SCIS & ISIS 2010, Dec. 8-12, 2010, Okayama Convention Center, Okayama, Japan

Operator-based Modeling for Nonlinear Ionic Polymer Metal Composite with Uncertainties

Mingcong Deng[a], Aihui Wang[b], Mamoru Minami[b], and Akira Yanou[b]

[a]Graduate School of Engineering, Tokyo University of Agriculture and Technology
2-24-16 Nakacho, Kagonei-shi, Tokyo 184-8588, Japan
[b]Graduate School of Natural Science and Technology, Okayama University
3-1-1 Tsushima-Naka, Okayama 700-8530, Japan
Email: deng@cc.tuat.ac.jp

***Abstract*—The ionic polymer metal composite (IPMC) is a novel smart polymer material, many potential application for low-mass high-displacement actuators in biomedical and robotic systems have been shown. Although several nonlinear models have been proposed in precision displacement control, none has finally been adopted absolutely for identification of some physical parameters is still difficult. In this paper, for a IPMC, some physical parameters are small enough for influence of displacement deformation, which are considered as uncertainties of model. To avoid the influence of the unknown uncertainties, an improved operator-based nonlinear model with uncertainties is proposed. To confirm the effectiveness of the proposed model, an operator-based nonlinear control for the IPMC setup is designed. The effectiveness of the proposed model is confirmed by simulation results of the operator-based nonlinear robust control system and experimental results of a IPMC setup.**

I. INTRODUCTION

Various electroactive polymers (EAP) materials, also called artificial muscles, are being developed to enable effective, miniature, light and low power actuators. The ionic polymer metal composite (IPMC) belongs to the category electroactive polymers (EAP), which is one of the most promising EAP actuators for applications. An IPMC sample consists of a thin ion-exchange membrane (e.g., Nafion) plated on both surfaces with a noble metal as electrodes. Because IPMCs are capable of producing large deformation under a low driving voltage, they have been shown to have many potential applications as biomimetic robotic distributed sensors, actuators, transducers and artificial muscles [1, 2].

The IPMC is usually broken up into three categories of different model types: black-box, gray-box, and white-box. The black-box models have no prior system knowledge of the system at all. The gray-box models have some system knowledge or structure. The white-box models are obtained by physical system derivation. It can be said that most black- and gray-box models were developed to study certain response characteristics or phenomena in the material, which are linear models. The white-box models, on the other hand, attempt to model physical processes taking place within the actuator, which are usually nonlinear models [3, 4]. To linear models, linear quadratic regulator (LQR), proportional integral and derivative (PID), adaptive fuzzy algorithm and impedance control scheme have been designed in position control [5, 6, 7]. Moreover, the IPMC shows mainly nonlinear behaviors in position control, but identification of some physical parameters is still a difficult issue.

Precision position control is critical in ensuring precise and safe operation of IPMC actuators in bio/micromanipulation. Considering an application as a robotic manipulator, IPMC has to move arbitrarily from one specified position to another. It needs a skilful operator to control manually based on his or her experiences to stop the swing immediately at the right position. To resolve this problem, operator-based robust right coprime factorization is used in this paper. It is well known that coprime factorization has been a promising approach for analysis, design, stabilization and control of nonlinear system [8, 9]. Especially, robust right coprime factorization has attracted much attention due to its convenient in researching input-output stability problems of nonlinear system with uncertainties [10, 11, 12, 13]. In this paper, for the IPMC, some physical parameters are small enough for the influence of displacement deformation, which can be considered as uncertainties in the model, to avoid the influence of the unknown uncertainties, an improved operator-based nonlinear model with uncertainties is proposed, robust right coprime factorization and its application to a nonlinear IPMC control setup are investigated. Experimental and simulation verification shows that the proposed model can capture the major nonlinearities of IPMC, the feasibility of the proposed control method is also confirmed through simulation and experiment results.

The outline of the paper is given as follows. In Section II, an improved nonlinear model with uncertainties of IPMC is obtained and problem statement is introduced; An operator-based nonlinear model with uncertainties is modeled for the IPMC in Section III; The experimental and simulation results are shown in Section IV, and Section V is the conclusion.

