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Dynamical Analyses of Humanoid's Walking by using Extended Newton-Euler Method

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Abstract: Biped locomotion created by a controller based on Zero-Moment Point [ZMP] known as reliable control method looks different from human's walking on the view point that ZMP-based walking does not include falling state, and it's like monkey walking because of knee-bended walking profiles. However, the walking control that does not depend on ZMP is vulnerable to turnover. Therefore, keeping the event-driven walking of dynamical motion stable is important issue for realization of human-like natural walking. In this paper, walking model of humanoid including slipping, bumping, surface-contacting and point-contacting of foot is discussed, where dynamical equation is derived by Newton-Euler method with constraint condition, which is named as "Extended Newton-Euler Method". Then, we further discuss walking stabilizer named "Visual Lifting Stabilization" to enhance standing robustness and prevent the robot from falling down. Simulation results indicate that this strategy helps stabilize pose and bipedal walking even though ZMP is not kept inside convex hull of supporting area.

Keywords: Bipedal, Dynamical, Extended Newton-Euler Method

1 INTRODUCTION

Human beings have acquired an ability of stable bipedal walking in evolving repetitions so far. From a view point of making a stable controller for the bipedal walking based on knowledge of control theory, it looks not easy because of the dynamics with high nonlinearity and coupled interactions between state variables with high dimensions. Therefore how to simplify the complicated walking dynamics to help construct stable walking controller has been studied intensively. Avoiding complications in dealing directly with true dynamics without approximation, inverted pendulum has been used frequently for making a stable controller [1]-[3], simplifying the calculations to determine input torque. Further, linear approximation having the humanoid being represented by simple inverted pendulum enables researchers to realize stable gait through well-known control strategy [4]-[6].

Our research has begun from a view point of [7] as aiming to describing gait's dynamics as correctly as possible, including point-contacting state of foot and toe, slipping of the foot and bumping. However, our model differs from [7] and [8] is that we discuss the dynamics of whole-body humanoid that contains head, waist and arms. And that what the authors think more important is that the dimension of dynamical equation will change depending on the walking gait's varieties, which has been discussed by [9] concerning one-legged hopping robot. In fact, this kind of dynamics with the dimension number of state variables varying by the result of its dynamical time transitions that are out of the arena of control theory that discusses how to control a system with fixed states' number. Further the tipping over motion has been called as non-holonomic dynamics that includes a joint without inputting torque, i.e., free joint.

Meanwhile, landing of the heel or the toe of lifting leg in the air to the ground makes a geometrical contact. [10] mentioned how to represent robot's motion contacting with environment that can handle constraint motion with friction by algebraic equation and applied it to human figures. Based on these references, we derive the dynamics of humanoid which is simulated as a serial-link manipulator including constraint motion and slipping motion by using Extended Newton-Euler Method [11].

The conventional method of NE could be applied to a robot having an open loop serial linkage structure, but the motion of hand was limited to motions without contacting external world. NE method has not been formulated although it was very important for a robot that works under a premise that it must be contacted with the environment when the robot was doing some grinding work or assembling work. For this point, the extended NE method proposed in [11] is is same as the research of [10][12], in terms of that the constraints are strictly satisfied. Meanwhile, the constraint force which can be included in the iterative calculation of NE method by calculating the constraint force by a substitution method [13].

2 THE INVERSE DYNAMICS SOLUTION BY NEWTON-EULER METHOD DURING RESTRAINED MOTION

Here, we consider the inverse dynamics solution of constrained motion of a tip link of straight chain link manipu-

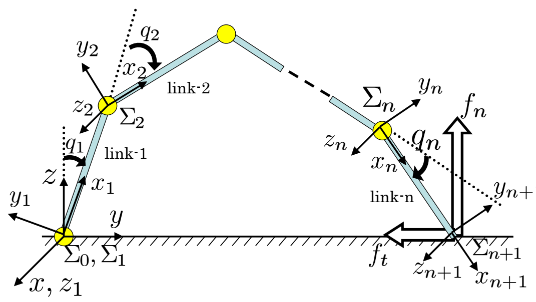

Fig. 1. n-link manipulator whose hand position is constraint by non elastic environment, which is a floor in this figure

lator which constituted by undeformed rigid links while it is contacting the undeformed environment. Considering the manipulator with n rigid links shown in Fig.1, which has a straight chain link structure and n degrees of freedom, and affected by friction force f_t and drag force f_n to hand from the ground. We will derive the equation of motion based on a coordinate system Σ_i fixed to the link i. Σ_0 is a work coordinate system fixed to the floor. The constraint condition can be defined as Eq.(1) when the hand is restrained to a restraint surface, and the $\boldsymbol{r}(\boldsymbol{q})$ is the position vector of the hand, and $\boldsymbol{q}$ is joint angle vector.

