

Precise and Efficient Control Method Using Position/Force Bracing Redundancy

Wenhao Gu, Tomohide Maeba, Mamoru Minami, Akira Yanou

Abstract—Inspired by human’s handwriting motion with the elbow or wrist contacting to a table, we thought that manipulator can save energy by bracing manipulator’s link with table, and can raise the hand’s trajectory tracking accuracy also. First we propose a dynamical model of robot whose plural links contact with table. In the model, the constrained forces are included and expressed as a function of the state and generalized forces by using the equation of constraints, what is important is that this is an algebraic function. By taking advantage of the redundancy of input generalized forces to the constrained force, a new control method which can control the hand’s trajectory and position/force of bracing point simultaneously was proposed. We named the redundancy appearing during bracing motion as “Bracing Redundancy”, which is different from well-known kinematical redundancy. Simulations results have certified that this method can improve accuracy and save energy comparing well known control strategy that utilizes Jacobean transpose.

keywords—manipulator; position/foce control; save energy;

I. INTRODUCTION

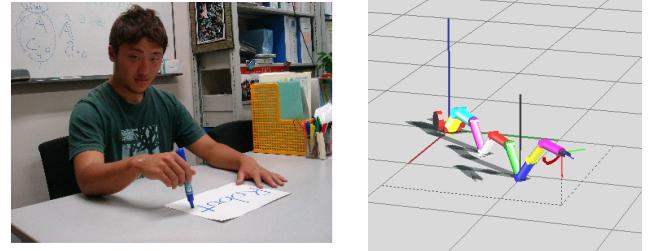
Handwriting motion is one of the examples of human’s skillful behavior that is thought to aim at exploiting the contact constraint of the elbow with the table for reducing inputting energy by countering the gravity effects with reaction forces. By supporting the elbow or wrist by the desk, we have known from our experiences that we can save energy for writing task and can write characters correctly. This suggests that robots may execute tasks with less energy and improved accuracy by exploiting constrained contact with environment.

In this paper we present a dynamical model of manipulator whose plural links contact with environment, e.g. tables and floors. This bracing behavior can usually be found in human’s common motions, such as handwriting, as shown Fig.1(a).

On the other hand, there have been several researches discussing effectiveness and accuracy of the redundant manipulator contacting with environments. West and Asada [10] presented a general kinematics contact model for the design of hybrid position/force controllers for constrained manipulator. And then a multi-contact kinematic model to control manipulator’s contact motion was also presented by Oussama Khatib in [11], [12], in which they assumed the contact environment as a spring model. Contrarily in this paper we will discuss purely rigid contact model without

This work was not supported by any organizatio

Wenhao Gu, Tomohide Maeba, Mamoru Minami, Akira Yanou are with Graduate School of Natural Science and Technology, Okayama University Tsushima-naka3-1-1, Okayama, JAPAN. {guwenhao, maeba, minami, yanou}@suri.sys.okayama-u.ac.jp



(a) Human’s writing motion

(b) Contacting strategy

Fig. 1. The sketch picture of Manipulator with bracing elbows

contacting deformation of environment.

The constrained force is derived from the equation of dynamics and the constrained equation as an explicit algebraic function of states and input generalized forces[16], which means force information can be obtained by calculation rather than by force sensing . Equation (1), which has been pointed out by Peng [17] in the analysis of biped walking robot, denotes also the kinematical algebraic relation of the controller, when robot’s end-effector being in touch with a surface in 3-D space:

$$f_n = a(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{b}^T(\mathbf{q})\boldsymbol{\tau}, \quad (1)$$

where, f_n is exerting force on the constrained surface. \mathbf{q} are angle of joint. $a(\mathbf{q}, \dot{\mathbf{q}})$ and $\mathbf{b}^T(\mathbf{q})$ are scalar function and row vector defined in following section, and $\boldsymbol{\tau}$ is input torque. This algebraic equation has been known, but it was the first time in robotics to be applied to the sensing function of exerting force by Peng [17].