II. NONLINEAR MODEL AND PROBLEM STATEMENT

A. Nonlinear Model of IPMC

Displacement control models of IPMCs fall into two general categories: linear models, and nonlinear models. Linear models have no prior knowledge or some knowledge of the system. Nonlinear models have a comprehensive knowledge of the physics system derivation. From [14, 15], a nonlinear dynamic model of IPMC can be obtainedd:

$$\dot{v} = -\frac{v+Y(v)(R_a+R_c)-u}{(C_1(v)+C_a(v))(R_a+R_c)} \quad (1)$$

$$y = \frac{3\alpha_0\kappa_e(\sqrt{2\Gamma(v)}-v)}{Y_e h^2} \quad (2)$$

where, v is the state variable, u is the control input voltage, y is the curvature output, R_a is the electrodes resistance, R_c is the ion diffusion resistance. $\Gamma(v)$, $C_1(v)$ and $C_a(v)$ are functions of the state variable and some parameters,

$$\Gamma(v) = \frac{b}{a^2}(\frac{ave^{-av}}{1-e^{-av}} - ln(\frac{ave^{-av}}{1-e^{-av}})-1) \quad (3)$$

where,

$$a = \frac{F(1-C^-\Delta V)}{RT} \quad (4)$$

$$b = \frac{F^2C^-(1-C^-\Delta V)}{RT\kappa_e} \quad (5)$$

F is Faraday's constant, C^- is the anion concentrations, ΔV is the volumetric change, R is the gas constant, T is the absolute temperature, and κ_e is the effective dielectric constant of the polymer.

$$C_1(v) = S\kappa_e\frac{\dot{\Gamma}(v)}{\sqrt{2\Gamma(v)}} \quad (6)$$

$S = WL$ is the surface area of the IPMC, L, W and h donote the length, the width and the thickness of the IPMC respectively.

$$C_a(v) = \frac{q_1SF}{RT}\frac{K_1C^{H^+}e^{-\frac{vF}{RT}}}{(K_1C^{H^+}+e^{-\frac{vF}{RT}})^2} \quad (7)$$

$K_1 = \frac{k_1}{k_{-1}}$, k_1 and k_{-1} are the chemical rate constants for forward and reverse directions of electrochemical surface process, q_1 is some constant, and C^{H+} is the concentration of H^+.

$$Y(v) = Y_1v + Y_2v^2 + Y_3v^3 \quad (8)$$

Y_1, Y_2 and Y_3 are the coefficients of polynomial.

In (4) and (5), in general, ΔV is little enough, then $|C^-\Delta V| \to 0$, so,

$$a \approx \frac{F}{RT} \quad (9)$$

$$b \approx \frac{F^2C^-}{RT\kappa_e} \quad (10)$$

The IPMC can operate in a humid environment or a dry environment, in this paper, the IPMC setup is investigated in a dry environment, then $C^{H+} \to 0$, so

$$C_a(v) \approx 0 \quad (11)$$

In (1), in general, $|\text{Y}(v)| \ll |v|$, R_a and R_c are bounded, so, $Y(v)$ can be ignored and as model error in the displacement control. In addition to some physical constants, such as q_1, R, F, other parameters (T, L, W, h, Y_e, R_a, ΔV, Y_1, Y_2, Y_3, R_c, K_1, C^-, κ_e, α_0) must be measured or identified through the experiments, which will also create error. In this paper, influence of variables ignored and measurement error of parameters are considered as uncertainties of model, so we can obtain:

$$\dot{v} = -\frac{v-u}{C_1(v)(R_a+R_c)} \quad (12)$$

$$y = \frac{3\alpha_0\kappa_e\sqrt{2\Gamma(v)}}{Y_eh^2} + \Delta P \quad (13)$$

ΔP is uncertainties consisting of measurement error of parameters and model error of the IPMC.