$$C(\boldsymbol{r}(\boldsymbol{q})) = 0 \tag{1}$$

Here, we can assume that $C(\boldsymbol{r}(\boldsymbol{q}))$ is differentiable respecting to $\boldsymbol{r}$ and $\boldsymbol{q}$. In Fig.1, there is depicted on the assumption that the robot is in contact with a floor environment, but the following discussions are not limited to discussions about the floor constraint.

First, as a forward dynamics computation of the Newton-Euler method, we can calculate the joint angular velocity ${}^i\boldsymbol{\omega}_i$ of link i toward the tip link from the root link, the joint angular acceleration ${}^i\dot{\boldsymbol{\omega}}_i$, the translational acceleration at the origin of Σ_i, ${}^i\ddot{\boldsymbol{p}}_i$, and the translational acceleration in the center of mass by the following equation. The upper left subscript shows the reference coordinate system, the lower right subscript shows the target link.

$$
\begin{aligned}
{}^i\boldsymbol{\omega}_i &= {}^{i-1}\boldsymbol{R}_i^{\mathrm{T}}\,{}^{i-1}\boldsymbol{\omega}_{i-1} + {}^i\boldsymbol{z}_i\dot{q}_i && (2)\\
{}^i\dot{\boldsymbol{\omega}}_i &= {}^{i-1}\boldsymbol{R}_i^{\mathrm{T}}\,{}^{i-1}\dot{\boldsymbol{\omega}}_{i-1} + {}^i\boldsymbol{z}_i\ddot{q}_i + {}^i\boldsymbol{\omega}_i \times ({}^i\boldsymbol{z}_i\dot{q}_i) && (3)\\
{}^i\ddot{\boldsymbol{p}}_i &= {}^{i-1}\boldsymbol{R}_i^{\mathrm{T}}\Big\{{}^{i-1}\ddot{\boldsymbol{p}}_{i-1} + {}^{i-1}\dot{\boldsymbol{\omega}}_{i-1} \times {}^{i-1}\hat{\boldsymbol{p}}_i \\
&\quad + {}^{i-1}\boldsymbol{\omega}_{i-1} \times ({}^{i-1}\boldsymbol{\omega}_{i-1} \times {}^{i-1}\hat{\boldsymbol{p}}_i)\Big\} && (4)\\
{}^i\ddot{\boldsymbol{s}}_i &= {}^i\ddot{\boldsymbol{p}}_i + {}^i\dot{\boldsymbol{\omega}}_i \times {}^i\hat{\boldsymbol{s}}_i + {}^i\boldsymbol{\omega}_i \times ({}^i\boldsymbol{\omega}_i \times {}^i\hat{\boldsymbol{s}}_i) && (5)
\end{aligned}
$$

${}^{i-1}\boldsymbol{R}_i$ is a rotation matrix from Σ_{i-1} to Σ_i, and ${}^i\boldsymbol{z}_i = [0, 0, 1]^{\mathrm{T}}$ is a unit vector of rotation axis of the link i, ${}^{i-1}\hat{\boldsymbol{p}}_i$ is a position vector from the origin point of Σ_{i-1} to Σ_i, ${}^i\hat{\boldsymbol{s}}_i$ is a position vector from the origin point of Σ_i to the center of mass of link i. As the initial value, we set ${}^0\boldsymbol{\omega}_0 = \boldsymbol{0}$, ${}^0\dot{\boldsymbol{\omega}}_0 = \boldsymbol{0}$, ${}^0\ddot{\boldsymbol{p}}_0 = [0, 0, g]^{\mathrm{T}}$, ${}^0\ddot{\boldsymbol{s}}_0 = \boldsymbol{0}$. Here g is the gravity acceleration.

