High Accuracy / Low-Energy Consumption Effects of Bracing Hyper-Redundant Manipulator

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***Abstract*—Hindrances interfering realistic and practical utilization of hyper-redundant manipulator are thought to be the facts that the higher redundant degrees make the weight of the structure heavier, resulting in a conflict between the required high-redundancy for dexterous manipulation and heavy weight stemming from the high redundancy. To overcome this contradicting realities of hyper-redundant manipulator, we discuss a realizability whether the contacting and bracing motion of intermediate links with environment may simultaneously reduce energy-consumption and raise hand's trajectory tracking accuracy, inspired by human's handwriting motion with the elbow or wrist contacting to a table. We first propose a dynamical model of hyper-redundant manipulator whose plural intermediate links are being braced with environment, second how the contacting can save energy and raise trajectory tracking accuracy. The simulation result shows that less-energy consumption and high trajectory tracking accuracy are achieved by conventional PD controller, compared with non-contacting condition.**

I. INTRODUCTION

It is a common sense that robot's dexterous manipulation needs high redundancy. However the weight of manipulator increases as the degree of redundancy rises, resulting in that the controlling of manipulator becomes difficult. Since increasing of redundancy amplifies the influence of gravity toward each link, and that the effect is highly nonlinear. Then the motion controller should be at with high motion control gains and large-sized motors with adequate output, which may cause the whole robot system unstable.

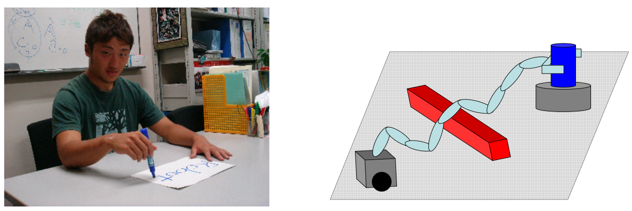
For solving this problem we obtain some inspirations about effective motion control strategies by observing human's handwriting motion . Writing a character on a paper with contacting one's elbow as shown Fig. 1(a), which is one of examples of human's skillful behavior thought to be exploiting the contact constraint of the elbow with the table for reducing inputting energy by countering the gravity effects with reaction forces. By supporting the elbow or wrist by the desk, we have known from our experiences that we can save energy for writing task and can write characters correctly. This suggests that robots may execute tasks with less energy and improved accuracy by exploiting constrained contacting with environments.

On the other hand, hyper-redundant manipulator had been researched intensively and those efforts had been introduced by Chirijian and Burdick[1] more than ten years ago, where the structure of the discussed hyper-redundant manipulator could not move in 3D space but restricted in the 2D space on a surface of table. Though considerable researches have discussed how to utilize the redundancy [2]-[5], for example avoiding obstacles [6]-[9] or optimizing the configuration concerning practical criteria [10],[11], etc.. It seems that the merits of the hyper-redundancy has not been utilized enough effectively and practically.

Here we think that higher redundant degree requires heavier structure's weight, having the robot system difficult to work in 3D task space. Moreover heavier the manipulator, easier its end effector falls down by gravity influence, which has the control precision of the end-effector getting worse.

To solve this problem, we consider that human being can do work precisely using less force by putting his arm's weight on his elbow like Fig.1(a). And in this paper, such effective motion strategy being energy-saving and accurate will be discussed especially how to utilize aggressively the contacting behavior with the environments, which is shown in Fig.1(b) depicting 10-links redundant manipulator whose intermediate links contact to the floor at two points, while the hand is required to draw a circle in 3D task space.

In this research, first we propose a new dynamical model of manipulator with multi-elbow, comprising manipulator's dynamics and geometrical constraint conditions, realized through the synthesization of multi-constraint condition of elbows and manipulator's motion equation. Even though a controller used for end-effector's trajectory tracking task is simple PD controller, it has been evaluated how the contacting strategy and improve tracking accuracy and energy-saving performances.



(a) Human's writing motion (b) Contacting strategy

Fig. 1. The sketch picture of Hyper-Redundant Manipulator with elbows

II. CONSTRAINT MOTION

A. *Manipulator's Model with Hand's Constraint*

To make the explanation of constraint motion with multi-elbow be easily understandable, we discuss firstly about the model of the manipulator whose end-effector is contacing rigid genvironment withot elasticity. Equation of motion of manipulator comprosing rigid structure of s links, and also contact relation between manipulator's end-effector and definition of constraint surface should be introduced firstly. Here contact friction is ignored. L represents lagrangian, $\boldsymbol{q} \in R^s$ represents the general coordinate, $\boldsymbol{\tau} \in R^s$ represents the general input. u is the unknown constant of lagrange, f_t is the friction. Manipulator hand's lagrange condition equation due to [17],[18],[19],[20] can be expressed as follows