B. Problem Statement

According to (3), (6), (11) and (12), the following equations are established,

$$\dot{v} = -\frac{(v-u)\sqrt{2b(\frac{ave^{-av}}{1-e^{-av}} - ln(\frac{ave^{-av}}{1-e^{-av}})-1)}}{S\kappa_eb(R_a+R_c)(1-\frac{1-e^{-av}}{ave^{-av}})\frac{e^{-av}(1-e^{-av}-av)}{(1-e^{-av})^2}} \quad (14)$$

$$y = \frac{3\alpha_0\kappa_e\sqrt{2b(\frac{ave^{-av}}{1-e^{-av}} - ln(\frac{ave^{-av}}{1-e^{-av}})-1)}}{aY_eh^2} + \Delta P \quad (15)$$

Defining a new state variable $x = av$, the following equations can be obtained,

$$\dot{x} = -\frac{(x-au)\sqrt{2b(\frac{xe^{-x}}{1-e^{-x}} - ln(\frac{xe^{-x}}{1-e^{-x}})-1)}}{S\kappa_eb(R_a+R_c)(1-\frac{1-e^{-x}}{xe^{-x}})\frac{e^{-x}(1-e^{-x}-x)}{(1-e^{-x})^2}} \quad (16)$$

$$y = \frac{3\alpha_0\kappa_e\sqrt{2b(\frac{xe^{-x}}{1-e^{-x}} - ln(\frac{xe^{-x}}{1-e^{-x}})-1)}}{aY_eh^2} + \Delta P \quad (17)$$

According to the above analysis, we can see that the IPMC is a nonlinear model with one input and one output. For the model, there exist some uncertainties because of errors of model and measurement. The influence of the uncertainties is unknown but bounded. To avoid the influence of the unknown uncertainties, operator-based robust right coprime factorization approach is used to model nonlinear IPMC with uncertainties.

III. OPERATOR-BASED MODELING FOR IONIC POLYMER METAL COMPOSITE

Right factorization is to factorize a given plant operator P as a composition of two operators N and D in the following form

$$P = ND^{-1} \quad (18)$$

If there are some operators A and B, the two operators N and D together satisfy the Bezout identity

$$AN + BD = I \quad (19)$$

where I is the identity operator, then the right factorization is said to be coprime shown in Fig. 1 [9, 10].

Considering the nonlinear system with bounded uncertainties, the robust control problem by using robust right coprime factorization approach has been researched in [11, 12, 13]. Assume that the uncertainties are given as ΔP, where ΔP is

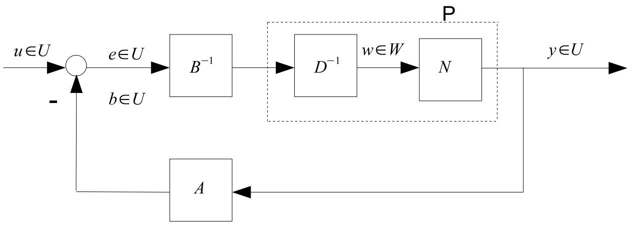

Fig. 1. A nonlinear system based on right coprime factorization

unknown but bounded. The right factorization of the nonlinear system is the following form

$$\tilde{P} = P + \Delta P = (N + \Delta N)D^{-1} \tag{20}$$

From [9, 10], we can see that if the following conditions are satisfied,

$$AN + BD = L \tag{21}$$
$$A(N + \Delta N) + BD = \tilde{L} \tag{22}$$
$$\|(A(N + \Delta N) - AN)L^{-1}\| < 1 \tag{23}$$

then the stability of the uncertain system $\tilde{P}$ is guaranteed, where L and $\tilde{L}$ are unimodular operators and $\|\cdot\|$ is Lipschitz operator norm, shown in Fig. 2.