Then, based on the inverse dynamics calculation, the Newton equation and the Euler equation of link i are derived recursively from bottom link to top link as Eq.(6)∼Eq.(8).

$${}^{n+1}\boldsymbol{f}_{n+1} = -{}^0\boldsymbol{R}_{n+1}^{\mathrm{T}}\left\{\frac{\left(\frac{\partial C}{\partial \boldsymbol{r}}\right)}{\left\|\frac{\partial C}{\partial \boldsymbol{r}}\right\|}f_n - \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|}f_t\right\} \tag{6}$$

$${}^i\boldsymbol{f}_i = {}^i\boldsymbol{R}_{i+1}\,{}^{i+1}\boldsymbol{f}_{i+1} + m_i\,{}^i\ddot{\boldsymbol{s}}_i \tag{7}$$

$$
\begin{aligned}
{}^i\boldsymbol{n}_i &= {}^i\boldsymbol{R}_{i+1}\,{}^{i+1}\boldsymbol{n}_{i+1} + {}^i\boldsymbol{I}_i\,{}^i\dot{\boldsymbol{\omega}}_i + {}^i\boldsymbol{\omega}_i \times ({}^i\boldsymbol{I}_i\,{}^i\boldsymbol{\omega}_i)\\
&\quad + {}^i\hat{\boldsymbol{s}}_i \times (m_i\,{}^i\ddot{\boldsymbol{s}}_i) + {}^i\hat{\boldsymbol{p}}_{i+1} \times ({}^i\boldsymbol{R}_{i+1}\,{}^{i+1}\boldsymbol{f}_{i+1}) && (8)
\end{aligned}
$$

The ${}^i\boldsymbol{f}_i$, ${}^i\boldsymbol{n}_i$ in Σ_i show the force and moment exerted on link i from link $(i-1)$. And ${}^i\boldsymbol{I}_i$ denotes the inertia matrix of the center of gravity of link i. Because ${}^{n+1}\boldsymbol{f}_{n+1}$ that is a force transmitting from top link to the floor will be the reaction force of drag force and friction force, we can calculate it as shown in Eq.(6). About constraint motion, we will make two assumptions as follows. (i)The drag force f_n and the friction force f_t on external contact portion are orthogonal. (ii)f_t is determined in proportion to drag force: $f_t = Kf_n$ (K is the coefficient of friction force : $0 < K \leq 1$). The drag force f_n can be determined by the method described in the next chapter. Equation of motion of all links can be obtained by repeating the Newton and Euler's equation in Eq.(7) and Eq.(8) from hand to root link. Giving the Σ_i to all joints that have rotation axes about the ${}^iz_i$-axis, the relationship between ${}^0\boldsymbol{n}_i$ and joint driving force τ_i can be calculated as follows.

$$\tau_i = {}^i\boldsymbol{z}_i^{\mathrm{T}}\,{}^i\boldsymbol{n}_i + D_i\dot{q}_i \tag{9}$$

Here, D_i represents the viscous friction coefficient of joint i.

3 DRAG FORCE F_N

3.1 Derivation of Drag Force f_n

In this chapter we describe a method of deriving the drag force f_n. A condition of hand constraint state of manipulator robot is represented by Eq.(1), and its equation of motion is represented by Eq.(10).

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} - (\boldsymbol{j}_c - \boldsymbol{j}_t K)f_n = \boldsymbol{\tau} \tag{10}$$

$\boldsymbol{M}(\boldsymbol{q})$ is inertia matrix with $n \times n$, $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and $\boldsymbol{g}(\boldsymbol{q})$ are vector with $n \times 1$ representing the term of centrifugal force/Coriolis force and the gravity term, D is a diagonal matrix $\boldsymbol{D} = diag[D_1, D_2, \cdots, D_n]$ with $n \times n$ representing the viscous friction coefficients of joints, $\boldsymbol{\tau}$ is a input torque vector with $n \times 1$, and $\boldsymbol{q} = [q_1,\ q_2,\ \cdots,\ q_n]^{\mathrm{T}}$ is the joint angle vector with $n \times 1$. Besides, $\boldsymbol{j}_c$ and $\boldsymbol{j}_t$ are defined as follows.

$$\boldsymbol{j}_c \triangleq \left(\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^{\mathrm{T}}}\right)^{\mathrm{T}}\left(\frac{\partial C}{\partial \boldsymbol{r}}\right)\Big/\left\|\frac{\partial C}{\partial \boldsymbol{r}}\right\|,\ \ \boldsymbol{j}_t \triangleq \left(\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^{\mathrm{T}}}\right)^{\mathrm{T}}\frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|} \tag{11}$$

Making second-order differentiation of Eq.(1) by time t to determine the constraint condition $\ddot{\boldsymbol{q}}$, we have

$$\dot{\boldsymbol{q}}^T \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C}{\partial \boldsymbol{q}^T} \right) \right] \dot{\boldsymbol{q}} + \left(\frac{\partial C}{\partial \boldsymbol{q}^T} \right) \ddot{\boldsymbol{q}} = 0. \tag{12}$$

