

# Control of Hyper-Redundancy Mobile Manipulator with Multi-Elbows braced for High Accuracy / Low-Energy Consumption

\*Wenhao GU(Okayama University), Hidemi KATAOKA(Okayama University)  
Fujia YU(Okayama University)Tomohide MAEBA(Okayama University)  
Mamoru MINAMI(Okayama University), Akira YANOU(Okayama University)

**Abstract**—To overcome a conflict between required high-redundancy for flexible manipulation and heavy weight caused by the high-redundancy structure, we proposed a realistic idea that contacting and bracing motion of intermediate links with environment may solve the conflict. This idea is inspired by human’s handwriting motion with the elbow or wrist contacting to a table. Moreover considering the flexible motion availability of the manipulator, a mobile robot which would give the mobile manipulator an ability to reach where it wanted can be designed as the basement of hyper-redundant manipulator. Thus in this paper we first proposed a dynamical model of hyper-redundant mobile manipulator whose plural elbows of intermediate links are being braced with environment. Then some simulations were done to confirm the effect of our system and the outcome of simulations indicates that our system indeed can improve accuracy and save energy.

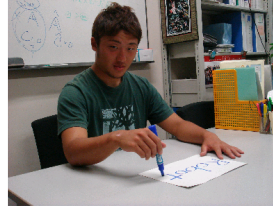
**keywords**—hyper-redundant manipulator; bracing motion; save energy;

## I. INTRODUCTION

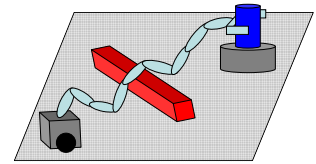
As we known hyper-redundant manipulator’s flexible operation needs high redundancy. However with the increase of redundancy, the weight of manipulator will increase at the same time, which will cause the controlling of manipulator becomes difficult. Since increasing of redundancy amplifies the influence of gravity toward each link, and that the effect is highly nonlinear. Then the motion controller should be with high motion control gains and large-sized motors with adequate output, which may cause the whole robot system unstable.

For solving this problem we obtain some inspirations about effective motion control strategies by observing human’s handwriting motion. Writing on a paper with one’s elbow contacting, as Fig.I(a) shows, is a familiar action of human being. Known from our experiences, we can save energy for writing task and can write correctly by the elbow or wrist supported by the desk. Suggested by this typical example, we came up with an idea that robots may also execute tasks with less energy and higher accuracy by using constrained contacting with environments, as Fig.I(b)shows. Furthermore, we can see this contacting strategy can also prevent the manipulator from overturning.

On the other hand, considering some work in our life cannot be done if the manipulator is stationary. For example, as Fig.I(c) shows, in the manufacture of ships, we need to use manipulator to paint the ship surface. In this case, a mobile vehicle which can keep the manipulator moving while



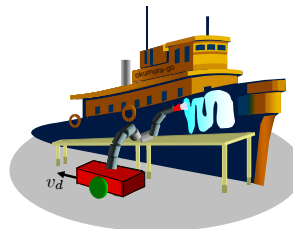
(a) Human’s writing motion



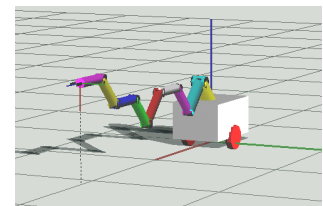
(b) Contacting strategy

working is necessary.

So in this research, we propose a model of mobile manipulator with multi-elbow and a mobile basement which is shown in Fig.I(d) and this redundant manipulator’s plural intermediate links can contact to the ground. Then we calculate the dynamics model according to the systemization of multi- constraint condition of elbows and equation of manipulator’s motion. Based the dynamics model, we made the program simulation. At last we do three experiments. The first one is to confirm that if this mobile manipulator doesn’t has multi-elbows, it will turnover easily. The second one is to prove the simulation is correctly done. The third one is to verify this proposed model indeed has the high accuracy and energy-saving performances. By the outcomes of this three experiments, we can see all the tentative ideas are proved to be correct.



(c) Manipulator to paint the ship surface



(d) Model of mobile manipulator

## II. MODELLING OF HYPER-REDUNDANT MOBILE MANIPULATOR WITH CONSTRAINT

### A. Manipulator's Model with Hand's Constraint

To make the explanation of constraint motion with multi-elbows be easily understandable, we discuss firstly about the model of the manipulator whose end-effector is contacting with rigid environment without elasticity. Equation of motion of manipulator is composed of rigid structure of  $s$  links, and also contact relation between manipulator's end-effector and definition of constraint surface should be introduced firstly.

