Extended Self-tuning Generalized Predictive Control with Computation Reduction Focused on Closed-loop Characteristics ⋆

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Abstract: This paper proposes a design method of extended self-tuning generalized predictive control (GPC) with computation reduction focused on closed-loop characteristics. The authors have extended GPC by coprime factorization and proposed the extended controller for constructing a strongly stable system. Moreover, the proposed controller is able to be designed to make the same steady state output as pre-designed system's steady state output even if feedback loop is cut. Although self-tuning controller is one of the control methods for systems with uncertainty, there is a problem that the computation of self-tuning GPC increases as design engineer takes long prediction horizon in the design parameters. Therefore this paper considers computation reduction for extended self-tuning GPC focused on closed-loop characteristics. The validity of the proposed method is shown by numerical simulation.

Keywords: Generalized predictive control; Self-tuning control; Coprime factorization; Closed-loop gain; Data reduction.

1. INTRODUCTION

Generalized Predictive Control (GPC) has been proposed by Clarke et al. [1987]. The control law is derived by minimizing an objective function. The objective function includes three design parameters, named as prediction horizon, control horizon and weighting factor of control input. The control input is calculated repeatedly by receding each horizon to future time. The control strategy has been acceptable to many of practical engineers and applied widely in industry (Camacho and Bordons [1995]). Although safety is most important in applying to industry, it is desirable to make the controller stable in addition to the closed-loop stability because the output of open-loop system with unstable feedforward controller becomes divergent if the feedback loop is cut. That is, a strongly stable controller is essential and the stable controller can keep the boundedness of its output even when the feedback loop is cut by an accident. It means that the system output can also keep its boundedness in the case that the controlled plant is stable.

So far many papers concerning on the stability of GPC have been published by Demircioglu and Clarke [1992], Kouvaritakis et al. [1997], Gossner et al. [1997], and most of the papers are devoted to obtain the design methods ensuring the stability of closed-loop system although less attention is paid to the design of strongly stable GPC. Hence in order to obtain a strongly stable GPC, we used conventional GPC designed by an existing method and it was extended by coprime factorization (Vidyasagar [1988]). Then the extended GPC can be designed to derive a stable controller by using newly introduced parameter (Inoue et al. [1999], Yanou et al. [2009]), that is, strongly stable system can be obtained. But the steady state of system output in the case that feedback loop was cut has not been considered in the methods. When the controlled plant is stable, the steady state of strongly stable system's output is bounded even if feedback loop is cut. And it is desirable for safety that the open-loop steady state output becomes as close to the closed-loop steady state output as possible even if feedback loop was cut. Therefore, a new design method has been proposed for strongly stable GPC focused on closed-loop characteristics by Okazaki et al. [2011]. The method has the feature that the open-loop steady state output becomes equal to the closed-loop steady state output even if feedback loop was cut.

In the case of designing the self-tuning controller, its computation should be reduced in the viewpoint of system cost, implementation and so on. In this paper, the calculation amount can be reduced by solving Diophantine equation for derivation of an output prediction equation only once (Saudagar et al. [1995]). And this paper explores a design method of extended self-tuning GPC with computation reduction focused on closed-loop characteristics. In view point of safety, it is suitable for temperature controller in process control that the output fluctuation remains small value if the feedback loop is cut by an accident. And it is also desirable to apply self-tuning controller in order to maintain the control performance for characteristic variation of controlled plant with aged dete-

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rioration. Therefore the temperature control of aluminum plate model is selected as a numerical example for the verification of the e ectiveness of the proposed method.

2. PROBLEM STATEMENT

We consider a single-input single-output system.

$$A[z^{-1}]y(t) = z^{-k_m}B[z^{-1}]u(t) \quad (1)$$

Where $y(t)$ and $u(t)$ denote output and input, k_m is time delay. $A[z^{-1}]$ and $B[z^{-1}]$ are n-order and m-order polynomials respectively.

$$A[z^{-1}] = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}$$
$$B[z^{-1}] = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}$$

In the first step the output prediction for the deviation system of (1) is calculated. The steady state values y_∞ and u_∞ for $y(t)$ and $u(t)$ are derived as follows.

$$A[z^{-1}]y_\infty = z^{-k_m}B[z^{-1}]u_\infty \quad (2)$$

By using (2), the deviation system of (1) is obtained as follows.

$$A[z^{-1}]y(t) = z^{-k_m}B[z^{-1}]u(t) \quad (3)$$

The deviations $y(t)$ and $u(t)$ are defined as $y(t) = y(t) - y_\infty$ and $u(t) = u(t) - u_\infty$. Assuming that y_∞ is equal to be the reference signal w, the prediction of $y(t)$ can be derived by the following Diophantine equation.

$$1 = A[z^{-1}]E_N[z^{-1}] + z^{-N}F_N[z^{-1}] \quad (N \geq N_2) \quad (4)$$

Where $[N_1, N_2]$ is prediction horizon in the design parameters, N is an integer, $E_N[z^{-1}]$ and $F_N[z^{-1}]$ are polynomials with degree $N-1$ and $n-1$,

$$E_N[z^{-1}] = 1 + e_1 z^{-1} + \ldots + e_{N-1} z^{-(N-1)}$$
$$F_N[z^{-1}] = f_0^j + f_1^j z^{-1} + \ldots + f_{n-1}^j z^{-(n-1)}$$

In GPC (Clarke et al. [1987], Inoue et al. [1999], Okazaki et al. [2011]), although (4) is repeatedly solved for $j = N_1, \cdots, N_2$ to derive the output prediction, this paper solves it for $j = N$ according to Saudagar et al. [1995]. Therefore the computation of the control law is especially reduced in the case of designing adaptive controller.

