Development of an Inverted Pendulum Cart with a Sliding Mechanism for Posture Control

–Design and Manufacture of a Small Mobility and Experiment–

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Abstract: A lot of researches on inverted pendulum cart are conducted in recent years. And it is attracted as a personal vehicle which realizes energy saving for many practical applications, such as the Segway and the P.U.M.A. The personal mobility has the advantage of energy efficiency for transportation because the cart is small and light. However, since most conventional personal vehicles require a certain level of physical ability from the driver, it is not suitable for elderly and disabled people to drive. Therefore, an inverted pendulum cart with a sliding mechanism for posture control is developed as a personal mobility available for anyone. And we aim for realization of acceleration and deceleration while keeping the angle of the cart perpendicular. In this research, proposed inverted pendulum cart with a sliding mechanism for posture control was designed and manufactured, and the posture control performance was verified through experiment.

Keywords: Inverted pendulum, Posture control, Personal mobility

1. INTRODUCTION

An inverted pendulum is a mechanical system which performs the same operation as the play which a child stands an umbrella or a stick upside down on his/her palm by automatic control. Because of no actuators in pendulum mechanism part structurally, this is also a kind of a system with fewer actuators than degrees of freedom, so-called under actuated system [1]. The inverted pendulum cart is what adopted this inverted pendulum mechanism and controlled by using acceleration caused by intentionally imbalance for movement. An inverted pendulum cart was proposed by Yamafuji et al. [2] as example of application of modern control theory. And the realization of function due to a difference in structure has been proposed by a number of researchers until today. In addition, a lot of researches on inverted pendulum cart are conducted in recent years. It has been applied to a personal vehicle and a mobile robot [3], and a fast passing over stairs robot [4]. It is attracted as a personal vehicle which realizes energy saving for many practical applications, such as the Segway [5] and the P.U.M.A [6] and the MOBIRO [7]. Especially, Segway secured a no practical problem level of stability, with only two parallel wheels, it has demonstrated the reliability of control of wheel type inverted pendulum. These personal mobilities are small and light and have the advantage of energy efficiency for transportation per person [8]. This is a big merit of personal mobility. However, since most conventional personal vehicles require a certain level of physical ability because the driver controls posture in person, it is not suitable for elderly and disabled people.

Therefore, an inverted pendulum cart with a sliding mechanism for posture control is developed as a personal mobility available for anyone to realize the energy savings as a society. And we aim for realization of acceleration and deceleration while keeping the angle of the inverted pendulum cart perpendicular. Because the proposed inverted pendulum cart always keep the angle of the cart perpendicular, the driver does not need make moved the center of gravity for oneself when it accelerates, decelerates and curves. In this research, proposed inverted pendulum cart with a sliding mechanism was designed and manufactured, and construct electrical circuits system including a microcomputer. Finally, the posture control performance was verified through experiment.

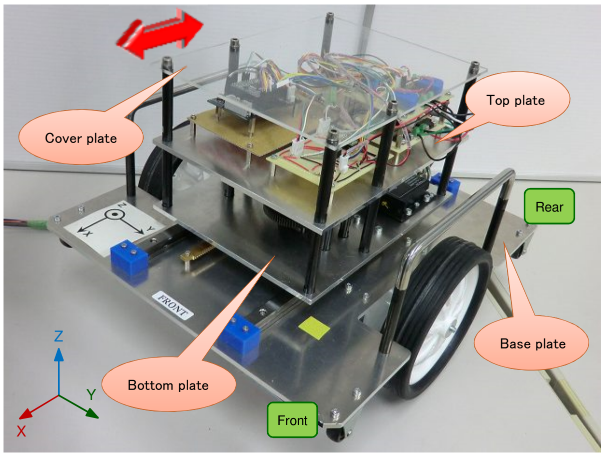
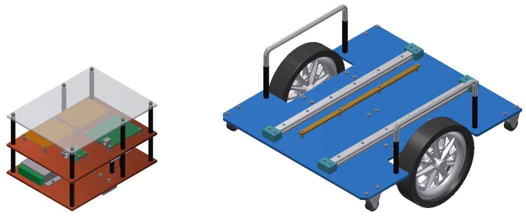