Here according to the kinematic relation, manipulator hand's position/posture vector  $\mathbf{r} \in R^s$  and scalar function, a single constraint condition  $C$  that is used to express the hypersurface can be expressed as

$$\mathbf{r} = \mathbf{r}(\mathbf{q}) \quad (1)$$

$$C(\mathbf{r}(\mathbf{q})) = 0 \quad (2)$$

Here Eq.(1) and Eq.(2) represent constraint is undeformed.

To move freely in the directions without constraint the freedom of manipulator's end-effector is left to be more than one, so here  $s > 1$ . If we set  $f_n$  to indicate the constraint force of manipulator hand, then the relation of  $u$  and  $f_n$  can be expressed as

$$u = f_n / \left\| \frac{\partial C}{\partial \mathbf{r}^T} \right\| \quad (3)$$

$\left\| \frac{\partial C}{\partial \mathbf{r}^T} \right\|$  shows Euclidean norm of vector  $\frac{\partial C}{\partial \mathbf{r}^T}$ . Then manipulator's equation of motion can be derived by combining Eq(??) with Eq(3) with viscous friction of joints [12].

$$\begin{aligned} & M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + D\dot{\mathbf{q}} \\ &= \boldsymbol{\tau} + \left\{ \left( \frac{\partial C}{\partial \mathbf{q}^T} \right)^T / \left\| \frac{\partial C}{\partial \mathbf{r}^T} \right\| \right\} f_n - \left( \frac{\partial \mathbf{r}}{\partial \mathbf{q}^T} \right)^T \frac{\dot{\mathbf{r}}}{\|\dot{\mathbf{r}}\|} f_t \end{aligned} \quad (4)$$

$M$  is inertia matrix of  $s \times s$ ,  $\mathbf{h}$  and  $\mathbf{g}$  are  $s \times 1$  vectors which indicate the effects from coriolis force, centrifugal force and gravity,  $D$  is a  $s \times s$  matrix which indicates the coefficient of joints' viscous friction, expressed as  $D = \text{diag}[D_1, D_2, \dots, D_s]$ .  $\mathbf{q}$  is the joint angle and  $\boldsymbol{\tau}$  is the input torque.

### B. Model with Multiple Constraints

Here we consider a motion of a manipulator having  $s$  links whose elbows are contact at  $p$  points with environments defined as

$$C_i(\mathbf{r}_i(\mathbf{q})) = 0, \quad (i = 1, 2, \dots, p) \quad (5)$$

where  $\mathbf{r}_i$  is the equation of position and posture of link  $i$  contacting with constraint, like Eq(1).

$$\mathbf{r}_i = \mathbf{r}_i(\mathbf{q}) \quad (6)$$

The Eq(4) describes a motion of the manipulator whose hand is constrained. Under the situation with the  $i$ -th link contacting, then we can define two vectors concerning  $i$ -th constraint condition  $C_i$  as follows,

$$\left( \frac{\partial C_i}{\partial \mathbf{q}^T} \right)^T / \left\| \frac{\partial C_i}{\partial \mathbf{r}^T} \right\| = \mathbf{j}_{c_i}^T \quad (7)$$

$$\left( \frac{\partial \mathbf{r}_i}{\partial \mathbf{q}^T} \right)^T \frac{\dot{\mathbf{r}}_i}{\|\dot{\mathbf{r}}_i\|} = \mathbf{j}_{t_i}^T \quad (8)$$

Accumulating all the above vectors ( $i = 1, 2, \dots, p$ ) where  $p$  is the number of contact point, so the next relations are redefined.

$$\mathbf{J}_c^T = [\mathbf{j}_{c_1}^T, \mathbf{j}_{c_2}^T, \dots, \mathbf{j}_{c_p}^T] \quad (9)$$

$$\mathbf{J}_t^T = [\mathbf{j}_{t_1}^T, \mathbf{j}_{t_2}^T, \dots, \mathbf{j}_{t_p}^T] \quad (10)$$

$$\mathbf{f}_n = [f_{n1}, f_{n2}, \dots, f_{np}]^T \quad (11)$$

$$\mathbf{f}_t = [f_{t1}, f_{t2}, \dots, f_{tp}]^T \quad (12)$$

$\mathbf{J}_c^T, \mathbf{J}_t^T$  are  $s \times p$  matrices,  $\mathbf{f}_n, \mathbf{f}_t$  are  $p \times 1$  vectors. Considering about  $p$  constraints of the intermediate links, the manipulator's equation of motion can be expressed as

$$\begin{aligned} & M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + D\dot{\mathbf{q}} \\ &= \boldsymbol{\tau} + \sum_{i=1}^p (\mathbf{j}_{c_i}^T f_{ni}) - \sum_{i=1}^p (\mathbf{j}_{t_i}^T f_{ti}) \\ &= \boldsymbol{\tau} + \mathbf{J}_c^T \mathbf{f}_n - \mathbf{J}_t^T \mathbf{f}_t \end{aligned} \quad (13)$$

