

Effectiveness of swing arm for walking/skating efficiencies

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Abstract: Humans tend to swing their arms when they walk or skate. In the control of biped robot, the arm and leg swing synchronously to imitate human. But the merit of arm swing has not been discussed yet. In this research, a model of the humanoid robot, including slipping, bumping, surface-contacting and point-contacting of the foot has been established, and its dynamical equation is derived by the Newton-Euler method. It must be good for people to swing their arms when they walk or ice skate, otherwise this behavior will be abandoned. There may be two benefits with arm swing, one is to improve efficiency, the other is to increase speed. We will analyze these two variable quantities. Therefore, we put forward the appropriate formula to control the swing arm and analyze the influence of the arm swing. The conclusion of the experiment is that although the swing arm reduces the efficiency, it can effectively speed up the walking speed.

Keywords: Humanoid, Arm swing, Bipedal walking, Dynamical, Newton-Euler Method

1 INTRODUCTION

Human beings have acquired the ability of stable bipedal walking in evolving repetitions so far. From the viewpoint of making a stable controller for the bipedal walking based on knowledge of conventional control theory, it looks not easy because of the dynamics with high nonlinearity and coupled interactions between links of humanoid with high dimensions. To avoiding complications in dealing directly with actual dynamics, an inverted pendulum model has been used frequently for making a stable controller[1]-[3], simplifying the calculations to determine input torque. Further, linear approximation having the humanoid represented by simple inverted pendulum enables researchers to realize steady gait through well-known control strategy such as ZMP preview control scheme and so on[4]-[6].

Our research has begun from the viewpoint as aiming to describe gait's dynamics as correctly as possible, including point-contacting state of foot and toe, slipping of the foot and bumping [7] [8]. Meanwhile, the landing of the lifting leg's heel or toe in the air to the ground makes a regular contact. Based on [9], the dynamics of humanoid can be modelled as a serial-link manipulator, including constraint motion and slipping motion by using the Extended Newton-Euler (NE) Method[10]. The NE method enables us to make a dynamical model of robots which is possible to calculate internal force and torque not generating real motion of robot manipulator. It seemed to be an advantage of the NE method that other methods do not have [11]. This merit can be applicable for propagations of constraint and impact force/torque when discussing humanoids walking based on strict dynamical models. In previous research [12], a walking model of the humanoid robot, including slipping, bumping, surface con-

tacting and point-contacting of the foot discussed, and its dynamical equation derived by the NE method.

Out-of-phase arm swing is a typical pattern during human bipedal walking or ice skating. The left-arm moves forward when the right leg and torso move forward, and vice versa for the opposing leg and arm. This arm motion, though natural, is not required for walking or skating motion. For example, we can walk even while executing specific manual tasks which constrain the arms from swinging (e.g., holding an object with two hands or carrying a suitcase). However, in the race walking or speed skating competition, all the players will choose to swing their arms. There must be some reasons for people to do that. Therefore, this paper researches the advantages of the arm swing.

2 DYNAMICAL WALKING MODEL

2.1 Humanoid Model

We discuss a bipedal robot whose definition is depicted in Fig.1. Table 1 lists length l_i [m], mass m_i [kg] of links and coefficient of joints' viscous friction d_i [N·m·s/rad], which are decided based on [13]. This model is simulated as a serial-link manipulator having ramifications and represents rigid whole body-feet including toe, torso, arms and so on-by 17 DoF. Though motion of legs is restricted in sagittal plane, it generates varieties of walking gait sequences since the robot has flat-sole feet and kicking torque on ankle. In this paper, one foot including link-1 is defined as "supporting-leg" and another foot including link-7 is defined as "free-leg" ("contacting-leg" when the free-leg contacts with floor) according to the walking state.