Then, we consider the mentioned nonlinear IPMC control

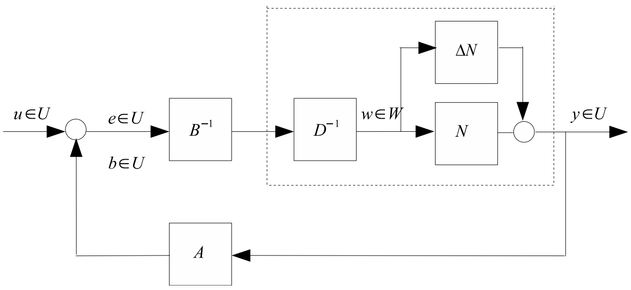

Fig. 2. A nonlinear system with uncertainties based on right coprime factorization

model by using robust right coprime factorization. For the model described by equations (16) and (17), there exist some uncertainties ΔP in the IPMC control model. The uncertainties are unknown but bounded. In Fig. 2, the uncertainties can be transformed into uncertain operator ΔN. That is, uncertain operator ΔN denotes the uncertainties caused by approximate calculation.

Denote N, D and ΔN as the following form

$$N(\omega(t)) = \frac{3\alpha_0\kappa_e\sqrt{2b(\frac{\omega(t)e^{-\omega(t)}}{1-e^{-\omega(t)}} - ln(\frac{\omega(t)e^{-\omega(t)}}{1-e^{-\omega(t)}}) - 1)}}{aY_eh^2} \tag{24}$$

$$D\left(\omega(t)\right) = \frac{S\kappa_e b(R_a + R_c)\dot{\omega}(t)(1 - \frac{1-e^{-\omega(t)}}{\omega(t)e^{-\omega(t)}})\frac{e^{-\omega(t)}(1-e^{-\omega(t)}-\omega(t))}{(1-e^{-\omega(t)})^2}}{a\sqrt{2b(\frac{\omega(t)e^{-\omega(t)}}{1-e^{-\omega(t)}} - ln(\frac{\omega(t)e^{-\omega(t)}}{1-e^{-\omega(t)}}) - 1)}} + \frac{\omega(t)}{a} \tag{25}$$

$$\Delta N(\omega(t)) = \Delta\sqrt{2b(\frac{\omega(t)e^{-\omega(t)}}{1-e^{-\omega(t)}} - ln(\frac{\omega(t)e^{-\omega(t)}}{1-e^{-\omega(t)}}) - 1)} \tag{26}$$

To satisfy robust right coprime factorization conditions, we suppose that

$$A(y(t)) = -\frac{aSY_eh^2(R_a + R_c)}{3\alpha_0}\dot{y}(t) \tag{27}$$
$$B(u(t)) = au(t) \tag{28}$$

So an operator-based nonlinear model with uncertainties is obtained.

IV. SIMULATION AND EXPERIMENT

In this section, some simulation and experimental results are given to illustrate the effectiveness of the proposed model. During the simulation and experiment, the uncertain factor in (26) is modeled as $\Delta = \frac{3\alpha_0\kappa_e\sqrt{2b}}{aY_eh^2}\times 5\%$. In fact, the uncertainties of model is smaller than ΔN, so robust stability of the system can be guaranteed. The simulation result of the IPMC with uncertainties and without uncertainties position control system based on right coprime factorization is shown in Fig. 3 respectively. From Fig. 3, we can see the nonlinear IPMC with uncertainties system using right coprime factorization is robust stable.