Fig. 1 External view of the inverted pendulum cart

2. CONTROL SYSTEM CONFIGURATION

2.1 Structure

The appearance of proposed inverted pendulum cart is shown in Fig.1. The length of the cart is 420 [mm], the width is 385 [mm], and the height is 280 [mm]. The parts are defined as base plate, bottom plate, top plate, cover plate. A rack gear, rails, and stoppers are mounted on base plate. Motor drivers are mounted on bottom plate



(a) Upside (b) Downside

Fig. 2 Structure of the inverted pendulum cart

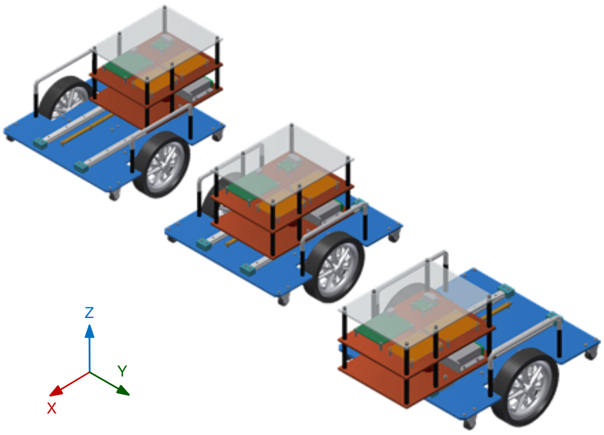


Fig. 3 Movement of the slider

and a microcomputer is mounted on top plate. Casters are mounted on base plate in order to decrease the load on the motor if the cart falls down. The casters will not touch the ground during feedback control, because the cart can incline until about 25 degrees.

An upper part of the body, which is named as slider, is shown in Fig.2(a) and a lower part is shown in Fig.2(b). A proposed sliding mechanism enables to control position of the center of gravity and keep the angle of the cart perpendicular, because entire cart is not only moved but also the slider shown in Fig.2(a) is moved to back and forth as Fig.3. The weight of the upper part is 3.5 [kg] and the weight of the lower part is 5.6 [kg], and total weight of the cart is 9.1 [kg]. And, range of motion of the slider, or length of the rack gear is about 280 [mm]. It is enough length for posture control by movement of the slider.

There are ball screw mechanism, rack and pinion mechanism, and belt pulley mechanism as power transmission mechanism [9]. Rack and pinion mechanism has feature that backlash is small, structure is simple, energy loss is small and torque is easily transmitted because rigidity is strong. This is the reason why rack and pinion mechanism is adopted as movement of the slider.

Two-stage speed reducer configured by combining spur gear shown in Fig.4 is manufactured. And enough reduction gear ratio not to lose the weight of the slider is set.