In this paper, we derive the equation of motion following by NE formulation. So we consider the structure of the supporting leg with two situations. When the supporting leg is

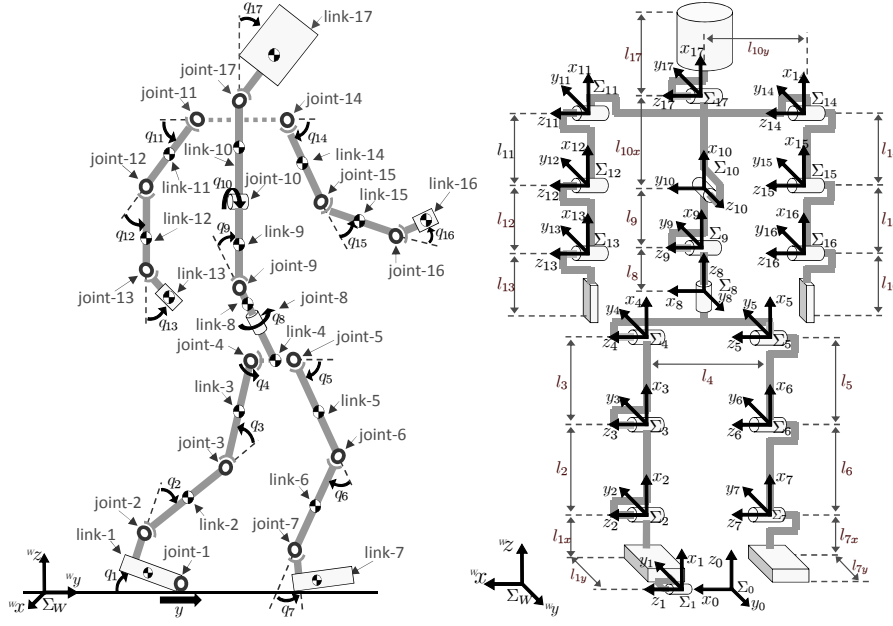


Fig. 1. Definition of humanoid's link, joint and coordinate system

Table 1. Physical parameters

Link	l_i	m_i	d_i
Head	0.24	4.5	0.5
Upper body	0.41	21.5	10.0
Middle body	0.1	2.0	10.0
Lower body	0.1	2.0	10.0
Upper arm	0.31	2.3	0.03
Lower arm	0.24	1.4	1.0
Hand	0.18	0.4	2.0
Waist	0.27	2.0	10.0
Upper leg	0.38	7.3	10.0
Lower leg	0.40	3.4	10.0
Foot	0.07	1.3	10.0
Total weight [kg]	—	64.2	—
Total height [m]	1.7	—	—

constituted of rotating joint, we first have to calculate relations of positions, velocities and accelerations between links as forward kinetics procedures from bottom link to top link. Serial link's angular velocity ${}^i\omega_i$, angular acceleration ${}^i\dot{\omega}_i$, acceleration of the origin ${}^i\ddot{p}_i$ and acceleration of the center of mass ${}^i\ddot{s}_i$ based on Σ_i fixed at i -th link are obtained as follows.

$${}^i\omega_i = {}^{i-1}R_i^T {}^{i-1}\omega_{i-1} + e_{z_i} \dot{q}_i \quad (1)$$

$${}^i\dot{\omega}_i = {}^{i-1}R_i^T {}^{i-1}\dot{\omega}_{i-1} + e_{z_i} \ddot{q}_i + {}^i\omega_i \times (e_{z_i} \dot{q}_i) \quad (2)$$

$${}^i\ddot{p}_i = {}^{i-1}R_i^T \left\{ {}^{i-1}\ddot{p}_{i-1} + {}^{i-1}\dot{\omega}_{i-1} \times {}^{i-1}\hat{p}_i + {}^{i-1}\omega_{i-1} \times ({}^{i-1}\omega_{i-1} \times {}^{i-1}\hat{p}_i) \right\} \quad (3)$$

$${}^i\ddot{s}_i = {}^i\ddot{p}_i + {}^i\dot{\omega}_i \times {}^i\hat{s}_i + {}^i\omega_i \times ({}^i\omega_i \times {}^i\hat{s}_i) \quad (4)$$

Then if the supporting leg is constituted of prismatic form object, that describes slipping motion along ${}^w y$ direction in Fig.1. The equations is switched as the following.