A strategy to control force and position proposed in this paper is also based on (1). Contrarily to Peng’s Method to use (1) as a force sensor, we used the equation for calculating $\boldsymbol{\tau}$ to achieve a desired exerting force f_{nd} . Actually, the strategy is based on two facts of (1) that have been ignored for a long time. The first fact is that the force transmission process is an immediately process being stated clearly by (1) is an algebraic function providing that the manipulator’s structure is rigid. Contrarily, the occurrence of velocity and position is time-consuming process. By using this algebraic relation, it’s possible to control the exerting force to the desired one without time lag. Another important fact is the input generalized forces have some redundancy against the constrained generalized forces in the constrained motion. Concerning (1), f_{nd} is a scalar and $\boldsymbol{\tau}$ is a vector, thus $\boldsymbol{\tau}$ to achieve f_{nd} has a redundancy. We named this as “Bracing

Redundancy". By taking advantage of the redundancy of input generalized forces to the constrained force and the fact that the force transmission from τ to f_{nd} is an immediate completion as shown in 1), a new control method that can control the position/force of bracing joint was proposed.

II. MODELLING OF HYPER-REDUNDANT MANIPULATOR WITH CONSTRAINTS

A. Manipulator's Model with Hand's Constraint

To make the explanation of constraint motion with multi-elbows be easily understandable, we discuss firstly about the model of the manipulator whose end-effector is contacting with rigid environment without elasticity. Equation of motion of manipulator is composed of rigid structure of s links, and also contact relation between manipulator's end-effector and definition of constraint surface should be introduced firstly. L represents Lagrangian, $\mathbf{q} \in R^s$ represents the general coordinate, $\tau \in R^s$ represents the general input. u is the unknown constant of Lagrange, f_t is the friction. Manipulator hand's Lagrange equation can be expressed as follows

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) - \left(\frac{\partial L}{\partial \mathbf{q}}\right) = \tau + \left(\frac{\partial C}{\partial \mathbf{q}^T}\right)^T u - \left(\frac{\partial \mathbf{r}}{\partial \mathbf{q}^T}\right)^T \frac{\dot{\mathbf{r}}}{\|\dot{\mathbf{r}}\|} f_t \quad (2)$$

Here according to the kinematic relation, manipulator hand's position/posture vector $\mathbf{r} \in R^s$ and scalar function, a single constraint condition C that is used to express the hypersurface can be expressed as

$$\mathbf{r} = \mathbf{r}(\mathbf{q}) \quad (3)$$

$$C(\mathbf{r}(\mathbf{q})) = 0 \quad (4)$$

Here (3) and (4) represent constraint is undeformed.

To move freely in the directions without constraint the freedom of manipulator's end-effector is left to be more than one, so here $s > 1$. If we set f_n to indicate the constraint force of manipulator hand, then the relation of u and f_n can be expressed as

$$u = f_n / \left\| \frac{\partial C}{\partial \mathbf{r}^T} \right\| \quad (5)$$

$\left\| \frac{\partial C}{\partial \mathbf{r}^T} \right\|$ shows Euclidean norm of vector $\frac{\partial C}{\partial \mathbf{r}^T}$. Then manipulator's equation of motion can be derived by combining (2) with (5) with viscous friction of joints.

$$\begin{aligned} M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \mathbf{D}\dot{\mathbf{q}} \\ = \tau + \left\{ \left(\frac{\partial C}{\partial \mathbf{q}^T} \right)^T / \left\| \frac{\partial C}{\partial \mathbf{r}^T} \right\| \right\} f_n - \left(\frac{\partial \mathbf{r}}{\partial \mathbf{q}^T} \right)^T \frac{\dot{\mathbf{r}}}{\|\dot{\mathbf{r}}\|} f_t \end{aligned} \quad (6)$$

M is inertia matrix of $s \times s$, \mathbf{h} and \mathbf{g} are $s \times 1$ vectors which indicate the effects from coriolis force, centrifugal force and gravity, \mathbf{D} is a $s \times s$ matrix which indicates the coefficient of joints' viscous friction, expressed as $\mathbf{D} = \text{diag}[D_1, D_2, \dots, D_s]$. \mathbf{q} is the joint angle and τ is the input torque.