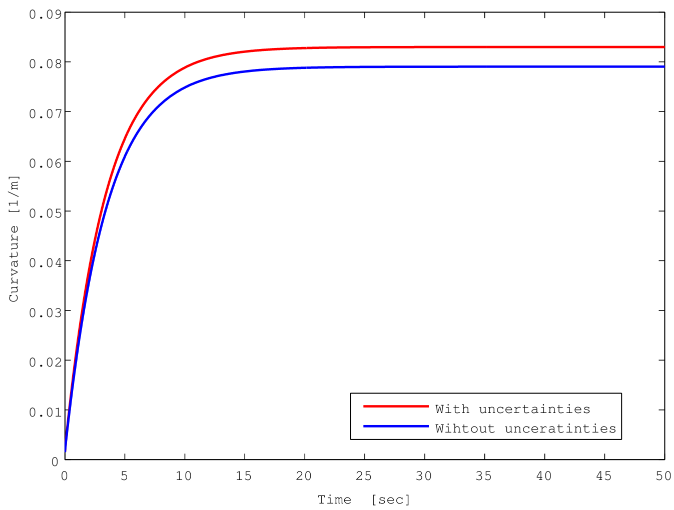

Fig. 3. The simulation result based on robust coprime factorization

Besides the robust stability of the IPMC system is guaranteed, the tracking performance of the system needs also to be considered. Here, the tracking condition is difficult to obtain for the operator N is a complex nonlinear function, such that we design a tracking system given in Fig. 4 [13], where the stabilizing system regarded as the plant is equal to the system in Fig. 2.

The tracking controller C is designed satisfying the conditions in [11]. Where, the controller C is shown as the following form:

$$u(t) = k_pe(t) + k_i\int e(\tau)d\tau \tag{29}$$

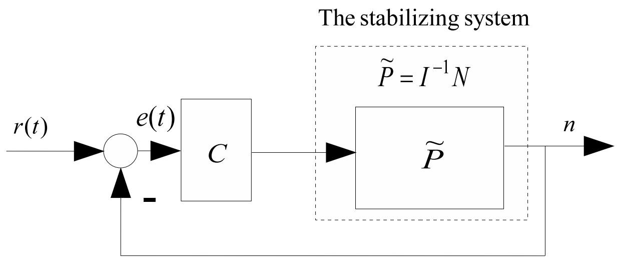

Fig. 4. The tracking control system

Fig. 5 shows the simulation result of system with tracking controller, the reference input of the curvature is $r_f = 1[1/m]$. Using the tracking controller, the IPMC control system can track the reference input.

Fig. 6 shows the experimental setup schematic illustration.

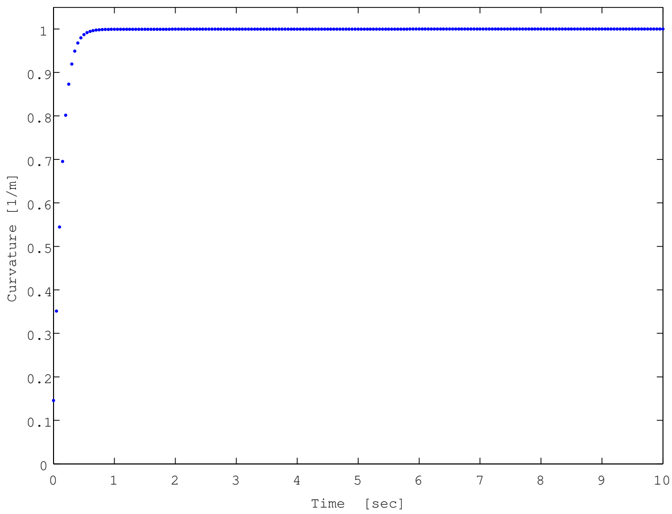

Fig. 5. The simulation result with tracking controller

Picture of experimental setup is shown in Fig. 7. In this experimental set, an IPMC sample of dimensions 50 mm $\times$ 10 mm $\times$ 0.2 mm is clamped at one end, and is subject to voltage excitation generated from the computer and board. A laser displacement sensor (ZX-LD40: 40$\pm$10mm) is used to measure the bending displacement d. The relationship between the bending curvature $1/\rho$ and the displacement d can be described by (seeing Fig. 8):

$$\frac{1}{\rho} = \frac{2d}{d^2 + h^2} \tag{30}$$

here, h is the vertical distance.

Some physical parameters have been identified by experiments, which are shown in Table 1.