$${}^i\omega_i = {}^{i-1}R_i^T {}^{i-1}\omega_{i-1} \quad (5)$$

$${}^i\dot{\omega}_i = {}^{i-1}R_i^T {}^{i-1}\dot{\omega}_{i-1} \quad (6)$$

$${}^i\ddot{p}_i = {}^{i-1}R_i^T \left\{ {}^{i-1}\ddot{p}_{i-1} + {}^{i-1}\dot{\omega}_{i-1} \times {}^{i-1}\hat{p}_i + {}^{i-1}\omega_{i-1} \times ({}^{i-1}\omega_{i-1} \times {}^{i-1}\hat{p}_i) \right\} + 2({}^{i-1}R_i^T {}^{i-1}\omega_{i-1}) \times (e_{z_i} \dot{q}_i) + e_{z_i} \ddot{q}_i \quad (7)$$

$${}^i\ddot{s}_i = {}^i\ddot{p}_i + {}^i\dot{\omega}_i \times {}^i\hat{s}_i + {}^i\omega_i \times ({}^i\omega_i \times {}^i\hat{s}_i) \quad (8)$$

Here, ${}^{i-1}R_i$ means orientation matrix, ${}^{i-1}\hat{p}_i$ represents position vector from the origin of $(i-1)$ -th link to the one of

i -th, ${}^i\hat{s}_i$ is defined as gravity center position of i -th link and e_{z_i} is unit vector that shows rotational axis of i -th link.

After the above forward kinetic has been calculated, contrarily inverse dynamical calculation from top to base link are calculated.

Finally, we get motion equation with one leg standing as:

$$M(q)\ddot{q} + h(q, \dot{q}) + g(q) + D\dot{q} = \tau, \quad (9)$$

Here, $\tau = [f_1, \tau_1, \tau_2, \dots, \tau_{17}]$ is input torque, $M(q)$ is inertia matrix, both of $h(q, \dot{q})$ and $g(q)$ are vectors which indicate Coriolis force, centrifugal force and gravity. When the supporting leg is slipping, the $D = \text{diag}[\mu_k, d_1, d_2, \dots, d_{17}]$ is a matrix which means coefficients between foot and ground, and $q = [y_0, q_1, q_2, \dots, q_{17}]^T$ means the relative position between foot and ground and that of joints. The vector q changes according to the state of the supporting foot as shown in Fig. 2.

2.2 Constraint Conditions for free-leg Model

When making free-leg contact with ground, the free leg appears with the position or angle to the ground being con-

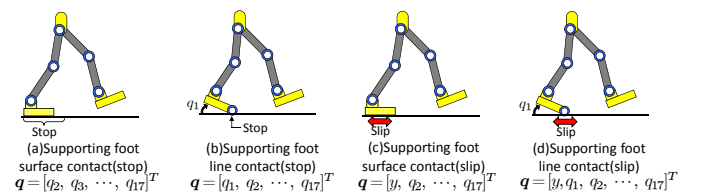


Fig. 2. Ground state of the supporting foot

strained. Also, when velocity of free leg's in traveling direction becomes less than 0.01[m/s], the free leg is constrained in acceleration by the static friction. The constraints of foot's z-axis position, heel's rotation and foot's y-axis position on floating foot are defined as C_1 , C_2 and C_3 respectively, these constraints can be written as follow, where $\mathbf{r}(\mathbf{q})$ means heel position of free leg or toe position of it in Σ_W .

$$\mathbf{C}(\mathbf{r}(\mathbf{q})) = \begin{bmatrix} C_1(\mathbf{r}(\mathbf{q})) \\ C_2(\mathbf{r}(\mathbf{q})) \\ C_3(\mathbf{r}(\mathbf{q})) \end{bmatrix} = \mathbf{0} \quad (10)$$

Then, equation of motion with external force f_{nz} , friction force f_t , external torque τ_n and external force f_{ny} corresponding to C_1 , C_2 and C_3 can be derived as:

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \mathbf{D}\dot{\mathbf{q}} \\ = \boldsymbol{\tau} + \mathbf{j}_{cz}^T f_{nz} - \mathbf{j}_t^T f_t + \mathbf{j}_r^T \tau_n + \mathbf{j}_{cy}^T f_{ny} \end{aligned} \quad (11)$$

where $\mathbf{j}_{cz}$, $\mathbf{j}_t$, $\mathbf{j}_r$ and $\mathbf{j}_{cy}$ are defined as:

