^T$ and f_n, and the friction force in the tangent direction of the surface, which is made up of $\boldsymbol{J}_t^T$ and Kf_n are added to the motion equation of the manipulator. The formula to calculate the constraint force f_n is given by (20) which guarantee that the motion of the elbow-bracing manipulator satisfies the constraint condition of (2), which is the merit of this paper.

In Fig.3, the definition of $\boldsymbol{B}$, $\boldsymbol{a}$ and $\boldsymbol{A}$ are shown in (16), (17) and (19). $\boldsymbol{B}^+$, $\boldsymbol{I} - \boldsymbol{B}^+\boldsymbol{B}$ are introduced in the former section.

IV. Simulation for 4-link elbow-bracing manipulator

A. The model of 4-link elbow-bracing manipulator

The model of 4 links manipulator shown in Fig. 4.

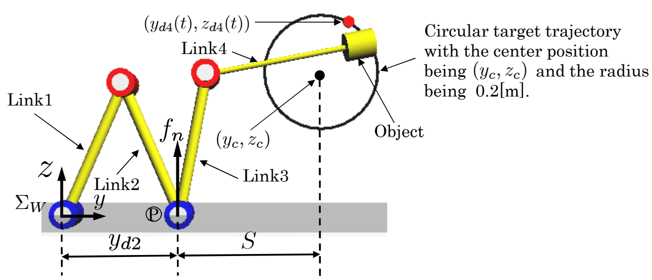

Fig. 4. Simulation model

Hand target trajectory is given as the following equations.

$$y_d(t) = 0.2 \cos \frac{2\pi}{10} t + y_c \tag{38}$$

$$z_d(t) = 0.2 \sin \frac{2\pi}{10} t + z_c \tag{39}$$

B. Simulation experiment

Link's weight is $m_i = 1.0$ [kg], link's length is $l_i = 0.5$ [m], viscous friction coefficient of joint is $D_i = 2.9$ [N · m · s/rad], torque constant is $K_i = 0.2$ [N · m/A], resistance is $R_i = 0.6$ [Ω], inductance is $L_i = 0.1$[H], inertia moment of motor is $I_{mi} = 1.64 \times 10^{-4}$ [kg · m^2], reduction ratio is $k_i = 3.0$ and viscous friction coefficient of reducer is $d_{mi} = 0.1$ [N · m · s/rad]$(i = 1, 2, 3, 4)$.

1) Analysis of energy-efficient: The energy consumption function of the i-th link is represented as the following equations.

$$E_i^*(T) = \int_0^{3T} v_i(t) i_i(t) dt, \tag{40}$$

$$E^*(T) = \sum_{i=1}^{4} E_i^*(T). \tag{41}$$

And the comparison of energy consumption for manipulators with bracing elbow and without bracing elbow is shown in Fig.5.

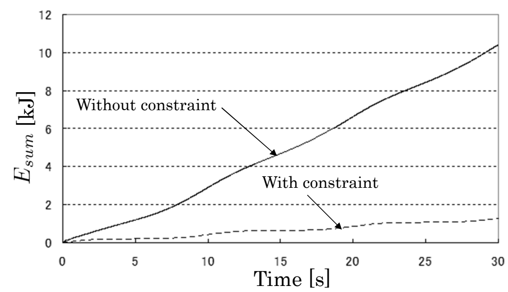

Fig. 5. Consumption energy with 2nd link bracing and without 2nd link bracing

In Fig.5, energy consumption is reduced by 1/10 when bracing elbow in comparison with no bracing, which displays the drastic effectiveness of bracing elbow.

2) Analysis of the factors which influence the consumption energy: As we known, when human is writing, he would change his elbow-bracing position to obtain the appropriate posture to write characters accurately and simultaneously less consumption energy. If the hand task is different, it's necessary to utilize the appropriate constraint force. Therefore, in this section, we mainly discuss the two factors which have great effect on the consumption energy of elbow-bracing manipulator, i.e. elbow-bracing position and constraint force.