The Fig .9 shows the open-loop displacement response of the IPMC to 1V or 2V step input. Open-loop step response showed a longer settling time. Figs. 10-12 show the displacement response of the IPMC using robust right coprime factorization, the desired outputs of displacement d are 4 [mm], 8 [mm] and 12 [mm] respectively. These experimental results show that the robust stability of the IPMC displacement

TABLE I
PARAMETERS IN THE MODEL

T	F	κ_e
290 K	96487 C mol^{-1}	$1.12 \times 10^{-6} Fm^{-1}$
R_a	R	Y_e
18 Ω	8.3143 J $mol^{-1}K^{-1}$	0.56 GPa
R_c	C^-	α_0
60 Ω	980 mol	0.12 J C^{-1}
L	W	h
50 mm	10 mm	200 μm

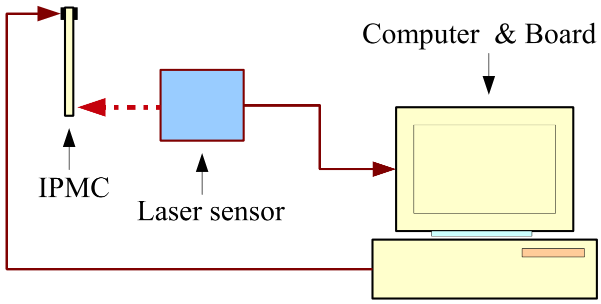

Fig. 6. The experimental setup schematic illustration

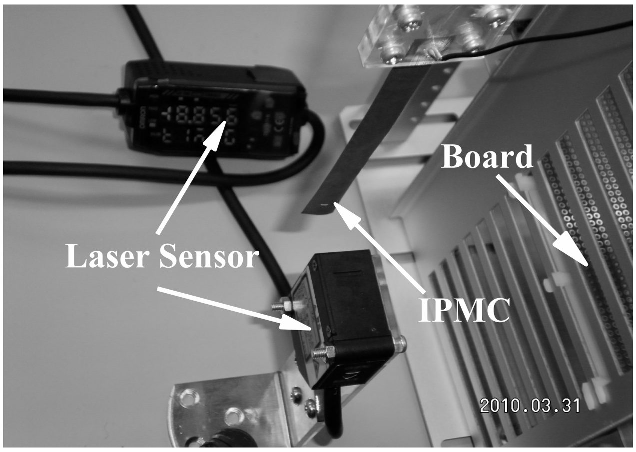

Fig. 7. Picture showing the experimental setup

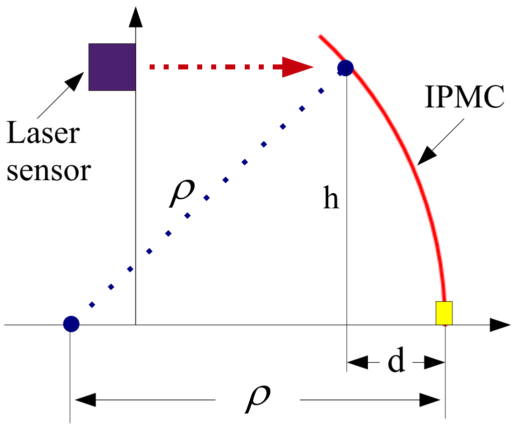

Fig. 8. Calculation of the displacement d based on the curvature

control system is guaranteed and tracking performance is satisfied by using the proposed method, the settling time becomes also more little than open-loop system. Owing to not considering hysteresis of IPMC actuator, there are oscillations in the system responses. As a continuous research, to avoid undesirable oscillations and instability, hysteresis of IPMC has been considered in [16].