$$\begin{aligned} \mathbf{j}_{cz}^T &= \left(\frac{\partial \mathbf{r}}{\partial \mathbf{q}^T} \right)^T \left(\frac{\partial C_1}{\partial \mathbf{q}^T} / \left\| \frac{\partial C_1}{\partial \mathbf{r}^T} \right\| \right), \quad \mathbf{j}_t^T = \left(\frac{\partial \mathbf{r}}{\partial \mathbf{q}^T} \right)^T \frac{\dot{\mathbf{r}}}{\|\dot{\mathbf{r}}\|}, \\ \mathbf{j}_r^T &= \frac{\partial C_2}{\partial \mathbf{r}^T} / \left\| \frac{\partial C_2}{\partial \mathbf{r}^T} \right\|, \quad \mathbf{j}_{cy}^T = \left(\frac{\partial \mathbf{r}}{\partial \mathbf{q}^T} \right)^T \left(\frac{\partial C_3}{\partial \mathbf{q}^T} / \left\| \frac{\partial C_3}{\partial \mathbf{q}^T} \right\| \right) \end{aligned} \quad (12)$$

By differentiating Eq.(10) second-order by time t and simultaneous with Eq.(11), the following dynamics is obtained

$$\begin{aligned} \begin{bmatrix} \mathbf{M}(\mathbf{q}) & -(\mathbf{j}_{cz}^T - \mathbf{j}_t^T K) & -\mathbf{j}_r^T & -\mathbf{j}_{cy}^T \\ \partial C_1 / \partial \mathbf{q}^T & 0 & 0 & 0 \\ \partial C_2 / \partial \mathbf{q}^T & 0 & 0 & 0 \\ \partial C_3 / \partial \mathbf{q}^T & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ f_{nz} \\ \tau_n \\ f_{ny} \end{bmatrix} \\ = \begin{bmatrix} \boldsymbol{\tau} - \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{g}(\mathbf{q}) - \mathbf{D}\dot{\mathbf{q}} \\ -\dot{\mathbf{q}}^T \left\{ \frac{\partial}{\partial \mathbf{q}} \left(\frac{\partial C_1}{\partial \mathbf{q}^T} \right) \right\} \dot{\mathbf{q}} \\ -\dot{\mathbf{q}}^T \left\{ \frac{\partial}{\partial \mathbf{q}} \left(\frac{\partial C_2}{\partial \mathbf{q}^T} \right) \right\} \dot{\mathbf{q}} \\ -\dot{\mathbf{q}}^T \left\{ \frac{\partial}{\partial \mathbf{q}} \left(\frac{\partial C_3}{\partial \mathbf{q}^T} \right) \right\} \dot{\mathbf{q}} \end{bmatrix} \end{aligned} \quad (13)$$

3 ANALYSIS OF ARM SWING

It must be good for people to swing their arms when they walk or ice skate, otherwise this behavior will be abandoned. There may be two benefits, one is to improve efficiency, the other is to increase speed. We will analyze these two variable quantities. Therefore, we need the appropriate formula to control the swing arm and analyze the influence of the arm swing.

3.1 The input of arms

In order to obtain the desired arm swing by specifying as few variables as possible, the sine wave is selected as the trajectory of the shoulder joints($joint-11$, $joint-14$). The elbows($joint-12$, $joint-15$) and wrists($joint-13$, $joint-16$) are always in line with the arms. The formulas are as follows.

$$\begin{cases} q_{d11} = A \sin(\omega t + \varphi) \\ q_{d14} = A \sin(\omega t + \varphi + \pi) \\ q_{d12,13,15,16} = 0 \end{cases} \quad (14)$$

$$\begin{cases} \dot{q}_{d11} = A \omega \cos(\omega t + \varphi) \\ \dot{q}_{d14} = A \omega \cos(\omega t + \varphi + \pi) \\ \dot{q}_{d12,13,15,16} = 0 \end{cases} \quad (15)$$

According to Eq.(14) and Eq.(15), we can set the action of the swing arm by three quantities (amplitude A , angular frequency ω and phase φ).

In this way, we can track the trajectory through the PD control to achieve the desired action. The rule of PD control is shown in the Eq.(16). Here $\boldsymbol{\tau}, \mathbf{q}_d, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ and $\dot{\mathbf{q}}$ are vectors composed of input torque, target angle, angle, target angular velocity and angular velocity corresponding to $joint-11-16$ respectively. The $\mathbf{k}_p = \text{diag}[k_{p11}, \dots, k_{p16}]$ and $\mathbf{k}_d = \text{diag}[k_{d11}, \dots, k_{d16}]$ are matrices which mean proportional and differential coefficients.