The proposed inverted pendulum cart needs to acquire

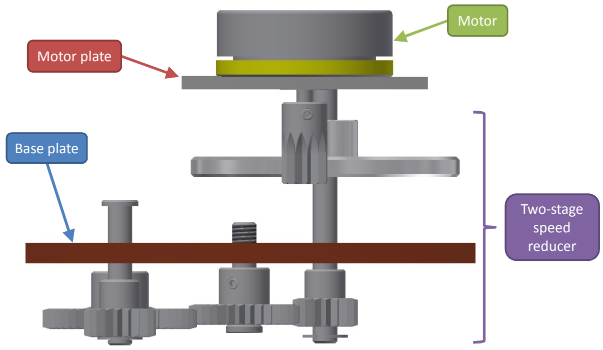


Fig. 4 Two-stage speed reducer

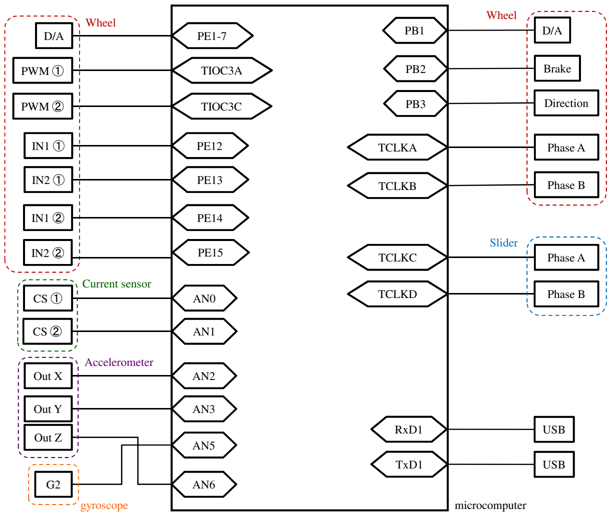


Fig. 5 Wiring diagram of microcomputer and peripheral devices

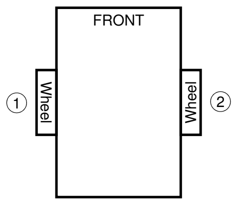


Fig. 6 Wheel number

the value of the accelerometer, the gyroscope and the encoder in order to obtain state variables for feedback control. Therefore, STK-7125 which has SH7125 microprocessor equipped with the number of ports required for A/D conversion is used. Wiring diagram of such as sensors and assignment of the number to wheels are shown in Fig.5 and Fig.6. And, D/A converter is required in order to transmit voltage to motor drivers. However, there is not D/A converter on microcomputer. Therefore, D/A converter is made by combining digital output parts.

Two wheel motors have an encoder respectively, and also a motor for the slider connect with encoder via gears. These encoders are counted by quad edge evaluation.

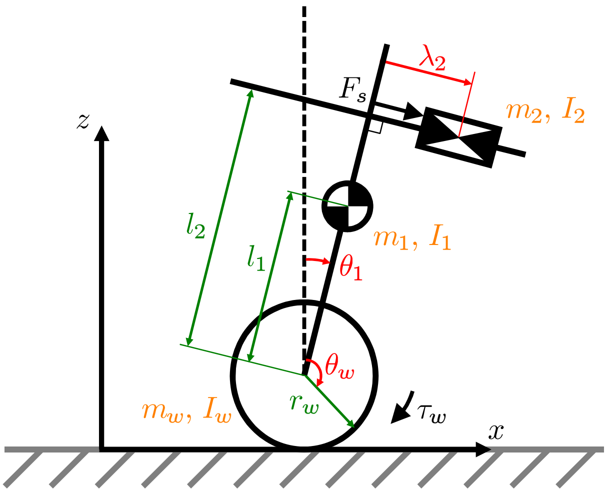


Fig. 7 Model of the inverted pendulum cart with a sliding mechanism

Table 1 Parameters

SYMBOLS	MEANINGS
m_1 [kg]	Weight of cart
m_2 [kg]	Weight of slider
m_w [kg]	Weight of wheel
r [m]	Radius of wheel
I_1 [kgm^2]	Inertia moment of cart
I_2 [kgm^2]	Inertia moment of slider
I_w [kgm^2]	Inertia moment of wheel
l_1 [m]	Height of COG of cart from center of wheel
l_2 [m]	Height of COG of slider from center of wheel
D_1 [Nms]	Coefficient of viscous friction for wheel axis
D_2 [Ns/m]	Coefficient of viscous friction for velocity of slider
D_w [Nms]	Coefficient of viscous friction or angular velocity of wheel angle
g [m/s^2]	Gravitational acceleration

2.2 Modeling

A model of dynamics of inverted pendulum cart is shown in Fig.7. Here, the state variables for control are the angle of the inverted pendulum cart, θ_1 [rad], the rotation angle of the tire wheel, θ_w [rad], and the displacement of the slider, λ_2 [m]. State variables are included derivative of above mentioned parameters. The control inputs are the torque of the motor wheel, τ_w [Nm], and the actuator force to move the seat of the cart, F_s [N]. The model parameters are described in Table 1.

Next, system dynamics of the inverted pendulum cart is derived. The kinetic energy T and the potential energy V is considered, and Lagrange L is obtained by Fig.7.

