Development of Redundant Robot Simulator for Avoiding Arbitrary Obstacles Based on Semi-Analytical Method of Solving Inverse Kinematics

Seiji Aoyagi, Member, IEEE, Kazuya Tashiro, Mamoru Minami, Member, IEEE, Masaharu Takano

Abstract— A method for solving obstacle avoidance for a redundant robot is proposed in the present paper. Extra degrees of freedom (DOF) of a redundant robot are effective for realizing an objective position and orientation of its end effector (referred to hereafter as "pose"), while the robot is avoiding obstacles. The path should be planned so as that the robot can avoid obstacles and realize the desired goal pose. The models of six elementary types of obstacles are assumed, taking account of real environment. The path planning method proposed herein is divided into three procedures as follows: 1) solving inverse kinematics by an analytical method for avoiding six elementary types of obstacles, 2) solving inverse kinematics by a semi-analytical method for realizing a goal pose, and 3) generating a path from a start pose to the goal one. A computational simulator for a redundant robot to avoid arbitrary obstacles based on these procedures is developed.

I. INTRODUCTION

F or the purpose of collision avoidance, a redundant robot having many rotational (or translational) joints is necessary [1]. Several researches are carried out to investigate how to decide the path of a redundant robot. In these researches, among many possible paths, one path is selected as a solution by minimizing some evaluation function [2-4], where it is focused on how to use redundant degrees of freedom (DOF) effectively. On the other hand, the present research assumes the situation that the robot configuration is rather firmly restrained by obstacles, and almost DOF are used for realizing an objective goal position and orientation of the end effector (referred to hereafter as "pose") and avoiding the obstacles, i.e., there are little extra DOF. Under this condition, it is focused on how to decide the joint angles so as to satisfy both the goal pose of the end effector and the restrained (specified) configuration of the arm, while avoiding the obstacles. Namely, inverse kinematics under the restraint by obstacles is important herein.

Newton method using a Jacobian matrix has been used for solving the inverse kinematics problem of a redundant robot.

Manuscript received April 7, 2007. This work was financially supported in part by the Kansai University Grant-in-Aid for the Faculty Joint Research Program, 2006-2007, "Study on Robot Vision Imitating Human Pattern Recognition Process".

S. Aoyagi, K. Tashiro, and M. Takano are with Department Systems Management Engineering, Kansai University, Suita, OSAKA 564-8680, JAPAN (Corresponding author: Seiji Aoyagi, phone: +81-6-638-0823; fax: +81-6-6330-3154; e-mail: aoyagi@iecs.kansai-u.ac.jp).

M. Minami is with Fukui University, Fukui 910-8507, JAPAN (e-mail: minami@rc.his.fukui-u.ac.jp).

However, this method needs large number of iterations and takes long computing time until a sufficient convergence of the solution is obtained. The computing time becomes longer as the total joint number, i.e., redundancy, becomes larger. Also, the solution cannot be obtained unless the initial value of the iterating calculation is taken as an appropriate value in the vicinity of the true value. Moreover, it gives only one solution depending on the initial value, while there are many solutions such as right hand configuration and left hand configuration, etc.

Considering these circumstances, an efficient semi-analytical method is proposed, which solves the inverse kinematics by utilizing analytical solution partially. In this method six elementary types of obstacles are considered. A computational simulator of robot motion is developed based on this method, which demonstrates that a 14 DOF robot can successfully pass thorough both a cylindrical hole and a gap, each of which is set on a different thick wall, and realize a final given pose of the end effector precisely.

II. OBSTACLE AVOIDANCE

Inverse kinematics of avoidance of six elementary obstacles is described hereinafter.

A. Model of Obstacles and Necessary DOF for Avoidance

Six types of obstacles are considered and modeled, as shown in Fig. 1. They are a point (contact with a ball), a straight line (contact with a pillar or a cylinder), a gap on a thin wall, a gap on a thick wall, a cylindrical hole on a thin wall, and a cylindrical hole on a thick wall. Any complex obstacle in the real world could be approximated by combining these elementally obstacles. Figure 2 is an example image that a stapler, a stick paste and two books on a desk. These objects can be modeled by combining these elementally obstacles.

The number of DOF which must be added for avoiding each model of obstacle is investigated. This number is decided based on how many DOF are restrained by the obstacle when the robot arm interferes with it. Models of obstacles and necessary additional DOF are shown in Fig. 1.