B. Bracing Dynamics

The manipulator's equation of motion has been derived, shown as (6). Because the coefficient value of the constraint

force and $(\partial C / \partial \mathbf{q})M^{-1}(\partial C / \partial \mathbf{q})^T$ is always positive definite, it is always invertible. f_n shown in (6) can be calculated. First, let

$$\left(\frac{\partial C}{\partial \mathbf{q}} \right) M^{-1} \left(\frac{\partial C}{\partial \mathbf{q}} \right)^T \triangleq m_c \quad (7)$$

then we can obtain,

$$\begin{aligned} f_n &= m_c^{-1} \left\| \frac{\partial C}{\partial \mathbf{r}} \right\| \left\{ -\dot{\mathbf{q}}^T \left[\frac{\partial}{\partial \mathbf{q}} \left(\frac{\partial C}{\partial \mathbf{q}} \right) \dot{\mathbf{q}} \right] \right. \\ &\quad + \left. \left(\frac{\partial C}{\partial \mathbf{q}} \right) M^{-1} (\mathbf{h} + \mathbf{g} + \left(\frac{\partial \mathbf{r}}{\partial \mathbf{q}} \right)^T \frac{\dot{\mathbf{r}}}{\|\dot{\mathbf{r}}\|} f_t) \right\} \\ &- m_c^{-1} \frac{\partial C}{\left\| \frac{\partial \mathbf{r}}{\partial \mathbf{q}} \right\|} \left\{ \left(\frac{\partial C}{\partial \mathbf{q}} \right) M^{-1} \right\} \tau \end{aligned} \quad (8)$$

It can be rewritten as

$$f_n = a(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{b}^T(\mathbf{q}) \left(\frac{\partial \mathbf{r}}{\partial \mathbf{q}} \right)^T \frac{\dot{\mathbf{r}}}{\|\dot{\mathbf{r}}\|} f_t - \mathbf{b}^T(\mathbf{q}) \tau \quad (9)$$

where,

$$\begin{aligned} m_c^{-1} \left\| \frac{\partial C}{\partial \mathbf{r}} \right\| \left\{ -\dot{\mathbf{q}}^T \left[\frac{\partial}{\partial \mathbf{q}} \left(\frac{\partial C}{\partial \mathbf{q}} \right) \dot{\mathbf{q}} \right] + \left(\frac{\partial C}{\partial \mathbf{q}} \right) M^{-1} (\mathbf{h} + \mathbf{g}) \right\} \\ \triangleq a(\mathbf{q}, \dot{\mathbf{q}}) \end{aligned} \quad (10)$$

$$m_c^{-1} \left\| \frac{\partial C}{\partial \mathbf{r}} \right\| \left\{ \left(\frac{\partial C}{\partial \mathbf{q}^T} \right) M^{-1} \right\} \triangleq \mathbf{b}^T(\mathbf{q}) \quad (11)$$

$a(\mathbf{q}, \dot{\mathbf{q}})$ is scalar. It is the sum of the term which does not include τ , $\mathbf{b}^T(\mathbf{q}) \in R^{1 \times l}$ is a vector that projects τ to constraint force f_n . Reviewing the dynamic equation (2) and constraint condition (4), it can be found that $l > 1$, the number of input generalized forces is more than that of the constrained forces. From this point and (9) we can claim that there is some redundancy of constrained force between the input torques τ , and the constrained force f_n . This condition is much similar to the kinematical redundancy.

III. CONTROLLER

In this section, we will introduce the control method, called bracing redundancy position/force control method, which we used in the simulation. Based on the argument in section 2, the parameters of the (9) are known and its state variables could be measured. Fig.2 shows the four links manipulators model. As Fig.2 shown, the second link is contacting with an environment, which means that the constraint condition $C = 0$ be known, $a(\mathbf{q}, \dot{\mathbf{q}})$ and $\mathbf{b}^T(\mathbf{q})$ in (9) could be calculated correctly. Then we propose the following equation.

$$\begin{aligned} \tau &= (\mathbf{b}^T)^+(\mathbf{q}) \{ a(\mathbf{q}, \dot{\mathbf{q}}) - f_{nd} \} \\ &+ M_1 M_1^+ \Delta \tau_1 + M_1 M_2 \Delta \tau_2, \end{aligned} \quad (12)$$

Where, M_1 and M_2 were defined as

$$M_1 = I - (\mathbf{b}^T)^+(\mathbf{q}) \mathbf{b}(\mathbf{q}) \quad (13)$$

$$M_2 = I - M_1^+(\mathbf{q}) M_1(\mathbf{q}), \quad (14)$$

I is an identity matrix of $s \times s$, f_{nd} is the desired constrained forces, $(\mathbf{b}^T)^+(\mathbf{q})$ is the pseudoinverse matrix of $\mathbf{b}^T(\mathbf{q})$,

$$(\mathbf{b}^T)^+(\mathbf{q}) = (\mathbf{b}^T)^T(\mathbf{q})\{\mathbf{b}^T(\mathbf{q})(\mathbf{b}^T)^T(\mathbf{q})\}^{-1} \quad (15)$$