$$
\begin{aligned}
L &= T - V. && (1)\\
T &= \frac{1}{2}m_w(r_w\dot{\theta_w})^2 + \frac{1}{2}I_w\dot{\theta_w}^{\,2}\\
&\quad +\frac{1}{2}m_1(r_w\dot{\theta_w} + l_1\dot{\theta_1}\cos\theta_1)^2\\
&\quad +\frac{1}{2}m_1(l_1\dot{\theta_1}\sin\theta_1)^2\\
&\quad +\frac{1}{2}m_2\{r_w\dot{\theta_w} + (l_2\cos\theta_1 - l_2\sin\theta_1)\dot{\theta_1}\\
&\quad +\dot{\lambda_2}\cos\theta_1\}^2\\
&\quad +\frac{1}{2}m_2\{(-l_2\sin\theta_1\\
&\quad -\lambda_2\cos\theta_1)\dot{\theta_1} - \dot{\lambda_2}\sin\theta_1\}^2\\
&\quad +\frac{1}{2}(I_1+I_2)\dot{\theta_1}^{\,2}. && (2)\\
V &= m_1 g l_1\cos\theta_1\\
&\quad +m_2 g(l_2\cos\theta_1 - \lambda_2\sin\theta_1). && (3)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
L &= \frac{1}{2}I_w^*\dot{\theta_w}^{\,2}\\
&\quad +\frac{1}{2}(m_1l_1^2 + m_2l_2^2 + m_2\lambda_2^2 + I_1 + I_2)\dot{\theta_1}^{\,2}\\
&\quad +\frac{1}{2}m_2\dot{\lambda_2}^{\,2}\\
&\quad +\{m_1l_1\cos\theta_1 + m_2(l_2\cos\theta_1\\
&\quad -\lambda_2\sin\theta_1)\}r_w\dot{\theta_1}\dot{\theta_1}\\
&\quad +m_2r_w\cos\theta_1\dot{\theta_w}\dot{\lambda_2}\\
&\quad +m_2l_2\dot{\theta_1}\dot{\lambda_2}\\
&\quad -m_1gl_1\cos\theta_1\\
&\quad -m_2g(l_2\cos\theta_1 - \lambda_2\sin\theta_1). && (4)
\end{aligned}
$$

Here,

$$
I_w^* = I_w + (m_1 + m_2 + m_w)r_w^2. \tag{5}
$$

By lagrange equation

$$
\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta_w}}\right) - \frac{\partial L}{\partial\theta_w} + D_w\dot{\theta_w} + D_1(\dot{\theta_w} - \dot{\theta_1}) = \tau_w, \tag{6}
$$

the equation is obtained as follows:

$$
\begin{aligned}
&I_w^*\ddot{\theta}_w + r_w\{(m_1l_1 + m_2l_2)\cos\theta_1\\
&-m_2\lambda_2\sin\theta_1\}\ddot{\theta}_1\\
&+m_2r_w\cos\theta_1\,\ddot{\lambda}_2\\
= \;&\tau_w - (D_w + D_1)\dot{\theta}_w + D_1\dot{\theta}_1\\
&+r_w\{(m_1l_1 + m_2l_2)\sin\theta_1\\
&+m_2\lambda_2\cos\theta_1\}\,\dot{\theta}_1^2\\
&-2m_2r_w\sin\theta_1\dot{\theta}_1\dot{\lambda}_2. && (7)
\end{aligned}
$$

And by lagrange equation

$$
\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta_1}}\right) - \frac{\partial L}{\partial\theta_1} + D_{\theta_1}\dot{\theta_1} = -\tau_w, \tag{8}
$$

the equation is obtained as follows:

$$\begin{aligned} & r_w\{(m_1 l_1 + m_2 l_2)\cos\theta_1 \\ & -m_2\lambda_2 \sin\theta_1\}\,\ddot{\theta}_w \\ & +(m_1 l_1^2 + m_2 l_2^2 + m_2\lambda_2^2 + I_1 + I_2)\,\ddot{\theta}_1 \\ & +m_2 l_2\,\ddot{\lambda}_2 \\ = \; & -\tau_w - D_1\dot{\theta}_1 + D_1\dot{\theta}_w - 2m_2\lambda_2\dot{\theta}_1\dot{\lambda}_2 \\ & +(m_1 l_1 + m_2 l_2) g \sin\theta_1 \\ & +m_2 g \lambda_2 \cos\theta_1. \end{aligned} \tag{9}$$

And by lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\lambda}_2}\right) - \frac{\partial L}{\partial \lambda_2} + D_2\dot{\lambda}_2 = F_s, \tag{10}$$

the equation is obtained as follows:

$$\begin{aligned} & m_2 r_w \cos\theta_1\,\ddot{\theta}_w + m_2 l_2\,\ddot{\theta}_1 + m_2\,\ddot{\lambda}_2 \\ = \; & F_s - D_2\dot{\lambda}_2 + m_2\lambda_2\dot{\theta}_1^{\,2} + m_2 g \sin\theta_1. \end{aligned} \tag{11}$$

Sliding mechanism provides possibility of quick acceleration with keeping the angle of the cart perpendicular. However, if motion of the slider and acceleration of cart are not synchronized, the angle of the cart may be disturbed for under actuated system. Such a situation destabilizes cart system. It is known that it is impossible to continue acceleration while keeping pendulum cart perpendicularly by state feedback control. This problem is caused due to linear projections in coordinate orthogonalized variables of state. Therefore, it is difficult to always keep only particular variables of state at zero.