Moreover, Eq (5) is differentiated by time  $t$  two times, then we can derive the constraint condition of  $\ddot{\mathbf{q}}$ .

$$\left[ \frac{\partial}{\partial \mathbf{q}} \left( \frac{\partial C_i}{\partial \mathbf{q}^T} \right) \dot{\mathbf{q}} \right] \dot{\mathbf{q}} + \left( \frac{\partial C_i}{\partial \mathbf{q}^T} \right) \ddot{\mathbf{q}} = 0 \quad (14)$$

To make sure that manipulator hand is contact with the undeformed constraint surface all the time, value of  $\mathbf{q}(t)$  in Eq(13) always has to satisfy Eq(5) which has no relationship with time  $t$ , if value of  $\ddot{\mathbf{q}}$  in Eq(14) have the same value with  $\ddot{\mathbf{q}}$  in Eq(13), then value of  $\mathbf{q}(t)$  in Eq(13) and Eq(5) always keeps the same value regardless of time.

### C. Robot/Motor Equation with Contact Constraint

To make sure that  $\ddot{\mathbf{q}}$  in Eq (14) are identical, constraint force  $\mathbf{f}_n$  is subordinately decided by simultaneous equation. Then Eq (14) should be transformed as follows

$$\begin{aligned} & (M + \mathbf{J}_m)\ddot{\mathbf{q}} - \mathbf{J}_c^T \mathbf{f}_n \\ &= \mathbf{K}_m \dot{\mathbf{q}} - \mathbf{h} - \mathbf{g} - (D + D_m)\dot{\mathbf{q}} - \mathbf{J}_t^T \mathbf{f}_t \end{aligned} \quad (15)$$

$$\begin{aligned} \left( \frac{\partial C_i}{\partial \mathbf{q}^T} \right) \ddot{\mathbf{q}} &= - \left[ \frac{\partial}{\partial \mathbf{q}} \left( \frac{\partial C_i}{\partial \mathbf{q}^T} \right) \dot{\mathbf{q}} \right] \dot{\mathbf{q}} \\ &= - \dot{\mathbf{q}}^T \left[ \frac{\partial}{\partial \mathbf{q}} \left( \frac{\partial C_i}{\partial \mathbf{q}^T} \right) \right] \dot{\mathbf{q}} \end{aligned} \quad (16)$$

Here we assume that friction force  $f_{ti}$  is dynamic friction and define it as  $f_t = k f_n$  ( $i = 1, 2, \dots, p$ ), here we define  $k = 0.3$ .

The inertia term  $(M + \mathbf{J}_m)$  is a  $(s+2) \times (s+2)$  matrix, the coefficient vector of constraint force  $\mathbf{j}_{c_i}^T$  is  $s \times 1$  vertical vector,  $\frac{\partial C_i}{\partial \mathbf{q}^T}$  is  $1 \times s$  horizontal vector, inductance term  $L$  is  $s \times s$  diagonal matrix, therefore, the matrix of the first term in left side in Eq (17) is a matrix of  $(2s+p+2) \times (2s+p+2)$ .

And  $\ddot{\mathbf{q}}_L, \ddot{\mathbf{q}}_R$  can be included in  $\ddot{\mathbf{q}}$  and  $\tau_L, \tau_R$  can be also obtained by the current  $i$  input from the motors of two wheels. So then eqnarray can be rewritten concisely using the definitions of Eq (9) and Eq (11) as follows,

$$\begin{bmatrix} M + J_m & -J_c^T + kJ_t^T & \mathbf{0} \\ \frac{\partial C}{\partial \dot{q}^T} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & L \end{bmatrix} \begin{bmatrix} \ddot{q} \\ f_n \\ \dot{i} \end{bmatrix} = \begin{bmatrix} K_m \dot{i} - h - g - (D + D_m) \dot{q} \\ -\dot{q}^T \left[ \frac{\partial}{\partial q} \left( \frac{\partial C}{\partial \dot{q}^T} \right) \right] \dot{q} \\ v - Ri - K_m \dot{q} \end{bmatrix} \quad (17)$$