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}) - (\frac{\partial L}{\partial \boldsymbol{q}}) = \boldsymbol{\tau} + (\frac{\partial C}{\partial \boldsymbol{q}^T})^T u - (\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^T})^T \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|} f_t \quad (1)$$

Here according to the kinematic relation, manipulator hand's position/posture vector $\boldsymbol{r} \in R^s$ and scalar function, a single constraint condition C which used to express the hypersurface can be expressed as

$$\boldsymbol{r} = \boldsymbol{r}(\boldsymbol{q}) \quad (2)$$

$$C(\boldsymbol{r}(\boldsymbol{q})) = 0 \quad (3)$$

The freedom of the end-effctor to move freely in the direction of non-constraint is left to be more than one, so here $s > 1$. If we set f_n to indicate the constraint force of manipulator hand, then the relation of u and f_n can be expressed as

$$u = f_n / \| \frac{\partial C}{\partial \boldsymbol{r}^T} \| \quad (4)$$

$\|\partial C / \partial \boldsymbol{r}^T\|$ shows Euclidean norm of vector $\partial C / \partial \boldsymbol{r}^T$. Then manipulator's motion equation can be derived from the combination of Eq(1) and Eq(4) with viscous friction of joints [12].

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ = \; & \boldsymbol{\tau} + \{(\frac{\partial C}{\partial \boldsymbol{q}^T})^T / \| \frac{\partial C}{\partial \boldsymbol{r}^T} \| \} f_n - (\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^T})^T \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|} f_t \quad (5) \end{aligned}$$

$\boldsymbol{M}$ is inertia matrix of $s \times s$, $\boldsymbol{h}$ and $\boldsymbol{g}$ are $s \times 1$ vectors which indicate the effects from coriolis force, centrifugal force and gravity, $\boldsymbol{D}$ is a $s \times s$ matrix which indicates the coefficient of joints' viscous friction, expressed as $\boldsymbol{D} = diag[D_1, D_2, \cdots, D_s]$. $\boldsymbol{q}$ is the joint angle and $\boldsymbol{\tau}$ is the input torque.

B. Model with Multiple Constraints

Here we consider a motion of a manipulator having s links whose elbows are contactig at p points to the environments defined as

$$C_i(\boldsymbol{r}_i(\boldsymbol{q})) = 0, \quad (i = 1, 2, \cdots, p) \quad (6)$$

where $\boldsymbol{r}_i$ is the equation of position and posture of link i contacting with constraint, like Eq(2).

$$\boldsymbol{r}_i = \boldsymbol{r}_i(\boldsymbol{q}) \quad (7)$$

The Eq(5) describes a motion of the manipulator whose hand is constraint. Under the situation with the i-th link contacting, then we can define following two vectors concering i-th constraint condition C_i as follows,

$$(\frac{\partial C_i}{\partial \boldsymbol{q}^T})^T / \| \frac{\partial C_i}{\partial \boldsymbol{r}^T} \| = \boldsymbol{j_c}_i^T \quad (8)$$

$$(\frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{q}^T})^T \frac{\dot{\boldsymbol{r}}_i}{\|\dot{\boldsymbol{r}}_i\|} = \boldsymbol{j_t}_i^T \quad (9)$$

Accumulating all the above vectors ($i = 1, 2, \cdots, p$) when p links are contacting, the next is redefined.

$$\begin{aligned} \boldsymbol{J_c}^T &= [\boldsymbol{j_c}_1^T, \; \boldsymbol{j_c}_2^T, \; \cdots, \; \boldsymbol{j_c}_p^T] \quad (10) \\ \boldsymbol{J_t}^T &= [\boldsymbol{j_t}_1^T, \; \boldsymbol{j_t}_2^T, \; \cdots, \; \boldsymbol{j_t}_p^T] \quad (11) \\ \boldsymbol{f_n} &= [f_{n1}, \; f_{n2}, \; \cdots, \; f_{np}]^T \quad (12) \\ \boldsymbol{f_t} &= [f_{t1}, \; f_{t2}, \; \cdots, \; f_{tp}]^T \quad (13) \end{aligned}$$

$\boldsymbol{J_c}^T, \boldsymbol{J_t}^T$ are $s \times p$ matrix,$\boldsymbol{f_n}, \boldsymbol{f_t}$ are $p \times 1$ vectors. Considering about p constraints of the intermediate links, the manipulator's equation of motion can be expressed as

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ = \; & \boldsymbol{\tau} + \sum_{i=1}^{p} (\boldsymbol{j_c}_i^T f_{ni}) - \sum_{i=1}^{p} (\boldsymbol{j_t}_i^T f_{ti}) \\ = \; & \boldsymbol{\tau} + \boldsymbol{J_c}^T \boldsymbol{f_n} - \boldsymbol{J_t}^T \boldsymbol{f_t}. \quad (14) \end{aligned}$$