Matsuno et al. proposed the control method to realize quick acceleration with keeping the angle of the cart perpendicular for inverted pendulum cart through this model in a previous study [10]. And, the effectiveness of proposed methods to keep the angle of the cart perpendicular is confirmed through simulations.

2.3 Control system

Control system of inverted pendulum cart is shown in Fig.8. Values of accelerometer and gyroscope are acquired by A/D conversion and the angle of the cart is calculated in microcomputer. Equally, values of wheel encoder and slider encoder are acquired by A/D conversion and amount of movement is calculated. Each wheel is controlled by applying the appropriate voltage based on calculated values to wheel motor driver and slider motor driver. Two direction arrow between slider motor driver and slider motor show using a moror with hall sensors. Data sampling period and control cycle are set as 10 [ms].

3. VERIFICATION OF POSTURE CONTROL PERFORMANCE

In order to verify posture control performance of the inverted pendulum cart with a sliding mechanism, experiments are conducted with the manufactured cart.

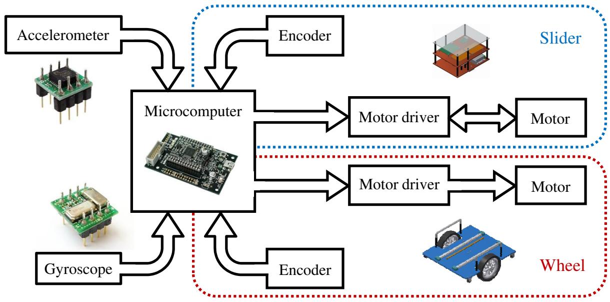


Fig. 8 Configuration of control system

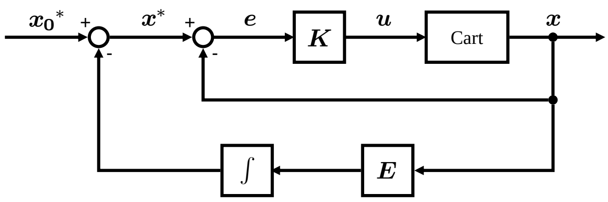


Fig. 9 Block diagram of state feedback

3.1 Control algorithm

Experiments are carried out by state feedback in this paper. The wheel torque, the slider torque and the slider displacement are drawn up by using the angle of the cart θ_1 obtained from accelerometer. Block diagram of state feedback is shown in Fig.9.

Here,

$$\boldsymbol{x} = [\theta_1 \quad \lambda_2 \quad \dot{\lambda}_2]^T, \tag{12}$$

$$\boldsymbol{x}^* = [\theta_1^* \quad \lambda_2^* \quad \dot{\lambda}_2^*]^T, \tag{13}$$

$$\boldsymbol{x}_0^* = [0 \quad 0 \quad 0]^T, \tag{14}$$

$$\boldsymbol{u} = [\tau_w \quad F_s]^T, \tag{15}$$

$$\boldsymbol{e} = \boldsymbol{x}^* - \boldsymbol{x}, \tag{16}$$

$$\boldsymbol{E} = \begin{bmatrix} 0 & 0 & 0 \\ \varepsilon & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{17}$$

$$\boldsymbol{K} = \begin{bmatrix} K_p & 0 & 0 \\ 0 & K_s & K_d \end{bmatrix}. \tag{18}$$

And, controller is shown as follows:

$$\begin{bmatrix} \tau_w \\ F_s \end{bmatrix} = \begin{bmatrix} K_p & 0 & 0 \\ 0 & K_s & K_d \end{bmatrix} \begin{bmatrix} \theta_1 \\ \lambda_2 \\ \dot{\lambda}_2 \end{bmatrix} \tag{19}$$