$$\boldsymbol{\tau} = \mathbf{k}_p(\mathbf{q}_d - \mathbf{q}) + \mathbf{k}_d(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) \quad (16)$$

3.2 The formula to analyze influence

We need an appropriate formula to evaluate the effect of arm swing on walking or ice skating. First, replace Eq.(7) into Eq.(9) and transform it into Eq.(18) as follows.

$$\boldsymbol{\tau}_b = \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \mathbf{D}\dot{\mathbf{q}} \quad (17)$$

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\boldsymbol{\tau} - \boldsymbol{\tau}_b) \quad (18)$$

Through Eq.(18), we can analyze the influence of input torque of one joint on angular acceleration of another joint. For example, $joint-8$'s angular acceleration is determined by follows:

$$\ddot{q}_8 = \sum_{i=1}^{17} M_{8,i}^{-1}(\tau_i - \tau_{bi}) \quad (19)$$

Then $M_{8,i}^{-1}$ in Eq.(19) is the influence coefficient of the τ_i on the $\ddot{q}_8$, which indicates how much influence the torque exerts and is only determined by the angle $\mathbf{q}$ at the current moment. And $M_{8,i}^{-1}(\tau_i - \tau_{bi})$ is the angular acceleration provided by $joint-i$.

But now we want to analyze not the impact on a certain joint, but the impact on the behavior of walking or skating.

Therefore, we define $\dot{\mathbf{p}} = [\dot{x}, \dot{y}, \dot{z}]^T$ as the velocity vector of the center of the hip($link = 4$) in the world coordinate system, so that we can use $\dot{\mathbf{p}}$ to represent the walking speed. Then $\ddot{\mathbf{p}} = [\ddot{x}, \ddot{y}, \ddot{z}]^T$ is the acceleration of walking or skating. The relationships between hip's $\dot{\mathbf{p}}$, $\ddot{\mathbf{p}}$ and humanoid's $\dot{\mathbf{q}}([\dot{q}_1, \dots, \dot{q}_4])$, $\ddot{\mathbf{q}}([\ddot{q}_1, \dots, \ddot{q}_4])$ are as follows. $\mathbf{J}_v$ is the Jacobian matrix from toe to hip whose size is 3×4 .

$$\dot{\mathbf{p}} = \mathbf{J}_v \dot{\mathbf{q}} \quad (20)$$

$$\ddot{\mathbf{p}} = \mathbf{J}_v \ddot{\mathbf{q}} + \dot{\mathbf{J}}_v \dot{\mathbf{q}} \quad (21)$$

Then replace Eq.(18) into Eq.(21), we get a formula similar to Eq.(18), but can analyze walking acceleration.

$$\ddot{\mathbf{p}} = \mathbf{J}_v \mathbf{M}^{-1}(\boldsymbol{\tau} - \boldsymbol{\tau}_b) + \dot{\mathbf{J}}_v \dot{\mathbf{q}} \quad (22)$$

The $\dot{\mathbf{J}}_v \dot{\mathbf{q}}$ of Eq.(22) is determined by $\mathbf{q}$ and $\dot{\mathbf{q}}$ at the current moment, it has nothing to do with angular acceleration $\ddot{\mathbf{q}}$ or torque $\boldsymbol{\tau}$.

$\ddot{p}_2 = \ddot{y}$ is the acceleration of walking direction, and $(\mathbf{J}_v \mathbf{M}^{-1})_{2,i}$ is the influence coefficient of τ_i on walking acceleration. The $(\mathbf{J}_v \mathbf{M}^{-1})_{2,i}(\tau_i - \tau_{bi})$ is the walking acceleration provided by $joint - i$.

4 SIMULATION

4.1 Full exploration experiment

In order to research the effect of different arm swing movements on the efficiency and speed of bipedal walking, a full exploration experiment was carried out by changing amplitude A (which range is $[0.2, 0.6]$), angular frequency ω (which range is $[0.45, 0.55]$) and phase φ (which range is $[0, \pi]$) in Eq.(14) and Eq.(15) to simulate a 20s walk each time while keeping the feedforward control of the legs($\tau_2 - \tau_7$) unchanged.

4.1.1 Effectiveness for Efficiency

Firstly, in order to better evaluate the efficiency of walking, we define the following evaluation functions:

$$g = E/y \quad (23)$$

Here $E[J]$ is the input energy and $y[m]$ is the walking distance. Therefore, the physical meaning of $g[J/m]$ is the energy consumed per meter. Obviously, the smaller the g , the better the efficiency.