In order to make manipulator be always constrained to restraint surface, the solution $\boldsymbol{q}(t)$ of Eq.(10) should satisfy the Eq.(1) regardless of the time t. When $\ddot{\boldsymbol{q}}$ in Eq.(10) and $\ddot{\boldsymbol{q}}$ which is satisfied to the Eq.(12) obtained by the time derivative of Eq.(1) take the same value, the $\boldsymbol{q}(t)$ in Eq.(10) will satisfy Eq.(1). Erasing the $\ddot{\boldsymbol{q}}$ by Eq.(10) and Eq.(12).

$$\begin{aligned} &\left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \boldsymbol{M}^{-1} \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right)^{\mathrm{T}} \frac{f_n}{\left\| \dfrac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\|} \\ &= \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \boldsymbol{M}^{-1} \left(\boldsymbol{j}_t K f_n + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g} - \boldsymbol{\tau} \right) \\ &-\dot{\boldsymbol{q}}^{\mathrm{T}} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} \end{aligned} \tag{13}$$

We got Eq.(13). Here, we define m_c as follow.

$$m_c \triangleq (\partial C / \partial \boldsymbol{q}^{\mathrm{T}}) \boldsymbol{M}^{-1} (\partial C / \partial \boldsymbol{q}^{\mathrm{T}})^{\mathrm{T}} \tag{14}$$

$\boldsymbol{M}^{-1}$ is nonsingular, $\partial C / \partial \boldsymbol{q}^{\mathrm{T}} = (\partial C / \partial \boldsymbol{r}^{\mathrm{T}})(\partial \boldsymbol{r} / \partial \boldsymbol{q}^{\mathrm{T}})$, C is a curved surface to satisfy the $\partial C / \partial \boldsymbol{r}^{\mathrm{T}} \neq \boldsymbol{0}$. Here, assuming the $\partial \boldsymbol{r} / \partial \boldsymbol{q}^{\mathrm{T}}$ is full row rank, and to be considered with the exception of singular configuration, there will be $m_c \neq 0$, since $\partial C / \partial \boldsymbol{q}^{\mathrm{T}} \neq \boldsymbol{0}$. By using m_c, Eq.(13) can be rewritten as Eq.(15).

$$\begin{aligned} m_c f_n = &\left\| \frac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \Big\{ \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \boldsymbol{M}^{-1} (\boldsymbol{j}_t K f_n + \boldsymbol{D}\dot{\boldsymbol{q}} \\ &+ \boldsymbol{h} + \boldsymbol{g} - \boldsymbol{\tau}) - \dot{\boldsymbol{q}}^{\mathrm{T}} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} \Big\} \end{aligned} \tag{15}$$

Here, we define $\boldsymbol{d}^{\mathrm{T}}$ as follow.

$$\boldsymbol{d}^{\mathrm{T}} \triangleq \left\| \frac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \boldsymbol{M}^{-1} \tag{16}$$

So, Eq.(15) can be rewritten as follow.

$$\begin{aligned} m_c f_n = &\boldsymbol{d}^{\mathrm{T}} \boldsymbol{j}_t K f_n - \boldsymbol{d}^{\mathrm{T}} \boldsymbol{\tau} + \boldsymbol{d}^{\mathrm{T}} \left\{ \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g} \right\} \\ &- \left\| \frac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \dot{\boldsymbol{q}}^{\mathrm{T}} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} \end{aligned} \tag{17}$$

Here, we define a as follow.

$$a \triangleq \boldsymbol{d}^{\mathrm{T}} \left\{ \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g} \right\} - \left\| \frac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \dot{\boldsymbol{q}}^{\mathrm{T}} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} \tag{18}$$

We will get Eq.(19).

$$m_c f_n = \boldsymbol{d}^{\mathrm{T}} \boldsymbol{j}_t K f_n - \boldsymbol{d}^{\mathrm{T}} \boldsymbol{\tau} + a \tag{19}$$

And we define A as follow.

$$A \triangleq m_c - \boldsymbol{d}^{\mathrm{T}} \boldsymbol{j}_t K \tag{20}$$

We can get Eq.(21).

$$A f_n = a - \boldsymbol{d}^{\mathrm{T}} \boldsymbol{\tau} \tag{21}$$

Drag force f_n of constraint point can be determined by the algebraic equation of the input torque $\boldsymbol{\tau}$ when $A \neq 0$.