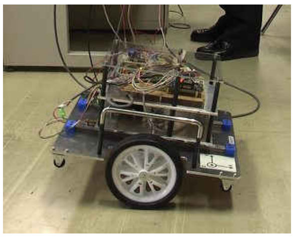
Each gain is decided heuristically as follows:

$$K_s = 0.8 \tag{20}$$

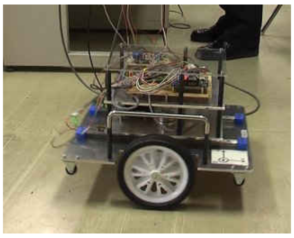
$$K_d = 0.03 \tag{21}$$

$$\epsilon = 7.0 \times 10^{-5} \tag{22}$$

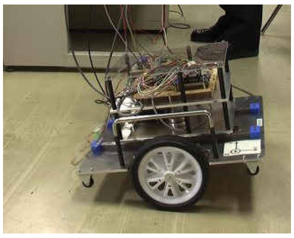
Since the threshold values by which wheels begin to move are different from right and left, The wheel gain is



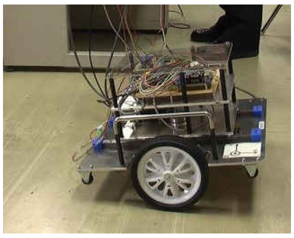
(a) 5.65 [s]



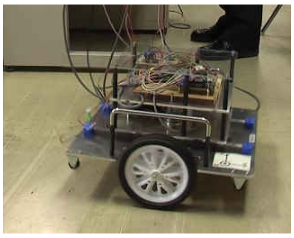
(b) 5.75 [s]



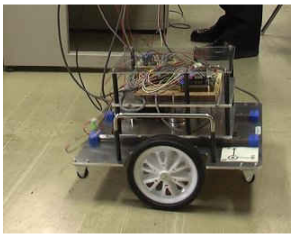
(c) 5.8 [s]



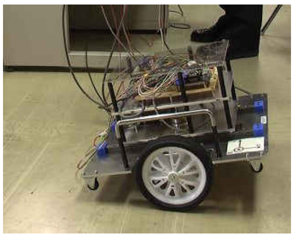
(d) 5.85 [s]



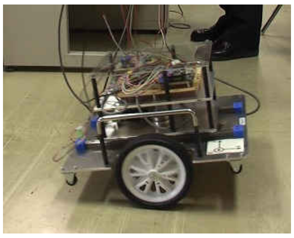
(e) 6.0 [s]



(f) 6.35 [s]



(g) 6.4 [s]



(h) 6.65 [s]

Fig. 10 First experiment for posture control

set as follows:

$$K_{p_L} = 1.4 \quad (23)$$

$$K_{p_R} = 2.0 \quad (24)$$

Here, the left gain is K_{p_L}, the right gain is K_{p_R}.

3.2 Experimental result

The situation of experiments are shown in Fig.10. The angle of the inverted pendulum cart, θ_1, the torque of the motor wheel, τ_w, and the rotation angle of the tire wheel, θ_w on first experiment are shown in Fig.11(a)-(c). And, the angle of the inverted pendulum cart, θ_1, the actuator force to move the seat of the cart, F_s, and the displacement of the slider, λ_2 on second experiment are shown in Fig.11(d)-(f).

3.3 Discussion

As shown in Fig.11, intentional movement can be realized until fall in any experiment. Therefore, although the operation which tends toward a stable state is seen, this is not maintainable.

When attention is paid to Fig.11(d) and Fig.11(f), F_s also changes in accordance with λ_2 at 12 [s] approximately. Because motor gear separates from rack gear on the both edges, torque is cut off in this moment. After that, the slider hit and rebound from the stopper, and gears engage and return to the initial value. This means that the moderating ratio of the slider is large enough.

Moreover, the fact that the angle of the inverted pendulum cart calculated only one accelerometer may affects error of θ_1. Since accelerometer also detect acceleration by motion of the cart [11] as above-mentioned. By using a gyroscope together, it appears that this detection error decreases.

4. CONCLUSION

In this report, proposed inverted pendulum cart with a sliding mechanism for posture control is introduced as a personal mobility available for anyone. And a small mobility was designed and manufactured. Rack and pinion mechanism is adopted to this mobility and rotational motion of the motor is converted to linear motion. Thus, it is possible for not only the movement of the cart but also posture control by seesaw of the slider. Finally, posture control performance was verified through experiment. It is a future work that it keeps standing a little longer time.