As it deems validly that there is no meaningful difference of initial values after $3s$ [17], energy consumption $E^*(T)$ after $t = 3$[s] defined by following equations are utilized to evaluate the influence of elbow-bracing position and constraint force.

$$E_i^*(T) = \int_3^{3T} v_i(t) i_i(t) dt, \tag{42}$$

$$E^*(T) = \sum_{i=1}^{4} E_i^*(T). \tag{43}$$

The target trajectory for the end-effector of the manipulator is a circle whose center is $(0.8[m], 0.5[m])$, and radius is $0.2[m]$, and the motion cycle T is $10[s]$. Moreover, the elbow-bracing position y_2 is changing as $0.2, 0.3 \cdots 1.0[m]$, and the constraint force f_n is changing as $10, 15, 20 \cdots 50[N]$. The weight of the hand payload is $0.6[Kg]$. The 3-D figure of the consumption energy during time $t = [3, 30][s]$ with both elbow-bracing position and constraint force changing is shown in Fig.6. When the elbow-bracing position is $y_2 = 0.8[m]$

and constraint force $f_n = 40[N]$, the consumption energy is the minimum which is $2.584[KJ]$. And if the point which is made up of constraint force and elbow-bracing position is more closer to the point $(40[N], 0.8[m])$, the consumption energy is lower compared with other points. By observing Fig.6 form tangent plane $f_n = 10, 20 \cdots 50[N]$, we can obtain the 2-D figure of the consumption energy with elbow-bracing position changing, as shown in Fig.7.

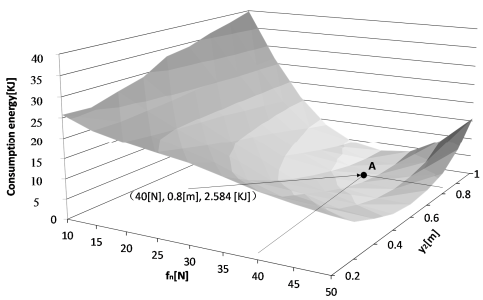

Fig. 6. Consumption energy with both elbow-bracing position and constraint force changing.(Point A represents the optimal f_n and y_2 in the case of the minimum consumption energy)

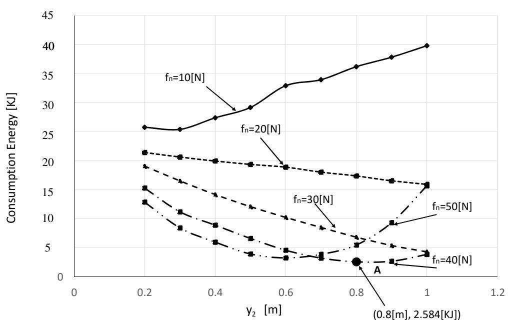

Fig. 7. Consumption energy with elbow-bracing position changing.(Point A represents the optimal f_n and y_2 in the case of the minimum consumption energy)

In Fig.7, when the constraint force is unchanged, the assumption energy is changing accordance with the elbow-bracing position changing as $0.2, 0.3 \cdots 1.0[m]$. From the tendency of these figures, we can get a conclusion that there exists a optimal elbow-bracing position which is changing accordance with f_n changing from $10, 20, 30, 40, 50[N]$, during which the minimum consumption energy can be obtained at $y_2 = 0.8[m]$ and $f_n = 40[N]$.

V. Conclusion

In this paper, first, the constraint dynamics of the elbow-bracing manipulator including motor is proposed. Then, by dividing the work space into two orthogonal domain, a position domain and a force domain, a controller whose control of constraint force and position has no interference has been discussed. Then, the factors, i.e. elbow-bracing position and constraint force which influence the consumption energy of the elbow-bracing manipulator has been analysed. Finally, the simulation results verify the effectiveness of energy-efficient for elbow-bracing manipulator using the controller proposed in this paper. What's more, the energy-efficient of the elbow-bracing manipulator can be achieved at the appropriate constraint force and optimal elbow-bracing position.

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