where,  $C$  is a vector of  $C = [C_1, C_2, \dots, C_p]^T$ . Furthermore by redefining as

$$M^* = \begin{bmatrix} M + J_m & -J_c^T + kJ_t^T & \mathbf{0} \\ \frac{\partial C}{\partial \dot{q}^T} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & L \end{bmatrix} \quad (18)$$

$$b = \begin{bmatrix} K_m \dot{i} - h - g - (D + D_m) \dot{q} \\ -\dot{q}^T \left[ \frac{\partial}{\partial q} \left( \frac{\partial C}{\partial \dot{q}^T} \right) \right] \dot{q} \\ v - Ri - K_m \dot{q} \end{bmatrix} \quad (19)$$

Then Eq (17) can be expressed as

$$M^* \begin{bmatrix} \ddot{q} \\ f_n \\ \dot{i} \end{bmatrix} = b \quad (20)$$

Here  $M^*$  has been confirmed to be nonsingular matrix before by us, and then calculate the inverse of  $M^*$ , finally the unknown value of  $\ddot{q}, f_n, \dot{i}$  can be determined based on the above simultaneous equation.

### III. EXPERIMENTS

#### A. Overturning Simulation

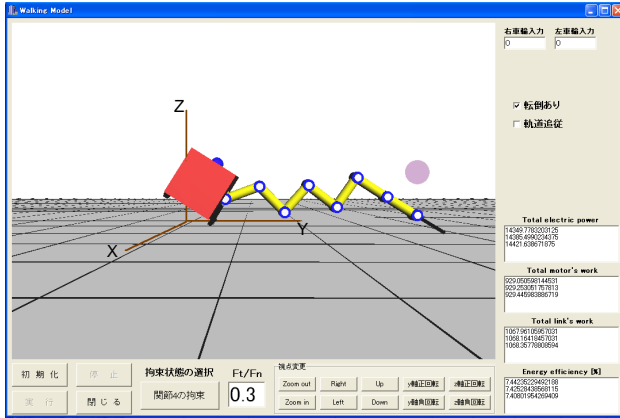


Fig. 1. Overturning Simulation

As the Fig.1 shows, we made the hyper-redundant manipulator program simulation. We can see that if this mobile manipulator doesn't have bracing elbow, the constraint force of wheels will possibly be negative, which means this mobile will turn over.

In this simulation, the force in  $z$  axis direction is  ${}^W f_{z_0}$ , the moment in  $x$  axis direction is  ${}^W n_{x_0}$ , and the length of mobile is  $2L$ , then when

$$\left( \frac{{}^W f_{z_0}}{2} + \frac{{}^W n_{x_0}}{L} \right) < 0 \quad (21)$$

the mobile manipulator will overturn towards the left. And when

$$\left( \frac{{}^W f_{z_0}}{2} - \frac{{}^W n_{x_0}}{L} \right) < 0 \quad (22)$$

the mobile manipulator will overturn towards the right. But from the simulation, we can see if we add the manipulator with bracing elbow, the mobile manipulator will not turn over. So we can get the conclusion that this proposed model can prevent the mobile manipulator from overturning.

#### B. Target tracking Simulation

In this section, first we will prove the simulation made by us is correctly done and then by the confirmed simulation, we will verify our proposed model indeed has the high accuracy and energy-saving performances. Simulation's condition has been set as: mobile's mass is  $m_0 = 10[kg]$ , width is  $W = 0.5[m]$ , length is  $S = 0.8[m]$ , height is  $H = 0.3[m]$ , the radius of two wheels is  $r = 0.1[m]$ , each link's mass is  $m_i = 1[kg]$ , length is  $l_1 = 0[m], l_j = 0.3[m]$ , radius of cylindrical link is  $r_i = 0.01[m]$ , proportional gain is  $k_{p_i} = 1000$ , velocity gain is  $k_{d_i} = 1000$ , viscous friction coefficient of joint is  $D_i = 0.5$ , torque constant is  $K_i = 0.203$ , resistance is  $R_i = 1.1[\Omega]$ , inductance is  $L_i = 0.0017[H]$ , inertia moment of motor is  $I_{m_i} = 0.000164$ , viscous friction coefficient of reducer is  $d_{m_i} = 0.01$  and these parameters are given by actual motor's specifications. Initial condition of each link:  $q_1(0) = 0.0\pi, q_2(0) = 1.15\pi, q_3(0) = 0.5\pi, q_4(0) = -0.4\pi, q_5(0) = 0.25\pi, q_6(0) = 0.25\pi, q_7(0) = -0.5\pi, q_8(0) = 0.25\pi, q_9(0) = -0.25\pi, q_{10}(0) = 0.15\pi, \dot{q}_i = 0$ . And the trajectory has been set as a circle with radius is  $0.1[m]$ , center is  $(x, y, z) = (0, 1.35, 0.45)$ , target which is tracked by the manipulator hand will rotate in counterclockwise along this circle trajectory. Then the Simulation has been done following four situations shown as Fig.2 which are two different directions views of the model.