3.2 Differentiation of Jacobian Matrix

By deforming the second term in the right side of Eq.(18), we can obtain Eq.(22). Here we put the $(\partial \boldsymbol{r} / \partial \boldsymbol{q}^{\mathrm{T}}) = \boldsymbol{J}_p$ that is the Jacobian matrix for the $\boldsymbol{q}$ against the hand position $\boldsymbol{r}$.

$$\begin{aligned} &\dot{\boldsymbol{q}}^{\mathrm{T}} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} \\ &= \frac{d\boldsymbol{q}^{\mathrm{T}}}{dt} \left[\frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} \\ &= \frac{d}{dt} \left(\frac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \boldsymbol{J}_p \right) \dot{\boldsymbol{q}} \\ &= \left[\frac{d}{dt} \left(\frac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \right) \boldsymbol{J}_p + \frac{\partial C}{\partial \boldsymbol{r}^{\mathrm{T}}} \frac{d\boldsymbol{J}_p}{dt} \right] \dot{\boldsymbol{q}} \end{aligned} \tag{22}$$

Therefore, it is necessary to obtain the time derivative of the Jacobian matrix to determine the a of Eq.(18), and basing on the calculation method of $\dot{\boldsymbol{J}}_p(\boldsymbol{q})$ in [11], we can get the time derivative of the Jacobian matrix as follow.

$$\begin{aligned} \dot{\boldsymbol{J}} = [\; &\begin{matrix} ({}^0\boldsymbol{\omega}_1 \times {}^0\boldsymbol{R}_1 {}^1\boldsymbol{z}_1) \times {}^0\boldsymbol{p}_{n+1,1} + {}^0\boldsymbol{z}_1 \times (\boldsymbol{J}_{p(n+1)}\dot{\boldsymbol{q}} - \boldsymbol{J}_{p1}\dot{\boldsymbol{q}}) \\ {}^0\boldsymbol{\omega}_1 \times {}^0\boldsymbol{R}_1 {}^1\boldsymbol{z}_1 \end{matrix} \\ &\begin{matrix} \cdots ({}^0\boldsymbol{\omega}_n \times {}^0\boldsymbol{R}_n {}^n\boldsymbol{z}_n) \times {}^0\boldsymbol{p}_{n+1,n} + {}^0\boldsymbol{z}_n \times (\boldsymbol{J}_{p(n+1)}\dot{\boldsymbol{q}} - \boldsymbol{J}_{pn}\dot{\boldsymbol{q}}) \\ \cdots {}^0\boldsymbol{\omega}_n \times {}^0\boldsymbol{R}_n {}^n\boldsymbol{z}_n \end{matrix} \;] \end{aligned} \tag{23}$$

4 SOLUTION OF THE FORWARD DYNAMICS PROBLEM

It is not easy to calculate $\boldsymbol{M}(\boldsymbol{q})$, $\boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}})$, $\boldsymbol{g}(\boldsymbol{q})$ directly in Eq.(10) which is the equation of motion of the n links multi-joint manipulator when its n becomes larger. Following describes the solution of the forward dynamics problem by using the NE inverse dynamics solution as follows.

First, define $\boldsymbol{b} = \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}}$ and $\boldsymbol{\tau}_p$ as the left-hand side of Eq.(10).

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{b} - (\boldsymbol{j}_c - \boldsymbol{j}_t K) f_n = \boldsymbol{\tau}_p \tag{24}$$

When the inverse dynamics calculation expresses as $\boldsymbol{\tau}_p = INV(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{g}, f_n, K)$ shown in Eq.(2)$\sim$Eq.(9), the following equation can be obtained.

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{b} - (\boldsymbol{j}_c - \boldsymbol{j}_t K) f_n = INV(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{g}, f_n, K) \tag{25}$$

Here, $\boldsymbol{b} = INV(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{0}, \boldsymbol{g}, 0, K)$ can be obtained by substituting $\ddot{\boldsymbol{q}} = \boldsymbol{0}$, $f_n = 0$ in Eq.(25), then there will be $\boldsymbol{M}_i = \boldsymbol{M}(\boldsymbol{q})e_i = INV(\boldsymbol{q}, \boldsymbol{0}, \boldsymbol{e}_i, \boldsymbol{0}, 0, K)$ by substituting $\dot{\boldsymbol{q}} = \boldsymbol{0}$, $\ddot{\boldsymbol{q}} = \boldsymbol{e}_i$, $\boldsymbol{g} = \boldsymbol{0}$, $f_n = 0$ into Eq.(25). $\boldsymbol{M}_i$ is a vector representing the i-th column of the inertia matrix, $\boldsymbol{e}_i = [0, \cdots, 1(i), 0, \cdots, 0]^{\mathrm{T}}$ is an unit vector that the i-th element with the '1', element of $\boldsymbol{M}(\boldsymbol{q})$ is calculated for each column. Besides, $\boldsymbol{j}_t$ can be obtained through $\boldsymbol{j}_c$ shown in Eq.(33) and $\boldsymbol{\tau}$ defined in Eq.(34) as shown next.