Total DOF of [6 + restrained DOF number by obstacles] are required at the least for avoiding obstacles and realizing the pose. The path, however, cannot be achieved by using only this DOF number in many cases. Further extra DOF would be necessary to solve this problem.



Fig. 1. Model of six elementary obstacles and necessary additional DOF. (a) Avoidance of point or ball. (b) Avoidance of pillar. (c) Avoidance of gap on thin wall. (d) Avoidance of gap on thick wall. (e) Avoidance of cylindrical hole in thin wall. (f) Avoidance of cylindrical hole in thick wall.



Fig. 2. Modeling of actual object by elementally obstacles.

B. Inverse Kinematics of Avoiding Obstacle

The method for solving inverse kinematics of avoidance of elementary obstacles is described in this section, where it is assumed as follows: (1) a robot arm is composed of only rotational joints, taking account that translational joints are rather inappropriate for avoiding arbitrary obstacles, (2) each link is a cylinder, the radius of which is c. There is no offset in the robot mechanism, (3) the radius of any joint is smaller than that of a cylinder of the link. The links and joints from the base are called as " L_0 - J_1 - L_1 - J_2 - \cdots - J_n - L_n ".

B.1. Avoidance of a point (ball)

When the link L_M contacts with a ball of which radius and center are c_B and r_B respectively, the link L_M must be located apart from r_B by the distance of more than $c + c_B$ by rotating the joint J_k (see Fig. 3). Assuming the joint angles besides ϕ_k are known, ϕ_k realizing contact limit is solved by coordinate transformation matrices. There are four patterns of solutions, which are inside and outside of the cylinder of link L_M and right and left rotations of J_k . Therefore, it is necessary to select one solution satisfying the given condition among the four.



Fig. 3. Inverse kinematics of avoidance of a ball.

B.2. Avoidance of a straight line (cylinder)

When the link L_M is avoiding a straight line of which position is \mathbf{r}_s , the link L_M must be located apart from the line by the distance of more than c (in case of cylindrical obstacle, it is $c + c_B$) by rotating joint J_k (see Fig. 4).

Assuming the joint angles besides ϕ_k are known, ϕ_k realizing contact limit is solved by coordinate transformation matrices. Similarly to the case of avoiding a point (a ball), there are four patterns of solutions.



Fig. 4. Inverse kinematics of avoidance of a straight line.

B.3. Avoidance of a gap on a thin wall

When the link L_M is passing through a gap on a thin wall, one lateral DOF is restricted by the gap. Other DOF of the link are free (see Fig. 5). The link L_M is located between the gap by rotating the joint J_k , which is located before the wall. There are two patterns of solutions of ϕ_k , which are right and left rotations.



Fig. 5. Inverse kinematics of avoidance of a gap on a thin wall.

B.4. Avoidance of a gap on a thick wall

When the link L_M is passing through a gap on a thick wall, two DOF, which are lateral movement and lateral rotation, are restricted by the gap (see Fig. 6). Therefore, two additional DOF are necessary. The joint J_M is located in front of the gap by rotating the joint J_k , which is located before the wall. And, the link L_M is located so as to be parallel to the center plane of the gap by rotating the joint J_M . There are two patterns of solutions for J_k and J_M , respectively, and totally there are four patterns of solutions. However, only the case in which J_M is before the wall and J_{M+1} is after the wall is the real solution.



(1) The joint J_M is located in front of the gap by rotating J_k .

(2) The link L_M is located so as to be parallel to the center plane of the gap by rotating J_M .

Fig. 6. Inverse kinematics of avoidance of a gap on a thick wall.

B.5. Avoidance of a cylindrical hole in a thin wall

When the link L_M is passing through a cylindrical hole in a thin wall, two DOF of lateral movements are restricted by the hole (see Fig. 7). Therefore, two additional DOF are necessary. The link L_M is located so as that it penetrates the hole by rotating the joints J_k and J_M . First, ϕ_k is defined so as

that the plane, on which L_M is located, intersects the hole. Second, ϕ_M is defined so as that the L_M penetrates the hole. There are two patterns of solutions; however, only the case in which J_{M+1} is located after the wall is the real solution.



- (1) ϕ_k is defined so as that the plane on which L_M is located intersects the hole.
- (2) ϕ_{M} is defined so as that the L_{M} penetrates the hole.

Fig. 7. Inverse kinematics of avoidance of a cylindrical hole in a thin wall.