Moreover, Eq (6) is differentiated by time t two times, then we can derive the constraint condition of $\ddot{q}$.

$$\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_i}{\partial \boldsymbol{q}^T})\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}} + (\frac{\partial C_i}{\partial \boldsymbol{q}^T})\ddot{\boldsymbol{q}} = 0 \quad (15)$$

To make sure that manipulator hand is contacting with the constraint surface all the time, value of $\boldsymbol{q}(t)$ in Eq(14) has always to satisfy Eq(6) which has no relation with time t, if value of $\ddot{\boldsymbol{q}}$ in Eq(15) should have the same value with $\ddot{\boldsymbol{q}}$ in Eq(14), then value of $\boldsymbol{q}(t)$ in Eq(14) and Eq(6) always keep the same regardless of time.

C. Robot's Dynamics Including Motors

In this research, we want to evaluate the effects to increase the trajectory tracking accuracy and reduce the energy consumption used for countering the gravity force and other effects by bracing the intermediate links. Even though there is no robot's motion –robot is stopping– the energy is kept to be consuming since motors of joints have to generate torques to maintain the required robot's configuration against gravity influences. When the robot is in motion, other effects of dynamics will be added more to the gravity effect. To evaluate this kind of wasted energy consumption, we included the effects of electronic circuit of servo motor into the equation of motion of the manipulator to represent explicitely that the robot consumes energy even while stopping.

Here v_i represents motor's voltage, R_i does resistance, L_i, i_i do the inductance and electric current, θ_i does the angular phase of motor, τ_{gi} does the motor ouput torque, τ_{Li} does the load torque, v_{gi} does electromotive force, I_{mi} does the inertia moment of motor, K_{Ei} does the constant of electrmotive force, K_{Ti} does the constant of torque, d_{mi} does the viscous friction's coefficient of speed reducer. The relation of those variables are shown hereunder.

$$\begin{aligned} v_i(t) &= L_i \dot{i}_i + R_i i_i(t) + v_{gi}(t) && (16) \\ v_{gi}(t) &= K_{Ei}\dot{\theta}_i(t) && (17) \\ I_{mi}\ddot{\theta}_i &= \tau_{gi}(t) - \tau_{Li}(t) - d_{mi}\dot{\theta}_i && (18) \\ \tau_g(t) &= K_{Ti} i_i(t) && (19) \end{aligned}$$

From the relation of magnetic field and the coefficients above, $K_{Ti} = K_{Ei}(= K)$ holds for motors used. Combine Eq (17) and Eq (16), and also Eq (19) and Eq (18), we derive

$$\begin{aligned} v_i &= L_i \dot{i}_i + R_i i_i + K_i\dot{\theta}_i && (20) \\ I_{mi}\ddot{\theta}_i &= K_i i_i - \tau_{Li} - d_{mi}\dot{\theta}_i && (21) \end{aligned}$$

In the situation with motor and gear whose reduction ratio is k_i are installed onto manipulator,

$$\begin{aligned} \theta_i &= k_i q_i && (22) \\ \tau_{Li} &= \frac{\tau_i}{k_i} && (23) \end{aligned}$$

Combining Eq (20) and Eq (21) into equation with $\dot{i}_i$ and τ_i, following equations are got

$$\begin{aligned} L_i \dot{i}_i &= v_i - R_i i_i - K_i k_i \dot{q}_i && (24) \\ \tau_i &= -I_{mi}k_i^2\ddot{q}_i + K_i k_i i_i - d_{mi}k_i^2\dot{q}_i && (25) \end{aligned}$$

Then using vector and matrix to indicate Eq (24) and (25),

$$\begin{aligned} \boldsymbol{L}\dot{\boldsymbol{i}} &= \boldsymbol{v} - \boldsymbol{R}\boldsymbol{i} - \boldsymbol{K_m}\dot{\boldsymbol{q}} && (26) \\ \boldsymbol{\tau} &= -\boldsymbol{J_m}\ddot{\boldsymbol{q}} + \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{D_m}\dot{\boldsymbol{q}} && (27) \end{aligned}$$

here,

$$\begin{aligned} \boldsymbol{v} &= [v_1, v_2, \cdots, v_s]^T \\ \boldsymbol{i} &= [i_1, i_2, \cdots, i_s]^T \end{aligned}$$

and the definitions are shown as follow, which always have positive value.