$$\boldsymbol{j}_c = INV(\boldsymbol{q}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{0}, -1, 0) \tag{26}$$
$$\tilde{\boldsymbol{\tau}} \triangleq \boldsymbol{j}_c - \boldsymbol{j}_t = INV(\boldsymbol{q}, \boldsymbol{0}, \boldsymbol{0}, \boldsymbol{0}, -1, 1) \tag{27}$$
$$\boldsymbol{j}_t = \boldsymbol{j}_c - \tilde{\boldsymbol{\tau}} \tag{28}$$

Based on these equations and $\boldsymbol{d}^{\mathrm{T}}$, a, A that were calculated in Eq.(16), (18) and (20), we can calculate f_n in Eq.(21).

Here, we define the $\boldsymbol{b}_n = \boldsymbol{b} - (\boldsymbol{j}_c - \boldsymbol{j}_t K) f_n$, and substitute $\ddot{\boldsymbol{q}} = \boldsymbol{0}$ into $\boldsymbol{b}_n$. And by using the f_n obtained above, it is possible to obtain $\boldsymbol{b}_n = INV(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{0}, \boldsymbol{g}, f_n, K)$. Thus, the angular acceleration $\ddot{\boldsymbol{q}}$ of each link during restrained motion is calculated as follow.

$$\ddot{\boldsymbol{q}} = \boldsymbol{M}^{-1}(\boldsymbol{\tau} - \boldsymbol{b}_n) \tag{29}$$

By using the numerical integration of $\ddot{\boldsymbol{q}}$ of the given equation, it is possible to perform the forward dynamics calculations of manipulator that the tip link is constrained and contacting with object without explicitly requiring the equation of motion in Eq.(10).

5 DYNAMICAL WALKING MODEL BASED ON EXTENDED NEWTON-EULER METHOD

We discuss a biped robot whose definition is depicted in Fig.2. Table 1 indicates length l_i [m], mass m_i [kg] of links and joints' coefficient of viscous friction d_i [N·m·s/rad], which are decided based on [14]. This model is simulated as a serial-link manipulator having ramifications and represents rigid whole body—feet including toe, torso, arms and so on—by 18 degree-of-freedom.

Though motion of legs is restricted in sagittal plane, it generates varieties of walking gait sequences since the robot has flat-sole feet and kicking torque. In this paper, one foot including link-0 and link-1 is defined as "supporting-foot" and another foot including link-7 and link-8 is defined as "floating-foot" or "contacting-foot" according to the walking state.

5.1 Model of Supporting-foot Standing

First, we can calculate the relations of positions, velocities and accelerations between links as forward kinematics procedures from bottom link to top link using Eq.(2)-Eq.(5) in Chapter 2.

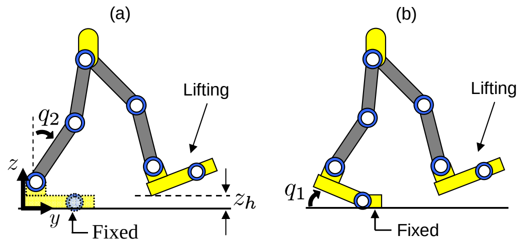

Fig. 3. Gaits including floating-foot

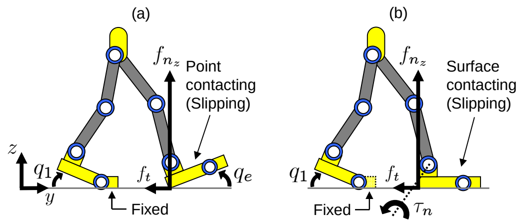

Fig. 4. Gaits including contacting-foot

However, velocity and acceleration of 4-th link transmit to 9-th link and ones of 11-th link transmit to 12-th, 15-th and 18-th link directly because of ramification mechanisms. After the above forward kinematic calculation has been done, contrarily inverse dynamical calculation procedures is the next from top to base link using Eq.(7)-Eq.(8) in Chapter 2.