$\Delta\tau_1$ and $\Delta\tau_2$ which can be used for other tasks besides realizing the second link contacting force f_{nd} , as (16) and (17) shown, here are used to control the position of bracing joint and end-effector.

$$\Delta\tau_1 = \mathbf{J}^T[\mathbf{K}_{p4}(\mathbf{r}_{4d} - \mathbf{r}_4) + \mathbf{K}_{d4}(\dot{\mathbf{r}}_{4d} - \dot{\mathbf{r}}_4)], \quad (16)$$

$$\Delta\tau_2 = \mathbf{J}^T[\mathbf{K}_{p2}(\mathbf{r}_{2d} - \mathbf{r}_2) + \mathbf{K}_{d2}(\dot{\mathbf{r}}_{2d} - \dot{\mathbf{r}}_2)], \quad (17)$$

where \mathbf{K}_{pi} and \mathbf{K}_{di} are coefficient matrices applied to

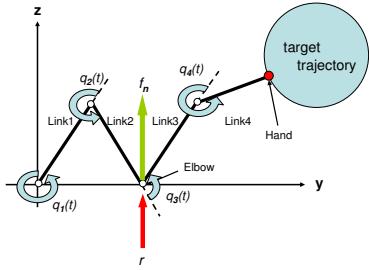


Fig. 2. Four links manipulators model

the position control by the redundant degree of freedom of $\mathbf{b}^T(\mathbf{q})$. $\mathbf{r}_{4d}(\mathbf{q})$ is the desired position vector of the end-effector and $\mathbf{r}_4(\mathbf{q})$ is the real position vector. So we can see $\Delta\tau_1$ is used to control the position of the manipulator's end-effector. $\mathbf{r}_{2d}(\mathbf{q})$ is the desired position vector of the bracing link along the constrained surface and $\mathbf{r}_2(\mathbf{q})$ is the real position vector of the second link. And $\Delta\tau_2$ is used to control the position of the manipulator's bracing joint.

We have gotten the function about f_n as (8). To confirm that hand position control through $\Delta\tau_1$ and bracing joint position control through $\Delta\tau_2$ should disappear in a null space of $\mathbf{b}^T(\mathbf{q})$, we combine (8) and (14) then we can get the following equation,

$$\begin{aligned} f_n &= a(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{b}^T(\mathbf{q})(\mathbf{b}^T)^+(\mathbf{q})\{a(\mathbf{q}, \dot{\mathbf{q}}) - f_{nd}\} \\ &+ \mathbf{b}^T(\mathbf{q})\mathbf{M}_1\mathbf{M}_1^+\Delta\tau_1 + \mathbf{b}^T(\mathbf{q})\mathbf{M}_1\mathbf{M}_2\Delta\tau_2. \end{aligned} \quad (18)$$

since

$$\mathbf{b}^T(I - (\mathbf{b}^T)^+\mathbf{b}^T)\mathbf{M}_1^+ = \mathbf{0} \quad (19)$$

$$\mathbf{b}^T(I - (\mathbf{b}^T)^+\mathbf{b}^T)\mathbf{M}_2 = \mathbf{0}, \quad (20)$$

the (18) can be written as

$$\begin{aligned} f_n &= a(\mathbf{q}, \dot{\mathbf{q}}) + f_{nd} - a(\mathbf{q}, \dot{\mathbf{q}}) \\ &= f_{nd}. \end{aligned} \quad (21)$$

From (21) we can see the real constraint force f_n is equal to desired force f_{nd} all the time by Position/Force control method.

IV. SIMULATION

In this section we will introduce the simulation to check the model discussed in this paper is right or not. Simulation's condition has been set as: each link's mass is $m_i = 1.0[kg]$, length is $l_i = 0.5[m]$, radius of cylindrical link is $r_i = 0.01[m]$, proportional gain is $k_{pi} = 300$, velocity gain is $k_{di} = 50$, viscous friction coefficient of joint is $D_i = 0.5$, torque constant is $K_i = 0.203$, resistance is $R_i = 1.1[\Omega]$, inductance is $L_i = 0.0017[H]$, inertia moment of motor is $I_{mi} = 0.000164[A]$, reduction ratio is $k_i = 3.0$, viscous friction coefficient of reducer is $d_{mi} = 0.01$ and these parameters are given by actual motor's specifications. And the trajectory has been set as a circle with radius is $0.1[m]$, center is $(x, y, z) = (0.0, 1.0, 0.6)$, target which is tracked by the manipulator hand will rotate in counterclockwise along this circle trajectory. The simulation controlled with Position/Force control method is shown as Fig.3