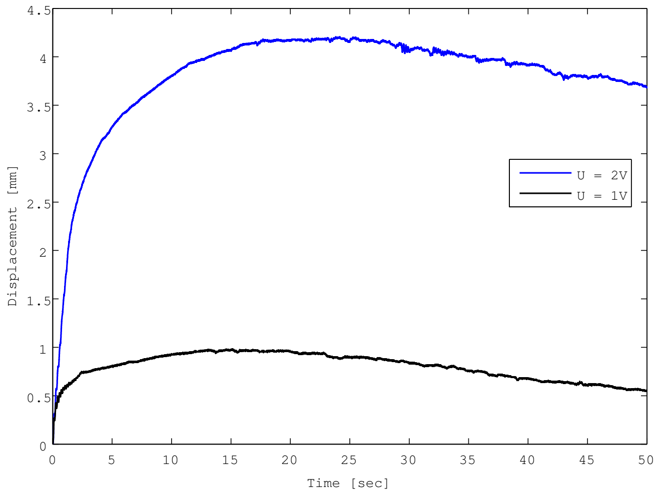

Fig. 9. The open-loop displacement response to 1V or 2V step input

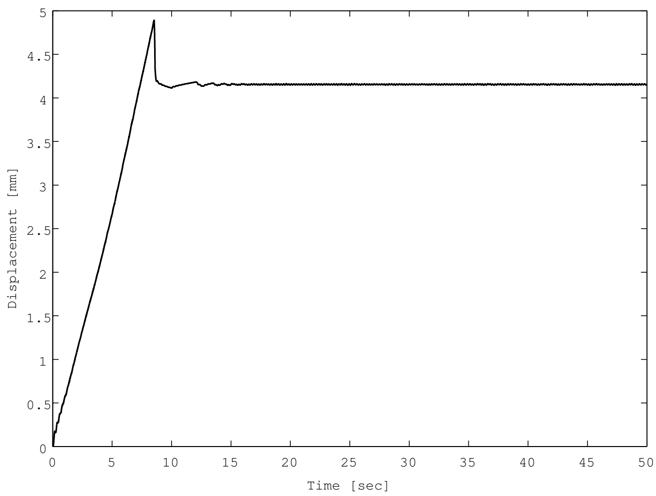

Fig. 10. The displacement response to desired output d = 4 mm

V. CONCLUSIONS

In this paper, considering errors of model and measurement, a nonlinear model with uncertainties of IPMC is obtained. Based on this model, an operator-based nonlinear model with uncertainties is modeled using robust right coprime factorization. For the operator-based nonlinear robust control system, a position tracking controller is also designed to realize the desired output tracking performance in a IPMC setup. The effectiveness of the proposed model is confirmed by simulation and experimental results.

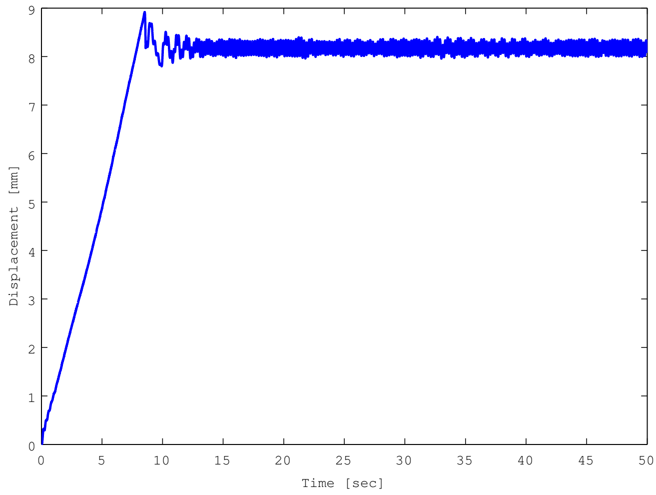

Fig. 11. The displacement response to desired output d = 8 mm

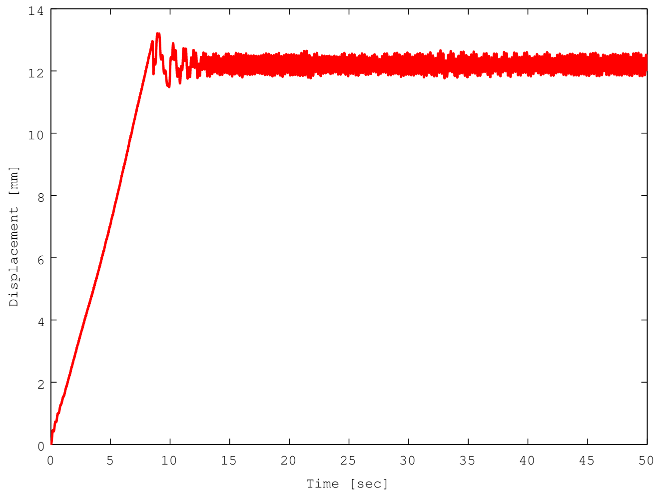

Fig. 12. The displacement response to desired output d = 12 mm

ACKNOWLEDGMENT

The authors would like to thank Mr. S. Wen, Mr. C. Jiang and Ms. N. Bu for their contributions of this work.