On the other hand, since force and torque of 5-th and 9-th links are exerted on 4-th link, effects onto 4-th link as:

$$^4\boldsymbol{f}_4 = {}^4\boldsymbol{R}_5{}^5\boldsymbol{f}_5 + {}^4\boldsymbol{R}_9{}^9\boldsymbol{f}_9 + m_4{}^4\ddot{\boldsymbol{s}}_4, \tag{30}$$
$$\begin{aligned} ^4\boldsymbol{n}_4 = {}& {}^4\boldsymbol{R}_5{}^5\boldsymbol{n}_5 + {}^4\boldsymbol{R}_9{}^9\boldsymbol{n}_9 + {}^4\boldsymbol{I}_4{}^4\dot{\boldsymbol{\omega}}_4 + {}^4\boldsymbol{\omega}_4 \times ({}^4\boldsymbol{I}_4{}^4\boldsymbol{\omega}_4) \\ & + {}^4\hat{\boldsymbol{s}}_4 \times (m_4{}^4\ddot{\boldsymbol{s}}_4) + {}^4\hat{\boldsymbol{p}}_5 \times ({}^4\boldsymbol{R}_5{}^5\boldsymbol{f}_5) \\ & + {}^4\hat{\boldsymbol{p}}_9 \times ({}^4\boldsymbol{R}_9{}^9\boldsymbol{f}_9). \end{aligned} \tag{31}$$

Similarly, force and torque of 12-th, 15-th and 18-th links transmit to 11-th link directly. Then, rotational motion equation of i-th link is obtained as Eq.(9) by making inner product of induced torque onto the i-th link's unit vector $\boldsymbol{z}_i$ around rotational axis. Finally, we get motion equation with one leg standing as:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} = \boldsymbol{\tau}, \tag{32}$$

where, $\boldsymbol{D} = diag[d_1, d_2, \cdots, d_{18}]$. If supporting-foot is surface-contacting and assumed to be without slipping, joint angle can be thought as $\boldsymbol{q} = [q_2, q_3, \cdots, q_{18}]^T$. This walking pattern is depicted in Fig. 3 (a). When heel of supporting-foot should detach from the ground before floating-foot contacts to the ground as shown in Fig. 3 (b), the state variable for the foot's angle q_1 be added to $\boldsymbol{q}$, thus $\boldsymbol{q} = [q_1, q_2, \cdots, q_{18}]^T$.

5.2 Model of contact constraints of contacting foot

Making floating-foot contact with ground, contacting-foot like Fig. 4 appears with contacting-foot's position z_h or angle q_e to the ground being constrained. Following by Eq.(1),

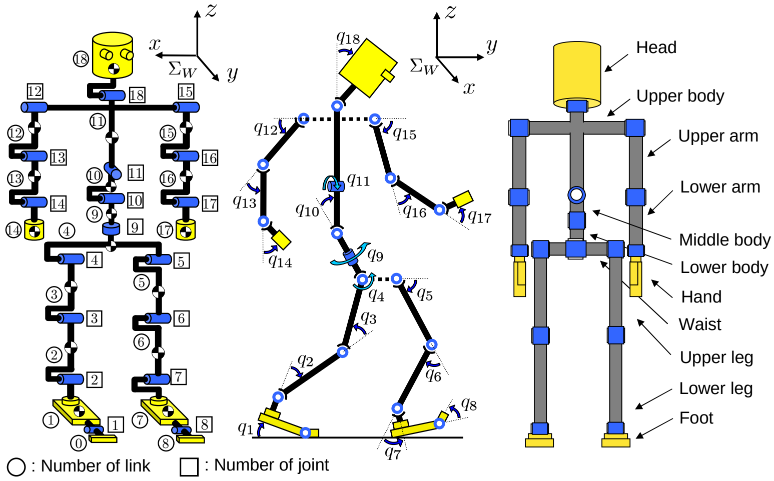

Fig. 2. Definition of humanoid's link, joint and angle number

Table 1. Physical parameters

Link	l_i	m_i	d_i
Head	0.24	4.5	0.5
Upper body	0.41	21.5	10.0
Middle body	0.1	2.0	10.0
Lower body	0.1	2.0	10.0
Upper arm	0.31	2.3	0.03
Lower arm	0.24	1.4	1.0
Hand	0.18	0.4	2.0
Waist	0.27	2.0	10.0
Upper leg	0.38	7.3	10.0
Lower leg	0.40	3.4	10.0
Foot	0.07	1.1	10.0
Total	1.7	63.8	

constraints of foot's position and heel's rotation can be defined as C_1 and C_2 respectively, these constraints can be written as follow, where $\boldsymbol{r}(\boldsymbol{q})$ means the contacting-foot's heel or toe position in Σ_W.