B.6. Avoidance of a cylindrical hole in a thick wall

When the link L_M is passing through a cylindrical hole in a thick wall, four DOF, which are two lateral movements and two rotational movements, are restricted by the hole (see Fig. 8). Therefore, four additional DOF are necessary. The problem is divided to two sub-problems as follows: 1) J_M is located on the hole axis by rotating J_i and J_j , 2) L_M is on the hole axis by rotating J_k and J_M . Since these two sub-problems are not independent to each other, they are carried out alternatively and iteratively until the satisfactory solution is obtained.



(1) J_M is located on the hole axis by rotating J_i and J_j .

(2) L_M is located on the hole axis by rotating J_K and J_M .

Fig. 8. Inverse kinematics of avoidance of a cylindrical hole in a thick wall.

Namely, under the state of fixing other joints besides the four joints of J_i , J_j , J_k , and J_M , this method is as follows:

- (1) (J_k, J_M) is decided appropriately.
- (2) (J_i, J_j) is decided so as that the joint J_M is located on the hole axis.
- (3) (J_k, J_M) is decided so as that the link L_M is located on the hole axis.
- (4) It is examined whether the link L_M is passing through the hole.
- (5) If the error is large, return to process (2).
- (6) If the error is small enough, the calculation is finished.

This calculation is surely converges provided that the relationship of i, j < k holds true.

III. METHOD FOR SOLVING POSITIONING PROBLEM WHILE AVOIDING OBSTACLES

Inverse kinematics of realizing a goal pose of its end effector is described in this section. An efficient method is proposed, which solves the inverse kinematics by utilizing analytical solution partially [5]. It is possible to solve the position inverse kinematics analytically by using 3 joints among *n* joints, where *n* is the number of DOF. Similarly it is possible to analytically solve orientation inverse kinematics by using wrist 3 joints. Also, it is possible to obtain the configuration analytically, which avoids collision such as passage through wall gaps, holes, etc., by using other remaining joints. These position, orientation, and configuration for avoidance are not independent from each others. Therefore, these three analytical syntheses are carried out iteratively until a sufficient convergence is obtained. Thus, this method is semi-analytical one.

For an example, the case is assumed that a 12 DOF robot passes thorough both a cylindrical hole and a gap, each of which is set on a different thick wall, as shown in Fig. 9. It is possible to synthesize the end effector's position analytically by using 3 joints ($\boldsymbol{\Phi}_3$) among 12 joints, and it is possible to synthesize analytically the end effector's orientation by using 3 wrist joints ($\boldsymbol{\Phi}_4$). Also, it is possible to synthesize the configuration analytically, which avoids a cylindrical hole by using other remaining joints ($\boldsymbol{\Phi}_1$) and a gap by using other further remaining joints ($\boldsymbol{\Phi}_2$), where $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$, $\boldsymbol{\Phi}_3$, $\boldsymbol{\Phi}_4$ are as follows:

$$\boldsymbol{\Phi}_{1} = (\phi_{1}, \phi_{2}, \phi_{4}, \phi_{5})^{T}, \quad (1)$$

$$\boldsymbol{\Phi}_{2} = (\phi_{7}, \phi_{8})^{T}, \quad (2)$$

$$\boldsymbol{\Phi}_{3} = (\phi_{3}, \phi_{6}, \phi_{9})^{T}, \quad (3)$$

$$\boldsymbol{\Phi}_{4} = (\phi_{10}, \phi_{11}, \phi_{12})^{T}. \quad (4)$$

Let the restraint condition for passage through the hole be described as the function H_1 , which is concretely the cluster of four equations to put through the link L_6 into the hole. Also, let the restraint condition for passage through the gap be



Fig. 9. Example case that a 12 DOF robot passes through a hole and a gap on two thick walls.

described as the function H_2 , which is the cluster of two equations to put through the link L_9 into the gap (see Fig. 9). And, let the objective position of the end effector (work piece) be described as $r_w^{(0)}$, and let the objective orientation be $E_w^{(0)}$. Since they are functions of ϕ_1 , ϕ_2 , --, ϕ_{12} , let us assume that they are expressed as follows:

$$\boldsymbol{H}_{1} = \boldsymbol{f}_{1}(\boldsymbol{\Phi}_{1}; \boldsymbol{\Phi}_{2}, \boldsymbol{\Phi}_{3}, \boldsymbol{\Phi}_{4}), \qquad (5)$$

$$\boldsymbol{H}_{2} = \boldsymbol{f}_{2}(\boldsymbol{\Phi}_{2}; \boldsymbol{\Phi}_{1}, \boldsymbol{\Phi}_{3}, \boldsymbol{\Phi}_{4}), \qquad (6)$$