$$\begin{aligned} \boldsymbol{L} &= diag[L_1, L_2, \cdots, L_s] \\ \boldsymbol{R} &= diag[R_1, R_2, \cdots, R_s] \\ \boldsymbol{K_m} &= diag[K_{m1}, K_{m2}, \cdots, K_{ms}] \\ \boldsymbol{J_m} &= diag[J_{m1}, J_{m2}, \cdots, J_{ms}] \\ \boldsymbol{D_m} &= diag[D_{m1}, D_{m2}, \cdots, D_{ms}] \\ K_{mi} &= K_i k_i,\ J_{mi} = I_{mi}k_i^2,\ D_{mi} = d_{mi}k_i^2 \end{aligned}$$

Now substitute Eq (27) into Eq (14), we get

$$\begin{aligned} &(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{J_m})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} \\ &= \boldsymbol{K_m}\boldsymbol{i} + \boldsymbol{J_c}^T\boldsymbol{f_n} - \boldsymbol{J_t}^T\boldsymbol{f_t} \qquad (28) \end{aligned}$$

Similar to the same relation between Eq (14) and Eq (15), the value of $\ddot{\boldsymbol{q}}$ in Eq (28) have to be identical to the the value of $\ddot{\boldsymbol{q}}$ in Eq (15) representing constrain condition.

D. Robot/Motor Equation with Contact Constraint

To make sure that $\ddot{\boldsymbol{q}}$ in Eq (28) and (15) be identical, constraint force $\boldsymbol{f_n}$ is subordinately decided by simultaneous equation. Then Eq (28),(15) should be transformed as follow

$$\begin{aligned} &(\boldsymbol{M} + \boldsymbol{J_m})\ddot{\boldsymbol{q}} - \boldsymbol{J_c}^T\boldsymbol{f_n} \\ &= \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T\boldsymbol{f_t} \qquad (29) \\ &(\frac{\partial C_i}{\partial \boldsymbol{q}^T})\ddot{\boldsymbol{q}} = -\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_i}{\partial \boldsymbol{q}})\dot{\boldsymbol{q}}\right]\dot{\boldsymbol{q}} \\ &= -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_i}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}} \qquad (30) \end{aligned}$$

Then Eqn (29),(30),(24) can be expressed as follow. Here we assumed that friction force f_{ti} is dynamic friction and define it as $f_t = 0.1 f_n (i = 1, 2, \cdots, p)$.

$$\begin{bmatrix} \boldsymbol{M} + \boldsymbol{J_m} & -\boldsymbol{j_{c1}}^T & \cdots & -\boldsymbol{j_{cp}}^T & 0 & \cdots & 0 \\ \frac{\partial C_1}{\partial \boldsymbol{q}^T} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial C_p}{\partial \boldsymbol{q}^T} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & L_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & L_s \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ f_{n1} \\ \vdots \\ f_{np} \\ \dot{i}_1 \\ \vdots \\ \dot{i}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{K_m}\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T\boldsymbol{f_t} \\ -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_1}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}} \\ \vdots \\ -\dot{\boldsymbol{q}}^T\left[\frac{\partial}{\partial \boldsymbol{q}}(\frac{\partial C_p}{\partial \boldsymbol{q}^T})\right]\dot{\boldsymbol{q}} \\ v_1 - R_1 i_1 - K_{m1}\dot{q}_1 \\ \vdots \\ v_i - R_s i_s - K_{ml}\dot{q}_s \end{bmatrix} \qquad (31)$$

The inertia term $(\boldsymbol{M} + \boldsymbol{J_m})$ is $s \times s$ matrix, the coefficient vector of constraint force $\boldsymbol{j_{ci}}^T$ is $s \times 1$ vertical vector, $\partial C_i / \partial \boldsymbol{q}^T$ is $1 \times s$ horizontal vector, inductance term $\boldsymbol{L}$ is $s \times s$ diagonal matrix, therefore, the matrix of the first term in left side in Eq (31) is matrix of $(2s + p) \times (2s + p)$.

Then Eq (31) can be rewritten concisely using the definitions of Eq (10),(12),(26) as follow,

$$\begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -\boldsymbol{J_c}^T & \boldsymbol{0} \\ \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{f_n} \\ \dot{\boldsymbol{i}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{K_m i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T \boldsymbol{f_t} \\ -\dot{\boldsymbol{q}}^T \left[\frac{\partial}{\partial \boldsymbol{q}} (\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T}) \right] \dot{\boldsymbol{q}} \\ \boldsymbol{v} - \boldsymbol{Ri} - \boldsymbol{K_m}\dot{\boldsymbol{q}} \end{bmatrix} \quad (32)$$

where, VC is a vector of $\boldsymbol{C} = [C_1, C_2, \cdots, C_p]^T$. Furthermore by redefining as