The experimental results are shown in Fig.3. The longitudinal axis represents the minimum value of g in all phases φ corresponding to the amplitude A and angular frequency ω of a point.

When the ω is close to the swing frequency of the leg (which is 5.0), the efficiency is the best, and the influence of the A is small.

When the frequency is different, the greater the A , the worse the walking efficiency, and even lead to fall down.

When the input torque of the arm is zero, there is almost no arm swing, and the energy consumed per meter g is $609.13[J/m]$. Obviously, the efficiency is the highest when the arm is not swinging.

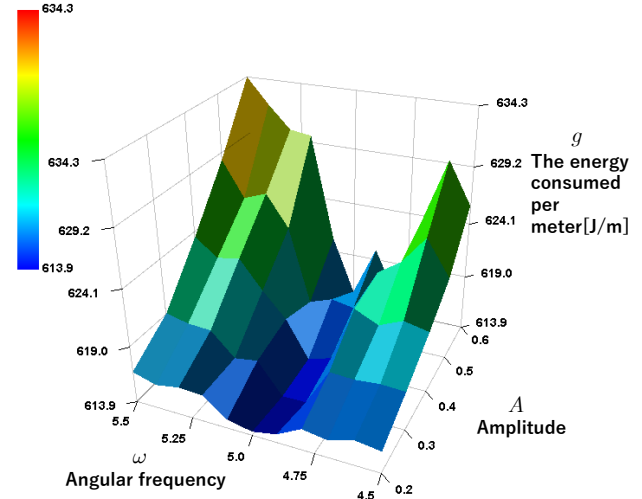


Fig. 3. The result of efficiency

4.1.2 Effectiveness for Speed

Next, we research the influence of arm swing on walking speed through full exploration experiment. We compare the walking speed indirectly by comparing the walking distance in 20 seconds under different arm swing conditions. Obviously, the further the distance, the faster the walking speed.

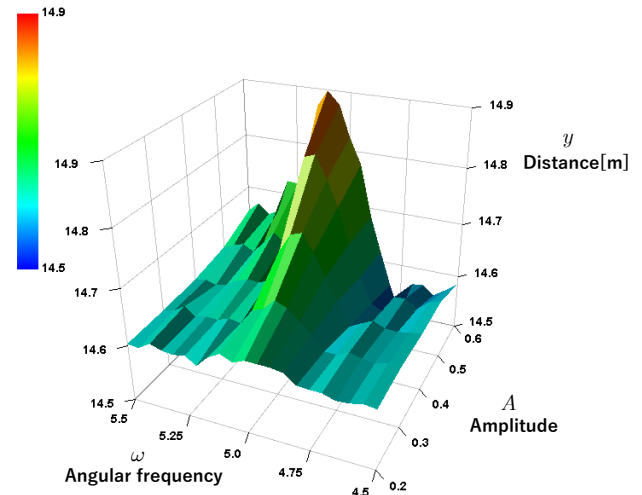


Fig. 4. The result of distance

The results of the experiment are shown in Fig.4. The longitudinal axis represents the maximum walking distance in all phases φ under the amplitude A and angular frequency ω corresponding to a point.

When the ω is close to the leg swing frequency (5.0), the walking speed is the fastest. At this time, the greater the A ,

the faster the walking speed.

When the frequency is different, the influence of the A on the walking speed is very small.

When the input torque of the arm is zero, the walking distance of the robot in 20 seconds is 14.73[m]. According to Fig.4, when ω is close to the leg swing frequency (5.0) and the A is large enough, the arm swing can accelerate the walking speed.

4.2 Analysis of data

We select the case(which $A = 0.6, \omega = 5.1, \varphi = 2.375$) with the highest speed among all the results in Fig.4 and analyze it with Eq.(22).

4.2.1 Walking appearance

The walking appearance is shown in Fig.5.