$$\boldsymbol{r}_{w}^{(\boldsymbol{\theta})} = \boldsymbol{f}_{3}(\boldsymbol{\Phi}_{3}; \boldsymbol{\Phi}_{1}, \boldsymbol{\Phi}_{2}, \boldsymbol{\Phi}_{4}), \qquad (7)$$

$$\boldsymbol{E}_{\boldsymbol{w}}^{(\boldsymbol{\theta})} = \boldsymbol{f}_{4}(\boldsymbol{\Phi}_{4}; \boldsymbol{\Phi}_{1}, \boldsymbol{\Phi}_{2}, \boldsymbol{\Phi}_{3}).$$
(8)

In these expressions, $f_1(\Phi_1; \Phi_2, \Phi_3, \Phi_4)$ means Φ_1 is variable, while Φ_2, Φ_3, Φ_4 are remained constant. These equations can be analytically solved. Let solutions be expressed as follows:

$$\boldsymbol{\Phi}_{1} = \boldsymbol{f}_{1}^{-1}(\boldsymbol{H}_{1}; \boldsymbol{\Phi}_{2}, \boldsymbol{\Phi}_{3}, \boldsymbol{\Phi}_{4}), \qquad (9)$$

$$\boldsymbol{\Phi}_2 = \boldsymbol{f}_2^{-1}(\boldsymbol{H}_2; \boldsymbol{\Phi}_1, \boldsymbol{\Phi}_3, \boldsymbol{\Phi}_4), \qquad (10)$$

$$\boldsymbol{\Phi}_{3} = \boldsymbol{f}_{3}^{-1}(\boldsymbol{r}_{w}^{(0)}; \boldsymbol{\Phi}_{1}, \boldsymbol{\Phi}_{2}, \boldsymbol{\Phi}_{4}), \qquad (11)$$

$$\boldsymbol{\Phi}_{4} = \boldsymbol{f}_{4}^{-1}(\boldsymbol{E}_{w}^{(0)}; \boldsymbol{\Phi}_{1}, \boldsymbol{\Phi}_{2}, \boldsymbol{\Phi}_{3}).$$
(12)

In this method, these four analytical syntheses are carried out iteratively until the sufficient convergence is obtained. If the convergence is not enough, the combination of joints for positioning the end effector and avoiding the obstacles (combination of joints for orientating end effector is fixed to the wrist 3 joints) is changed and the iterative analytical synthesis is carried out again. Flowchart of this calculation method is shown in Fig. 10.

The features of the method are as follows: 1) iteration number is very small and computing time is reduced to about one tenth of that computed by Newton method (see an example of Fig. 11), 2) the initial value of iterative



Fig. 10. Flowchart of inverse kinematics for avoiding obstacles and positioning end effector.



Fig. 11. Comparison between Newton method and Semi-analytical method.

calculation is defined analytically (it is surely in the vicinity of true value), 3) it gives all solutions, whereas Newton method gives only one solution, so one can choose the most adequate solution for his task among them.

In this section, only two elementary types of a hole in a thick wall and a gap on a thick wall are dealt with. However, other cases including other elementary obstacle types among the six can be surely solved in the same way (datails are omitted for want of space).

IV. PATH GENERATION

Path generation is herein described. There are infinite paths which satisfy both the initial and goal configurations. However, most of them are prevented by obstacles existing on its way. For the purpose of path generation, two methods are possible as follows:

- Setting the middle configurations, and solving inverse kinematics at these configurations. The robot joints are rotated linearly between these configurations.
- (2) One joint, which is located before the obstacle, is rotated gradually so as that the arm tip is pulled in the direction from the goal to the robot base, while other joints are used for keeping avoiding condition of all obstacles.

The former method can produce the smooth path. However, it takes much calculation time for positioning at each middle configuration. Moreover, there is a possibility of collision in contact with the obstacle between each middle point. On the other hand, the latter method can shorten the calculation time, since only the obstacle avoidance is dealt with. Moreover, it is possible to pull out the arm by small number of joints. However, the shape of the path is not known until the path is generated and it could be distorted. It is recommended for the path generation to use an appropriate method among the two, considering the demanded smoothness in the task, the structure of the robot, the kind of the obstacles, etc. totally. In the present paper, the latter method is employed.

The path is obtained by the method of avoiding obstacles sequentially from the goal configuration to the initial one. The path from the initial configuration to the goal one is easily obtained by tracking this obtained path reversely. The detail of the present method is as follows: first, the robot is put into the state of goal configuration avoiding all obstacles. Second the first joint J_1 is rotated gradually for pulling the arm tip in the direction from the goal to the robot base, while other joints are used for keeping avoiding condition of all obstacles.