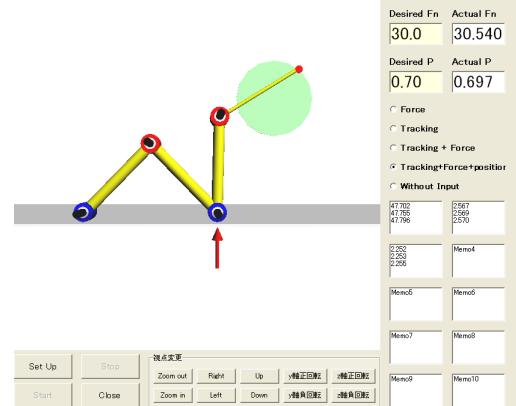
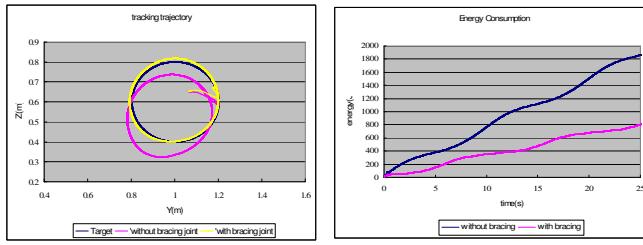


Fig. 3. The simulation controlled with Position/Force control method

A. Experiment One

We proposed a realistic idea that contacting and bracing motion of intermediate links with environment may improve the manipulator working precision and save energy. In this section, we want to confirm this assumption. So we conducted this experiment in two different conditions. One condition is the manipulator with bracing joint, the other is without.

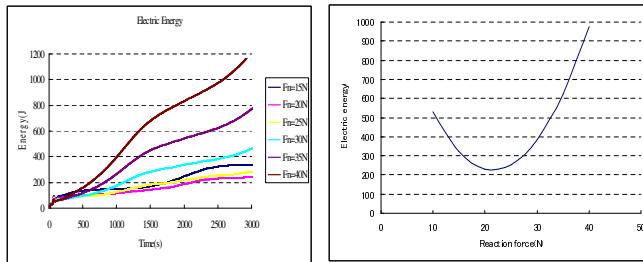
From Fig.4(a), We can see the manipulator with bracing joint can track the target more accurate than the one without bracing. Fig.4(b) shows the energy consumption in two conditions. We can see, doing the same work, the manipulator with bracing joint consumes less energy than the one without joint. These indicate the manipulator with bracing joint can improve accuracy and save energy.



(a) The tracking trajectory (b) The energy consumption
Fig. 4. The comparing of energy consumption and trajectory in two conditions

B. Experiment Two

In order to find the relationship between the energy consumption and reaction force, we conducted this experiment. Fig.5 is shown the energy consumption with different reaction force. From Fig.5(a), we can see the energy consumption is different as to the reaction force is different, which means the reaction force is one of the factors that affect the energy consumption. And Fig.5(b) is the function image about energy consumption and the reaction force at fifth minute. We thought when the reaction force of bracing joint is exactly equal to the gravity of manipulator acting on this joint, the energy consumption is least. The reason is when the reaction force of bracing joint is equal to the gravity of manipulator acting on this joint exactly; the contact constraint of the joint will reduce inputting energy and counter the gravity effects completely. So the energy consumption is least. Here we can get the conclusion that by Position/Force control method, we can control the reaction force value to save the energy consumption.



(a)The energy consumption at (b)The function image about en-
different reaction force ergy consumption

Fig. 5. The electric energy consumption with different reaction forces

V. CONCLUSION

We first derive a dynamical model of manipulator whose plural intermediate links are contacting with environment. Then we proposed a new control method called Position/Force control method. And we did two experiments. The outcome of first experiment indicates the manipulator with bracing joint can improve accuracy and save energy comparing to the one without bracing joint. By the second experiment, we find this new control method can save the energy by controlling the reaction force as the reaction force

is one of the factors that affect the energy consumption. What is more, this method can control the position of bracing joint. Even though we haven't thought out the appalction need to control the position, but we still think this method would be very useful. In the future, we will use this control method in hyper-redundancy manipulator, and we think it can control the hyper-redundancy manipulator better. At last, we will figure out how the reaction forces affect the energy consumption by mathematical derivation and also we want to find out the appalction need to control the force of brcing joint.