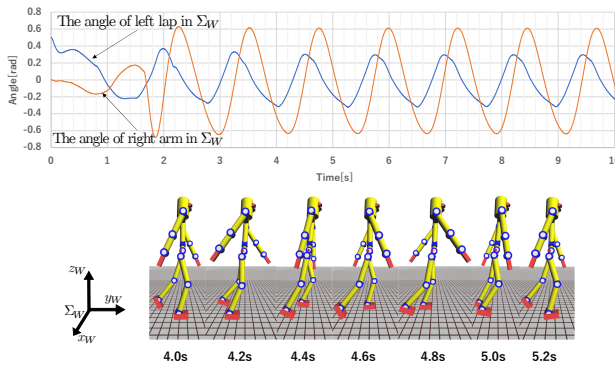


Fig. 5. Walking with arm swing($A = 0.6, \omega = 5.1, \varphi = 2.375$)

It can be understood from Fig.5 that ω and the frequency of the leg is completely consistent, while the φ is about 0.2 seconds later than the phase of the right lap. Which means, when the left leg starts to move forward, the right arm is still swinging backward, and when the left leg is recovered, the right arm will still swing forward for 0.2 seconds.

4.2.2 Analysis of dynamics

According to Fig.6, the order of the influence factors of the three joints of the right arm on walking is $(J_v M^{-1})_{2,12} > (J_v M^{-1})_{2,11} > (J_v M^{-1})_{2,13}$. The influence factor of elbow is opposite to the other two joints.

According to Fig.7, in fact, comparing with the other two joints, the shoulder plays an important role in walking, and almost all of influence are positive. The elbow only played a negative role.

Next, we sum the walking accelerations provided by the three joints to get the actual impact of the whole right arm on walking, as shown in Fig.8. Obviously, the reason why this arm swing mode have the best acceleration effect is that the whole process has only a positive impact on walking. When the angle of the right arm is positive, the acceleration provided by the arm is proportional to the angle of the arm.

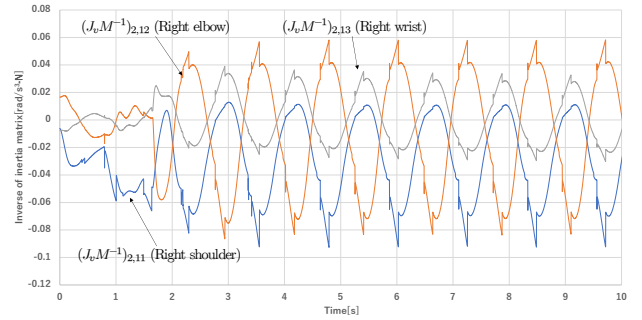


Fig. 6. The influence coefficient of the τ_i on walking direction

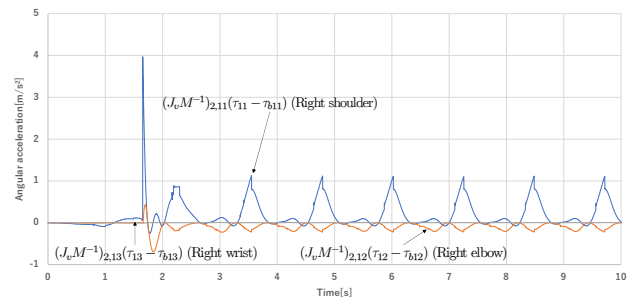


Fig. 7. The acceleration of walking direction provided by joint - i

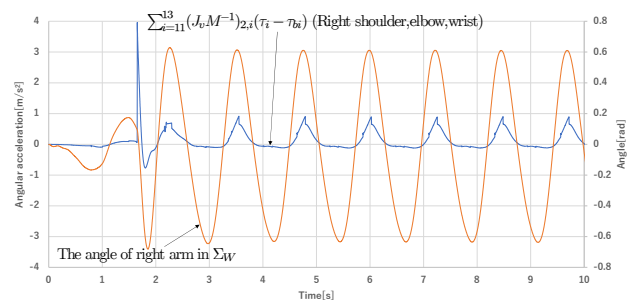


Fig. 8. The angle of the right arm and the sum acceleration of walking direction provided by three joints of right arm

5 CONCLUSION

We explore the influence of arm swing on the walking efficiency and speed. Although the arm swing will reduce the efficiency, the appropriate swing arm can effectively improve the walking speed. The so-called appropriate arm swing here means that ω and the frequency of the leg is completely consistent, φ is slightly delayed than the leg, and A is as large as possible within the allowable range of hardware. At this time, the whole arm swing has almost no negative effect on walking. The results explain why humans choose to swing their arms in race walking or speed skating competition.

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