$$\boldsymbol{M}^* = \begin{bmatrix} \boldsymbol{M}+\boldsymbol{J_m} & -\boldsymbol{J_c}^T & \boldsymbol{0} \\ \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \quad (33)$$

$$\boldsymbol{b} = \begin{bmatrix} \boldsymbol{K_m i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D}+\boldsymbol{D_m})\dot{\boldsymbol{q}} - \boldsymbol{J_t}^T \boldsymbol{f_t} \\ -\dot{\boldsymbol{q}}^T \left[\frac{\partial}{\partial \boldsymbol{q}} (\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^T}) \right] \dot{\boldsymbol{q}} \\ \boldsymbol{v} - \boldsymbol{Ri} - \boldsymbol{K_m}\dot{\boldsymbol{q}} \end{bmatrix} \quad (34)$$

Then Eq (32) can be expressed as

$$\boldsymbol{M}^* \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{f_n} \\ \dot{\boldsymbol{i}} \end{bmatrix} = \boldsymbol{b} \quad (35)$$

If $\boldsymbol{M}^*$ is confirmed to be nonsingular matrix, the unknown value of $\ddot{\boldsymbol{q}}$,$\boldsymbol{f_n}$,$\dot{\boldsymbol{i}}$ can be determined based on the above simultaneous equation. In the next section we will confirm whether $\boldsymbol{M}^*$ is non-singular or not.

III. NON-SINGULARITY OF $\boldsymbol{M}^*$

First, we consider a block matrix of $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}$. In general, if a whole matrix $\boldsymbol{A}$ is non-singular, the necessary and sufficient condition to guarantee that $\boldsymbol{A}$ is non-singular is that $\boldsymbol{A}_{22} - \boldsymbol{A}_{21}\boldsymbol{A}_{11}^{-1}\boldsymbol{A}_{12}$ is non-singular [13]. Then when $\boldsymbol{A}_{11}$ is non-singular, $det\boldsymbol{A}$ can be decomposed as,

$$det\boldsymbol{A} = det\boldsymbol{A}_{11} \cdot det(\boldsymbol{A}_{22} - \boldsymbol{A}_{21}\boldsymbol{A}_{11}^{-1}\boldsymbol{A}_{12}) \quad (36)$$

Using Eq (36), let us consider the singularity of $\boldsymbol{M}^*$. The $(s+1)\times(s+1)$ block matrix of upper-left in $\boldsymbol{M}^*$ is picked up first as

$$\begin{bmatrix} \boldsymbol{M}_0 & -\boldsymbol{j_{c}}_1^T \\ \frac{\partial C_1}{\partial \boldsymbol{q}^T} & 0 \end{bmatrix} \triangleq \boldsymbol{M}_1 \quad (37)$$
$$(\boldsymbol{M}_0 = \boldsymbol{M} + \boldsymbol{J_m})$$

Since $\boldsymbol{M}_0 > 0$ from $\boldsymbol{M} > 0$ and $\boldsymbol{J_m} > 0$, and definition of $\boldsymbol{j_{c}}_i^T$ in Eq (8), the determinant of $\boldsymbol{M}_1$ can be calculated as

$$\begin{aligned} det\boldsymbol{M}_1 &= det\boldsymbol{M}_0 \cdot det\{\frac{\partial C_1}{\partial \boldsymbol{q}^T}\boldsymbol{M}_0^{-1}\boldsymbol{j_{c}}_1^T\} \\ &= \frac{1}{\|\frac{\partial C_1}{\partial \boldsymbol{r}^T}\|} det\boldsymbol{M}_0 \cdot det\{\frac{\partial C_1}{\partial \boldsymbol{q}^T}\boldsymbol{M}_0^{-1}(\frac{\partial C_1}{\partial \boldsymbol{q}^T})^T\} \quad (38) \end{aligned}$$

C_1 is an independent constraint condition and $\boldsymbol{M}_0^{-1}$ is positive definite, so obviously part $\{(\partial C_1/\partial \boldsymbol{q}^T)\boldsymbol{M}^{-1}(\partial C_1/\partial \boldsymbol{q}^T)^T\}$ is also a constantly positive scalar value whether the value of $\boldsymbol{q}$ is plus or minus. Therefore, the fact that "$\boldsymbol{M}_1$ is a non-singular matrix", has been confirmed.