Note that the resultant path (the transition of the end effector's position) meanders owing to the path generation method mentioned above. However, the end effector's orientation can be arbitrarily controlled during the path except for the inside of the obstacle, since the orientation is realized by the wrist 3 joints, which are not used for the path generation.

V. SIMULATION RESULTS

Based on the proposed method, a computer simulator is developed for solving the inverse kinematics of a redundant robot under restraint by obstacles. Windows XP is adopted as an operating system and Visual C++ is adopted as a programming language. The CPU is Pentium 4 (1.70GHz) and the memory size is 256 MB. The total computing time for the solutions tested in this research is about 1 minute. The simulation results can be graphically shown in a computer display.

For an example, the case that a robot passes a cylindrical hole in the 1st thick wall (see Fig. 1(f)) and a gap on the 2nd thick wall (see Fig. 1(d)) is simulated. The result is shown



Fig. 12. Results of simulation in case of passing through hole and gap (Forward path is obtained by following these figures conversely). (a) Goal pose. (b) J10 passed through the 2nd wall. (c) Work piece passed through the 2nd wall. (d) J6 passed through the 1st wall. (e) J7 passed through the 1st wall. (f) J8 passed through the 1st wall. (g) J9 passed through the 1st wall. (h) J10 passed through the 1st wall. (i) Work piece passed through the 1st wall. (j) Initial pose.

in Fig. 12. This robot has 14 rotational joints, of which joint composition is RPP'PP'PP'PP'PP' RPR, where R, P, P' mean the rotational joint, the pivot joint, and the pivot joint whose axis is perpendicular to that of P joint, respectively.

This robot can independently do positioning its tip and avoiding obstacle by adding two further DOF to the necessary and sufficient DOF. As a result, the calculation time is shortened, and the movable range after the avoidance of obstacles is widened. Seeing this figure, the robot can surely avoid these two obstacles. This case is only one example. The method of avoiding obstacles proposed in this paper is thought to be effective to more various obstacles, which are combination of the six types of elementally obstacles as shown in the section 2.

VI. CONCLUSIONS

For the purpose of collision avoidance of any type of obstacles, an efficient method for solving inverse kinematics of a redundant robot is proposed. This method utilizes analytical solution both for collision avoidance and position/orientation inverse kinematics. This method is divided into three procedures; 1) position synthesis by 3 joints, 2) orientation synthesis by 3 wrist joints, and 3) collision avoidance by other remained joints. Each of these three procedures can be calculated analytically. After series of three procedures, the positioning error is occurred, since they are not independent from each other. Therefore, the calculation carried out iteratively until the sufficient convergence is obtained. A simulator based on this method is developed. An example is shown that a robot of 14 DOF can successfully pass through the hole and the gap.

ACKNOWLEDGMENT

This work was financially supported in part by the Kansai University Grant-in-Aid for the Faculty Joint Research Program, 2006-2007, "Study on Robot Vision Imitating Human Pattern Recognition Process".

REFERENCES

- T. Yoshikawa, "Analysis and Control of Robot Manipulators with Redundancy, Robotics Research", *The First International Symposium, (M.Brady and R. Paul ed.)*, MIT Press, Cambridge, Mass. pp.735-747, 1984.
- [2] Y. Nakamura, H. Hanafusa, and T. Yoshikawa, "Task-Priority Based Redundancy Control of Robot Manipulators", *The International Journal of Robotics Research*, Vol.6, No.2, pp.3-15, 1987.
- [3] H. Hirukawa, S. Kitamura, "A collision Avoidance Method for Robot Manipulators based on Safty First Algorithm and the Potential Function", *Journal of the Robotics Society of Japan*, Vol.5, No.3, pp.3-11, 1987.
- [4] M.Takano, "A New Effective Solution for Inverse Kinematics Problem(Synthesis) of a Robot with Any Type of Configuration", J. Fac. Eng., Univ. Tokyo(B), Vol.38, No.2, pp.107-135, 1985.
- [5] J.Kawamoto, T.HARA, K.Bando, M.Arakawa, S.Nakahara, K.Makihira, M.Takano, S.Aoyagi, "Method for Solving Inverse Kinematics of Redundant Robot under Restraint by Obstacles", *Proc. Int. Conf. on Machine Automation (ICMA2004)*, pp.113-118, 2004.