REFERENCES

[1] M. Shahinpoor and K. Kim, Ionic polymer-metal composites: I. Fundamentals, *Smart Materials and Structures*, vol. 10, pp. 819-833, 2001.

[2] M. Shahinpoor, Ionic polymer-conductor composites as biomimetic sensors, robotic actuators and artificial muscles-a review, *Electrochimica Acta*, vol. 48, pp. 2343-2353, 2003.

[3] H. Tang, T. Nie, Y. Tang, C. Yin, C. Tang, and Q. Wang, Constitutive relationship of ionic polymer-metal composite and static response character of its cantilever setup to voltage, *Journal of Central South University of Technology*, vol. 15, no. 3, pp. 387-391, 2008.

[4] M. Shahinpoor and K. Kim, Ionic polymer-metal composites: III. Modeling and simulation as biomimetic sensors, actuators, transducers, and artificial muscles, *Smart Materials and Structures*, vol. 13, no. 6, pp. 1362-1388, 2004.

[5] S. Oh and H. Kim, A study on the control of an IPMC actuator using an adaptive fuzzy algorithm, *Journal of Mechanical Science and Technology*, vol. 18, no. 1, pp. 1-11, 2004.

[6] N. Bhat and W. Kim, Precision position control of ionic polymer metal composite, *Proc. of the 2004 American Control Conference*, Boston, MA, pp.740-745, 2004.

[7] N. Bhat and W. Kim, Precision force and position control of ionic polymer metal composite, *Journal of Systems and Control Engineering*, vol. 218, no.6, pp. 421-432, 2004.
[8] M. Deng, A. Inoue and K. Ishikawa, Tracking of perturbed nonlinear plants using robust right coprime factorization approach, *Proc. of the 2004 American Control Conference*, Boston, MA, pp. 3666-3670, 2004.
[9] R. J. P. de Figueiredo and G. Chen, Nonlinear feedback control system: an operator theory approach, *New York: Academic Press, INC.*, 1993.
[10] M. Deng, A. Inoue and K. Ishikawa, Operator-based nonlinear feedback control design using robust right coprime factorization, *IEEE Trans. on Automatic Control*, vol. 51, pp. 645-648, 2006.
[11] G. Chen and Z. Han, Robust right coprime factorization and robust stabilization of nonlinear feedback control systems, *IEEE Trans. on Automatic Control*, vol. 43, no. 10, pp. 1505-1510, 1998.
[12] M. Deng, S. Bi, and A. Inoue, Robust nonlinear control and tracking design for multi-input multi-output nonlinear perturbed plants, *IET Control Theory & Applications*, vol. 3, no. 9, pp. 1237-1248, 2009.
[13] S. Wen, M. Deng, Experimental study of robust control to a nonlinear MIMO process using robust right coprime factorization, *ICIC Express Letters*, vol. 3, no. 4(a), pp. 1061-1066, 2009.
[14] Z. Chen, D. R. Hedgepeth and X. Tan, A nonlinear control-oriented model for ionic polymer-metal composite actuators, *Smart Materials and Structures*, vol. 18, 055008 (9 pp), 2009.
[15] Z. Chen, X. Tan, A control-oriented and physics-based model for ionic polymer-metal composite actuators, *IEEE/ASME Trans. on Mechatronics*, vol. 13, no. 5, pp. 519-529, 2008.
[16] A. Wang, M. Deng, and D. Wang, Operator-based robust nonlinear control for ionic polymer metal composite with uncertainties and hysteresis, *Proc. of the International Conference on Intelligent Roboticd and Applications*, Shanghai, China, pp. 135-146, 2010.