In the same way as mentioned in Eq (31), $\boldsymbol{M}_2$ is defined as

$$\begin{bmatrix} \boldsymbol{M}_1 & -\boldsymbol{j_{c}}_2^T \\ \frac{\partial C_2}{\partial \boldsymbol{q}^T} & 0 \end{bmatrix} \triangleq \boldsymbol{M}_2 \quad (39)$$

Also the determinant of Eq (39) can be calculated with the condition of non-singular matrix $\boldsymbol{M}_1$ and definition of $\boldsymbol{j_{c}}_i^T$ in Eqn (8),

$$\begin{aligned} det\boldsymbol{M}_2 &= det\boldsymbol{M}_1 \cdot det\{\frac{\partial C_2}{\partial \boldsymbol{q}^T}\boldsymbol{M}_1^{-1}\boldsymbol{j_{c}}_2^T\} \\ &= \frac{1}{\|\frac{\partial C_2}{\partial \boldsymbol{r}^T}\|} det\boldsymbol{M}_1 \cdot det\{\frac{\partial C_2}{\partial \boldsymbol{q}^T}\boldsymbol{M}_1^{-1}(\frac{\partial C_2}{\partial \boldsymbol{q}^T})^T\} \quad (40) \end{aligned}$$

just like the argument of $\boldsymbol{M}_1$, the fact that "$\boldsymbol{M}_2$ is a non-singular matrix", can be also be confirmed. According to these arguments mentioned above, after confirming $\boldsymbol{M}_1$,$\boldsymbol{M}_2$,$\cdots$,$\boldsymbol{M}_{p-1}$ are all non-singular matrix, we can define

$$\begin{bmatrix} \boldsymbol{M}_{p-1} & -\boldsymbol{j_{c}}_p^T \\ \frac{\partial C_p}{\partial \boldsymbol{q}^T} & 0 \end{bmatrix} = \boldsymbol{M}_p \quad (41)$$

and $\boldsymbol{M}_p$ is also non-singular matrix. Moreover, $|\boldsymbol{M}_p| \neq 0$,$|\boldsymbol{L}| \neq 0$, so $\boldsymbol{M}^*$ is also a non-singular matrix since $\boldsymbol{M}^*$ can be rewritten by referring Eq (31) and Eq (33) as,

$$\begin{bmatrix} \boldsymbol{M}_p & 0 \\ 0 & \boldsymbol{L} \end{bmatrix} = \boldsymbol{M}^*. \quad (42)$$

Based on the above discussion we have confirmed that we can calculate $\ddot{\boldsymbol{q}}^T$, $\boldsymbol{f_n}^T$, $\dot{\boldsymbol{i}}^T]^T$ from Eq (35). Since the variables included Eq (33) and Eq (34) such as $\boldsymbol{M}$, $\boldsymbol{h}$, $\boldsymbol{g}$ are fairly complicated to solve them analytically, remaing problem is how to calculate them. Next section will discuss that.

IV. FORWARD DYNAMICS CALCULATION

To calculate $\boldsymbol{M}^*, \boldsymbol{b}$ in Eq (35), we need to first calculate $\boldsymbol{M}, \boldsymbol{h}, \boldsymbol{g}$. Here we can notice that $\boldsymbol{M}, \boldsymbol{h}, \boldsymbol{g}$ are included Eq (28) that describes the dynamics of non-constraint,and those can be calculated numerically and recursively through forward dynamics calculation [14] by exploiting the inverse dynamics calculation called ''Newton Euler" Method [15]. Because $\boldsymbol{M}$ is 10×10 matrix when the hyper-redundant manipulator including 10 links, resulting in a large amount of computation to calculate each element of $\boldsymbol{M}$ by using lagrange method. This implies that analytical deriving Eq (28) is almost impossible by hand writing calculation, then we introduce Newton Euler method as follows.

First of all, Eq (28) should be set as hereunder.

$$\boldsymbol{M_J}\ddot{\boldsymbol{q}} + \boldsymbol{b_J} = \tilde{\boldsymbol{\tau}} \tag{43}$$

Here,

$$\begin{aligned} \boldsymbol{M_J} &= \boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{J_m} \\ \boldsymbol{b_J} &= \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + (\boldsymbol{D} + \boldsymbol{D_m})\dot{\boldsymbol{q}} \\ \tilde{\boldsymbol{\tau}} &= \boldsymbol{K_m}\boldsymbol{i} + \boldsymbol{J_c}^T\boldsymbol{f_n} - \boldsymbol{J_t}^T\boldsymbol{f_t} \end{aligned}$$

With forward motion analysis, Eq (43) should be calculated by Newton-Euler method from the bottom link to upper link until the manipulator 's hand, and also with the motion analysis of backward calculation in [21], we get equation of motion of i-th link Eq (44).

$$\tilde{\tau}_i = {}^i\boldsymbol{z}_i^{T\,i}\boldsymbol{n}_i + J_{mi}\ddot{q}_i + (D_i + D_{mi})\dot{q}_i \tag{44}$$