In order to separate the future and past values of $u(t)$ in (3), $E_N[z^{-1}]B[z^{-1}]$ for $j = N_1, \cdots, N_2$ is separated as

$$E_N[z^{-1}]B[z^{-1}] = R_j[z^{-1}] + z^{-j}S_j[z^{-1}]$$

$R_j[z^{-1}]$ and $S_j[z^{-1}]$ are polynomials with degree of $j-1$ and $n_3 = N + m - 1 - j$ as follows.

$$R_j[z^{-1}] = r_0 + r_1 z^{-1} + \ldots + r_{j-1} z^{-(j-1)}$$
$$S_j[z^{-1}] = s_0 + s_1^j z^{-1} + \ldots + s_{m_3}^j z^{-n_3}$$

Then j-ahead output $y(t+j)$ is derived by using $F_N[z^{-1}]$, $R_j[z^{-1}]$ and $S_j[z^{-1}]$.

$$y(t+j) = R_j[z^{-1}]u(t+j-k_m) + h_j(t)$$

Where

$$h_j(t) = F_N[z^{-1}]y(t) + S_j[z^{-1}]u(t-k_m)$$

Assuming that there is no perturbation, j-ahead prediction $y(t+j|t)$ is given as $y(t+j|t) = y(t+j)$. The objective function for the deviation system (3) is defined to derive the control law.

$$J = \sum_{j=N_1}^{N_2}\{y(t+j)\}^2 + \lambda\sum_{j=1}^{Nu}\{u(t+j-1)\}^2 \quad (5)$$

$[1, N_u]$ and λ are control horizon and weighting factor on control inputs in the design parameters respectively. For simplicity, it is assumed that $N_1 = k_m = 1$ and $N_u = N_2$ in this paper. And the objective function can also be rewritten by the following vector form.

$$J = \hat{\tilde{\boldsymbol{y}}}^T\hat{\tilde{\boldsymbol{y}}} + \lambda\tilde{\boldsymbol{u}}^T\tilde{\boldsymbol{u}} \quad (6)$$

The control law will be derived by minimizing the objective function J on $\tilde{\boldsymbol{u}}$. Here $\hat{\tilde{\boldsymbol{y}}}$, $\tilde{\boldsymbol{u}}$, $\boldsymbol{h}$ and $\boldsymbol{R}$ are defined.

$$\hat{\tilde{\boldsymbol{y}}} = \left[y(t+1|t), y(t+2|t), \cdots, y(t+N_2|t)\right]^T$$
$$\tilde{\boldsymbol{u}} = [u(t), u(t+1), \cdots, u(t+N_2-1)]^T$$
$$\boldsymbol{h} = [h_1(t), h_2(t), \cdots, h_{N_2}(t)]^T$$

$$\boldsymbol{R} = \begin{bmatrix} r_0 & 0 & \cdots & & 0 \\ r_1 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ r_{N_2-1} & \cdots & \cdots & r_1 & r_0 \end{bmatrix}$$

The output prediction vector $\hat{\tilde{\boldsymbol{y}}}$ and the objective function J are given as the following vector forms.

$$\hat{\tilde{\boldsymbol{y}}} = \boldsymbol{R}\tilde{\boldsymbol{u}} + \boldsymbol{h} \quad (7)$$
$$J = (\boldsymbol{R}\tilde{\boldsymbol{u}} + \boldsymbol{h})^T(\boldsymbol{R}\tilde{\boldsymbol{u}} + \boldsymbol{h}) + \lambda\tilde{\boldsymbol{u}}^T\tilde{\boldsymbol{u}} \quad (8)$$

Then the control input $u(t)$ minimizing J is obtained by solving $\partial J/\partial\hat{\boldsymbol{u}} = 0$ and extracting the first element of $\tilde{\boldsymbol{u}}$,

$$u(t) = \frac{F_p[z^{-1}] + (1 + z^{-1}S_p[z^{-1}])K}{1 + z^{-1}S_p[z^{-1}]}w - \frac{F_p[z^{-1}]}{1 + z^{-1}S_p[z^{-1}]}y(t) \quad (9)$$

Where

$$[p_1, \cdots, p_{N_2}] = -[1, 0, \cdots, 0](\boldsymbol{R}^T\boldsymbol{R} + \lambda\boldsymbol{I})^{-1}\boldsymbol{R}^T$$
$$F_p[z^{-1}] = \sum_{j=1}^{N_2} p_j F_N[z^{-1}], \quad K = \frac{A[1]}{B[1]}$$
$$S_p[z^{-1}] = \sum_{j=1}^{N_2} p_j S_j[z^{-1}]$$

The closed-loop system from (1) and (9) is given by

$$y(t) = \frac{z^{-1}B[z^{-1}]\{F_p[z^{-1}] + (1 + z^{-1}S_p[z^{-1}])K\}}{T[z^{-1}]}w \quad (10)$$

And the closed-loop characteristic $T[z^{-1}]$ is given as follows.