REFERENCES

- [1] G.S.Chirikjian, J.W.Burdick:“A Hyper-Redundant Manipulator”, IEEE Robotics & Automation Magazine, December 1994, pp.22-29 (1994)
- [2] Shugen Ma, Watanabe.M, Kondo.H:“Dynamic control of curve-constrained hyper-redundant manipulators”, Computational Intelligence in Robotics and Automation, 2001. Proceedings 2001 IEEE International Symposium, 29 July-1 Aug. 2001 pp.83-88.
- [3] Oda.N, Murakami.T, Ohnishi.K,:“A force based motion control strategy for hyper-redundant manipulator”, Industrial Electronics, Control and Instrumentation, IECON 97. 23rd International Conference, Volume 3, 9-14 Nov. 1997 pp.1385-1390 vol.3.
- [4] Hirose.S, Chu.R:“Development of a light weight torque limiting M-Drive actuator for hyper-redundant manipulator Float Arm ”, Robotics and Automation, 1999. Proceedings. 1999 IEEE International Conference , Volume 4, 10-15 May 1999 pp.2831-2836 vol.4 .
- [5] Glass K., Colbaugh R., Lim D. and Seraji H. “Real-time collision avoidance for redundant manipulators”, *IEEE Transactions on Robotics and Automation*, Vol.11, pp.448-457, 1995.
- [6] Homayoun Seraji and Bruce Bon, “Real-Time Collision Avoidance for Position-Controlled Manipulators”, *IEEE Transactions on Robotics and Automation*, Vol.15, No.4, pp.670-677, 1999.
- [7] Leon Zlajpah and Bojan Nemeć, “Kinematic Control Algorithms for On-line Obstacle Avoidance for Redundant Manipulator”, *International Conference on Intelligent Robots and Systems*, Lausanne, Vol.2, pp.1898-1903, 2002.
- [8] Kwang-Kyu Lee and Martin Buss, “Obstacle Avoidance for Redundant Robots Using Jacobian Transpose Method”, *International Conference on Intelligent Robots and Systems*, San Diego, pp.3509-3514, 2007.
- [9] Ryo Kurazume and Tsutomu Hasegawa, “A New Index of Serial-Link Manipulator Performance Combining Dynamic Manipulability and Manipulating Force Ellipsoid”, *IEEE Transactions on Robotics*, Vol.22, No.5, pp.1022-1028, 2006.
- [10] West.H.,and Asada.H.,“A Method for the Design of Hybrid Position/Force Controllers for Manipulators Constrained by Contact with the Environment”, Proc. of 1985 IEEE International Conference on Robotics and Automation, pp.251-260
- [11] Jaeheung Park and Oussama Khatib , “Multi-Link Multi-Contact Force Control for Manipulators”, Proc. of 2005 IEEE International Conference on Robotics and Automation, April 2005, pp.3624-3629
- [12] Anna Petrovskaya, Jaeheung Park and Oussama Khatib “Probabilistic Estimation of Whole Body Contacts for Multi-Contact Robot Control”, Proc. of 2007 IEEE International Conference on Robotics and Automation, April 2007, pp.568-573
- [13] Takeshi Ikeda and Mamoru Minami:“Research of Grinding Robot without Force Sensor by Using Algebraic Equation”(in Japanese), Transactions of the Japan Society of Mechanical Engineers(C), Vol.71, No.702, pp.270-277, 2005.
- [14] M.W.Walker and D.E.orin:‘Efficient Dynamic Computer Simulation of Robotic Mechanisms”, ASME J. of DSMC,104, pp.205-211, 1982
- [15] L.R.Hunt, R.Su and G.Meyer:“Global Transformation of Non Linear system,” IEEE Trans. AC., 28-1, pp.24-31, 1983
- [16] Takeshi Ikeda, Mamoru Minami, “Research of Grinding Robot without Force Sensor by Using Algebraic Equation(in Japanese).” Transactions of the Japan Society of Mechanical Engineers(C), Vol.71, No.702, pp.270-277, 2005.
- [17] Z. X. Peng, N. Adachi, “Position and Force Control of Manipulators without Using Force Sensors(in Japanese),” Trans. of JSME(C), Vol.57, No.537, pp.1625-1630, 1991.