- trajectory tracking motion with no bracing elbow;
  - trajectory tracking motion with one bracing elbow at joint 5;
  - trajectory tracking motion with one bracing elbow at joint 8;
  - trajectory tracking motion with two bracing elbows.
- Simulation results are shown in Fig.3(a)~Fig.3(d). Comparing the four graphs in axis  $Z$  direction, we can see that

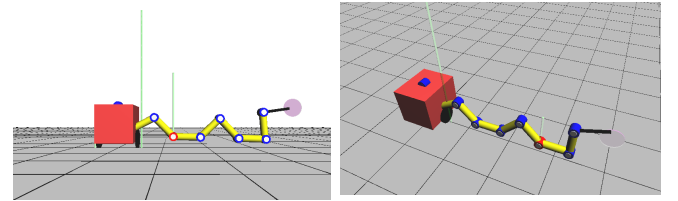


Fig. 2. The initial shape of simulation model

the distance between the target and track in the first graph is largest in which manipulator with no bracing elbow. And in the fourth graph is least in which manipulator with two bracing elbows. We can get that the manipulator hand can track the circle trajectory more accurately with more restraint elbows. Especially, in z axis, along with gravity 's direction. So this phenomenon can verify our proposed model indeed has the high accuracy performances.

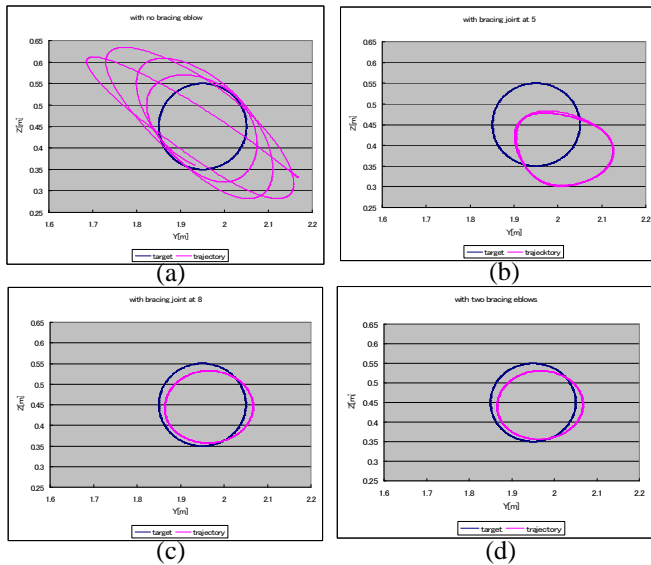


Fig. 3. The trajectory of the tracking

Fig.4 shows all the electric energy. Even though the friction force of bracing elbows may consume some energy. But from Fig.4 we can see the electric energy is still decreasing with constraint point increasing, which is the proof that constraint condition can make the model of hyper-redundant manipulator with mobile robot less-consumption. So our proposed model indeed can save energy.

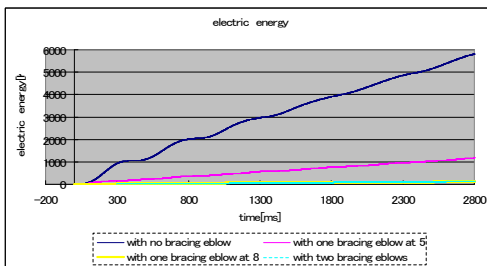


Fig. 4. The contrast of electric energy among four conditions

#### IV. CONCLUSION

In this paper we first proposed a dynamical model of hyper-redundant mobile manipulator whose plural elbows of intermediate links are being braced with environment. Then some simulations were done to confirm the effect of our system and the outcome of simulations indicates that our system indeed can improve accuracy and save energy. So we can believe our proposed model can be widely application

in the future.