The following are mentioned as improvements: stabilization of a posture, the angle of the inverted pendulum cart which used the accelerometer and the gyroscope together. Moreover, wireless in order not to affect tension by a power cable or a communication cable, digitization of each sensor, and so on are needed. We wrestle these assignments and aim to realize acceleration and deceleration with keeping the angle of the cart perpendicular in the future.

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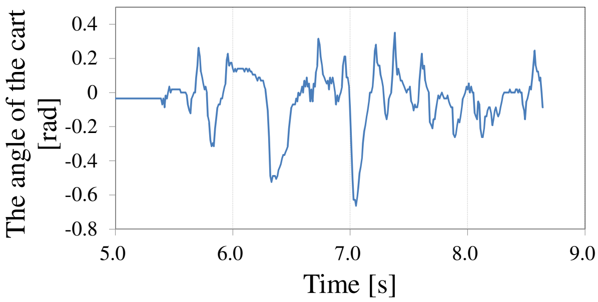
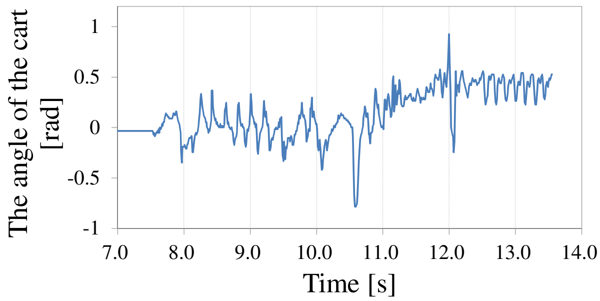
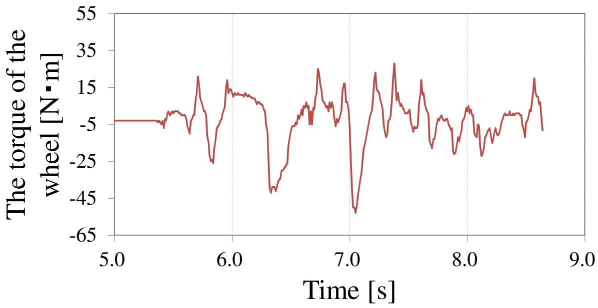


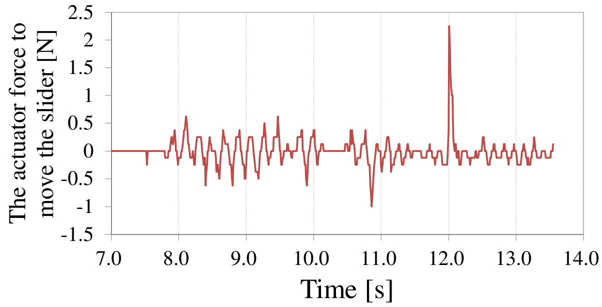
(a) θ_1 (1st)



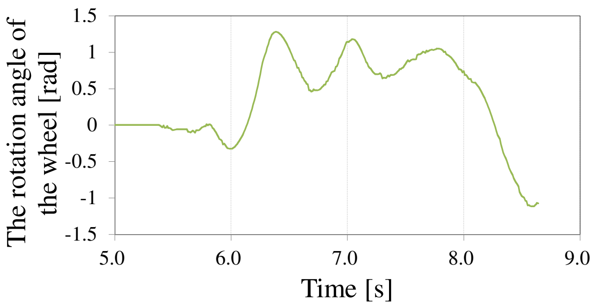
(d) θ_1 (2nd)



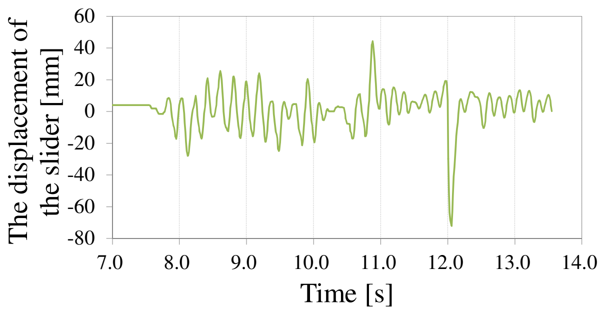
(b) τ_w (1st)



(e) F_s (2nd)



(c) θ_w (1st)



(f) λ_2 (2nd)

Fig. 11 Experimental results

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