Therefore, the motion Eq (43) can be used to inverse dynamics calculation $\tilde{\boldsymbol{\tau}} = [\tilde{\tau}_1, \tilde{\tau}_2, \cdot\cdot\cdot, \tilde{\tau}_n]^T$ in Eq (44). This inverse calculation can be described as $\tilde{\boldsymbol{\tau}} = \boldsymbol{p}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{g})$. Then considering Eq (43) and Eq (44),

$$\boldsymbol{M_J}\ddot{\boldsymbol{q}} + \boldsymbol{b_J} = \boldsymbol{p}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{g}) \tag{45}$$

Substitute $\ddot{\boldsymbol{q}} = \boldsymbol{0}$ into Eq (45):

$$\boldsymbol{b_J} = \boldsymbol{p}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{0}, \boldsymbol{g}) \tag{46}$$

so $\boldsymbol{b_J}$ can be calculated. Next substitute $\boldsymbol{g} = \boldsymbol{0}, \dot{\boldsymbol{q}} = \boldsymbol{0}, \ddot{\boldsymbol{q}} = \boldsymbol{e}_i (i = 1, 2, \cdot\cdot\cdot, s)$ into Eq (45), then the $\boldsymbol{b_J} = 0$:

$$\boldsymbol{m}_i = \boldsymbol{M_J}\boldsymbol{e}_i = \boldsymbol{p}(\boldsymbol{q}, \boldsymbol{0}, \boldsymbol{e}_i, \boldsymbol{0}) \tag{47}$$

here we can calculate $\boldsymbol{m}_i$ defined as the component vector of the i-th column in inertia matrix $\boldsymbol{M}$, $\boldsymbol{e}_i$ is a $l \times 1$ matrix in which the i-th element is 1 and others are all 0 like $\boldsymbol{e}_i = [0, 0, \cdot\cdot\cdot, 1_{(i)}, \cdot\cdot\cdot, 0, 0]^T$. So with Eq (47) $\boldsymbol{M_J} = [\boldsymbol{m}_1, \boldsymbol{m}_2, \cdot\cdot\cdot, \boldsymbol{m}_l]$ can be calculated one by one separately.

Thus up to now, we have calculated the $\boldsymbol{M_J}$ and $\boldsymbol{b_J}$. Back to the Eq (33), the $\boldsymbol{M}^*$ can be calculated while the constraint condition is given. Moreover, the inverse of $\boldsymbol{M}^*$ can be also calculated due to invertible for $\boldsymbol{M}^*$.

V. TRAJECTORY TRACKING SIMULATION

In this section we will introduce the trajectory tracking simulation results. The input voltage of PD controller has been set as follows.

$$\boldsymbol{v} = \boldsymbol{K_p}(\boldsymbol{q_d} - \boldsymbol{q}) + \boldsymbol{K_d}(\dot{\boldsymbol{q_d}} - \dot{\boldsymbol{q}}) \tag{48}$$

$\boldsymbol{K_p}$ and $\boldsymbol{K_d}$ are moth $s \times s$ diagonal matrix which indicates a position gain and a velocity gain, $\boldsymbol{q_d}$,$\dot{\boldsymbol{q_d}}$ are the desired joint angle and joint angular velocity, respectively.

Simulation's condition has been set as: each link's mass is $m_i = 0.1[kg]$, length is $l_1 = 0[m], l_j = 0.3[m]$,radius of cylindrical link is $r_i = 0.01[m]$,proportional gain is $k_{pi} = 500$,velocity gain is $k_{di} = 20$,viscous friction coefficient of joint is $D_i = 0.5$, torque constant is $K_i = 0.203$, resistance is $R_i = 1.1[\Omega]$, inductance is $L_i = 0.0017[H]$,inertia moment of motor is $I_{mi} = 0.000164$,reduction ratio is $k_i = 3.0$,viscous friction coefficient of reducer is $d_{mi} = 0.01$,friction is $f_{ti} = 0$ and these parameters are given by actucal motor's specifications. Initial condition of each link: $q_1(0) = 0, q_2(0) = 0.25\pi, q_3(0) = 0.5\pi, q_4(0) = -0.5\pi, q_5(0) = 0.25\pi, q_6(0) = 0.25\pi, q_7(0) = -0.5\pi, q_8(0) = 0.25\pi, q_9(0) = -0.25\pi, q_10(0) = 0.25\pi[rad]$,$\dot{q}_i = 0[rad/s]$. Moreover, the desired trajectory's parameters have been set as: $q_{d2}(t) = 0.25\pi, q_{d3}(t) = 0.5\pi, q_{d4}(0) = -0.5\pi, q_{d5}(0) = 0.25\pi, q_{d6}(0) = 0.25\pi, q_{d7}(0) = -0.5\pi, q_{d8}(0) = 0.25\pi[rad]$, and the trajectory has been set as a circle with radius is $0.1[m]$, center is $(x, y, z) = (1.5, 1.5, 0.4)$, target which is tracked by the manipulator hand will rotate in counterclockwise along this circle trajectory. The constraint condition is set as $\boldsymbol{C} = z = 0$. Simulation has been done under three situations as Fig.2.