Next we will consider the method how to input the suitable torque and force to control the mobile robot effectively and accurately without overturning. And also we can try some other ways to control this mobile manipulator. At last we found the whole dynamical model has some characters resembling a humanoid, such as the two wheels of mobile robot can be replaced by the feet of humanoid robot and the contacting of manipulator with ground is similar with the hands of humanoid robot touching ground. Therefore we will use this proposed dynamical model of hyper-redundant mobile manipulator as a foundation to discuss constraint motion of humanoid robot to environment.

#### REFERENCES

- [1] G.S.Chirikjian, J.W.Burdick: "A Hyper-Redundant Manipulator", IEEE Robotics & Automation Magazine, December 1994, pp.22-29 (1994)
- [2] F.Fahimi, H.Ashrafiuon and C.Nataraj: "Obstacle Avoidance for Spatial Hyper-Redundant Manipulators", The First Asian Conference on Multibody Dynamics 2002, July 31-August 2, 2002, Iwaki, Fukushima, Japan.
- [3] Shugen Ma, Watanabe.M, Kondo.H: "Dynamic control of curve-constrained hyper-redundant manipulators", Computational Intelligence in Robotics and Automation, 2001. Proceedings 2001 IEEE International Symposium, 29 July-1 Aug. 2001 pp.83-88.
- [4] Oda.N, Murakami.T, Ohnishi.K.: "A force based motion control strategy for hyper-redundant manipulator", Industrial Electronics, Control and Instrumentation, IECON 97. 23rd International Conference, Volume 3, 9-14 Nov. 1997 pp.1385-1390 vol.3.
- [5] Hirose.S, Chu.R: "Development of a light weight torque limiting M-Drive actuator for hyper-redundant manipulator Float Arm ", Robotics and Automation, 1999. Proceedings. 1999 IEEE International Conference, Volume 4, 10-15 May 1999 pp.2831-2836 vol.4 .
- [6] Glass K., Colbaugh R., Lim D. and Seraji H, "Real-time collision avoidance for redundant manipulators", IEEE Transactions on Robotics and Automation, Vol.11, pp.448-457, 1995.
- [7] Homayoun Seraji and Bruce Bon, "Real-Time Collision Avoidance for Position-Controlled Manipulators", IEEE Transactions on Robotics and Automation, Vol.15, No.4, pp.670-677, 1999.
- [8] Leon Zlajpah and Bojan Nemeec, "Kinematic Control Algorithms for On-line Obstacle Avoidance for Redundant Manipulator", International Conference on Intelligent Robots and Systems, Lausanne, Vol.2, pp.1898-1903, 2002.
- [9] Kwang-Kyu Lee and Martin Buss, "Obstacle Avoidance for Redundant Robots Using Jacobian Transpose Method", International Conference on Intelligent Robots and Systems, San Diego, pp.3509-3514, 2007.
- [10] Ryo Kurazume and Tsutomu Hasegawa, "Impedance Matching for Free Flying Robots", (in Japanese) Proc. 20th Annu. Conf. Robot. Soc. Jpn., p. 3J16, 2002.
- [11] Ryo Kurazume and Tsutomu Hasegawa, "A New Index of Serial-Link Manipulator Performance Combining Dynamic Manipulability and Manipulating Force Ellipsoid", IEEE Transactions on Robotics, Vol.22, No.5, pp.1022-1028, 2006.
- [12] Takeshi Ikeda and Mamoru Minami: "Research of Grinding Robot without Force Sensor by Using Algebraic Equation"(in Japanese), Transactions of the Japan Society of Mechanical Engineers(C), Vol.71, No.702, pp.270-277, 2005.
- [13] Nobuhide Suda, Shinzou Kojima: "Matrix Theory for Control Engineers (2)", System and Control, Vol.15, No.10, pp.864-878, 1971 (in Japanese).
- [14] M.W.Walker and D.E.orin: "Efficient Dynamic Computer Simulation of Robotic Mechanisms", ASME J.of DSCM, 104, pp.205-211, 1982
- [15] L.R.Hunt, R.Su and G.Meyer: "Global Transformation of Non Linear system," IEEE Trans. AC., 28-1, pp.24-31, 1983
- [16] K.Uematsu, T.Fukuda: "Control of Redundant Manipulator using Geometric Constraint Force on the Environment Surface", Trans. of the Japan Soc. of Mech. Eng., C, Vol.65, No. 637, JSME, pp.180-185 (1999) (in Japanese)