1) trajectory tracking motion with two elbows contacting at joint 4 and joint 7,

2) trajectory tracking motion with just one elbow contacting at joint 4;

3) trajectory tracking motion with no elbow contacting at all.

Simulation results are shown in Fig.3∼Fig.5. Fig.3 shows

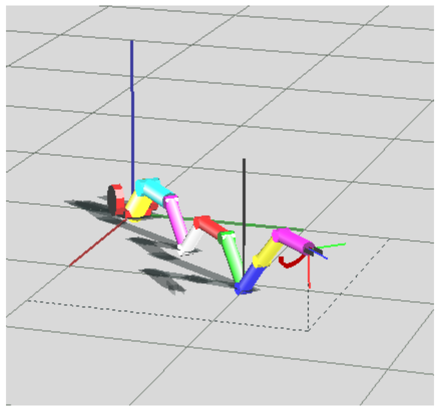


Fig. 2. The simulation of hyper-redundant manipulator

that manipulator hand's trajectory in xy coordinate under three different simulation condition, in the same way, Fig.3 shows that manipulator hand's trajectory in xz coordinate, and Fig.3 shows that manipulator hand's trajectory in yz coordinate, the simulation time is $t = 10[s]$. Fig.4 shows that all manipulator links' total amount of work, Fig.5 shows that all manipulator links' total amount of cost electric energy during the whole simulation. From Fig.3, we can tell that the manipulator hand can track the circle trajectory more accurately with more restraint elbows. Especially, in z axis, along with gravity's direction, motion of manipulator hand without restraint elbow is affected by the nutation of each link, the trajectory tracking has not been perfectly accomplished. Moreover, from Fig.4 and Fig.5, even in case of doing same work by manipulator, the cost electric energy is bigger in the motion done by the manipulator without restraint elbow.

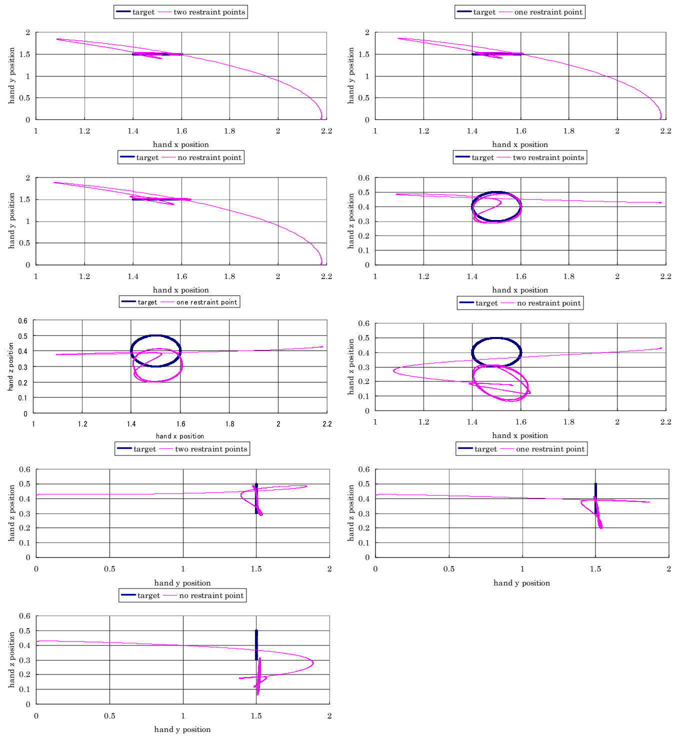


Fig. 3. Trajectory tracking of circle on x-y,x-z,y-z plane

Fig. 4. All link work

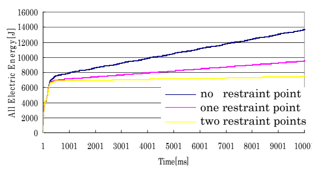


Fig. 5. All electric energy

VI. CONCLUSION

We first propose a dynamical model of hyper-redundant manipulator whose plural intermediate links is contacting with environment, second how the contacting raises benefits for saving energy and achieving trajectory tracking accuracy of the end effector. The simulation result shows that less-energy consumption and high trajectory tracking accuracy are achieved by simple PD controller. And the efficiency has been improved drastically, and the accuracy also confirmed to be refined, compared with non-contacting manipulator condition. We think the usage of contacting motion with environment is promising as a new robot motion control strategy.

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