$$\boldsymbol{C}(\boldsymbol{r}(\boldsymbol{q})) = \begin{bmatrix} C_1(\boldsymbol{r}(\boldsymbol{q})) \\ C_2(\boldsymbol{r}(\boldsymbol{q})) \end{bmatrix} = \boldsymbol{0} \tag{33}$$

Then, robot's equation of motion with external force f_n, friction force f_t and external torque τ_n corresponding to C_1 and C_2 can be derived based on Eq.(10) as:

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ &\quad = \boldsymbol{\tau} + \boldsymbol{j}_c^T f_n - \boldsymbol{j}_t^T f_t + \boldsymbol{j}_r^T \tau_n, \end{aligned} \tag{34}$$

where $\boldsymbol{j}_c$, $\boldsymbol{j}_t$ and $\boldsymbol{j}_r$ are defined as:

$$\boldsymbol{j}_c^T = \left(\frac{\partial C_1}{\partial \boldsymbol{q}^T}\right)^T \left(1 \middle/ \left\| \frac{\partial C_1}{\partial \boldsymbol{r}^T} \right\| \right), \quad \boldsymbol{j}_t^T = \left(\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}^T}\right)^T \frac{\dot{\boldsymbol{r}}}{\|\dot{\boldsymbol{r}}\|}, \tag{35}$$

$$\boldsymbol{j}_r^T = \left(\frac{\partial C_2}{\partial \boldsymbol{q}^T}\right)^T \left(1 \middle/ \left\| \frac{\partial C_2}{\partial \boldsymbol{q}^T} \right\| \right). \tag{36}$$

It is common sense that (i) f_n and f_t are orthogonal, and (ii) value of f_t is decided by $f_t = K f_n$ $(0 < K \leq 1)$. Based on Eq.(12) that differentiating Eq. (33) by time for two times, we can derive the constraint condition of $\ddot{\boldsymbol{q}}$.

$$\left(\frac{\partial C_i}{\partial \boldsymbol{q}^T}\right)\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^T \left\{ \frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C_i}{\partial \boldsymbol{q}^T}\right) \dot{\boldsymbol{q}} \right\} = 0 \quad (i = 1,\ 2) \tag{37}$$

Making the $\ddot{\boldsymbol{q}}$ in Eqs. (34) and (37) be identical, we can obtain the equation of contacting motion as follow.

$$\begin{bmatrix} \boldsymbol{M}(\boldsymbol{q}) & -(\boldsymbol{j}_c^T - \boldsymbol{j}_t^T K) & -\boldsymbol{j}_r^T \\ \partial C_1/\partial \boldsymbol{q}^T & 0 & 0 \\ \partial C_2/\partial \boldsymbol{q}^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ f_n \\ \tau_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{\tau} - \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{D}\dot{\boldsymbol{q}} \\ -\dot{\boldsymbol{q}}^T \left\{ \frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C_1}{\partial \boldsymbol{q}^T} \right) \right\} \dot{\boldsymbol{q}} \\ -\dot{\boldsymbol{q}}^T \left\{ \frac{\partial}{\partial \boldsymbol{q}} \left(\frac{\partial C_2}{\partial \boldsymbol{q}^T} \right) \right\} \dot{\boldsymbol{q}} \end{bmatrix} \tag{38}$$

Here, since motion of the foot is constrained only vertical direction, walking direction has a degree of motion. That is, contacting-foot may slip forward or backward depending on the foot's velocity in traveling direction.

6 ANALYSES OF BIPEDAL WALKING

To realize bipedal walking, four kinds of input torques were used [15]. which are: a torque for stabilization of pose, a periodical input to thigh of floating-leg (joint-5) to make the leg step forward, a periodical input to roll angle of body (joint-11), and a periodical input to make arms(joint-12,15) swing like humans. Fig.5 is the relation between angle q_{10} and angular velocity $\dot{q}_{10}$ of waist during 1000 steps' walking when the first step is by right foot. Although both trajectories being close the same constant cycle along with time passage, these trajectories in Fig. 5 are not limit cycle since trajectories have a certain width after 1000 walking steps. Changing the initial condition,we got Fig.6 whose first step is by left foot. We can find that Fig.5(a) and Fig.6(b), Fig.5(b) and Fig.6(a) are the same because of the symmetry motion of foots. Therefore, we can say that the model we made is appropriate.

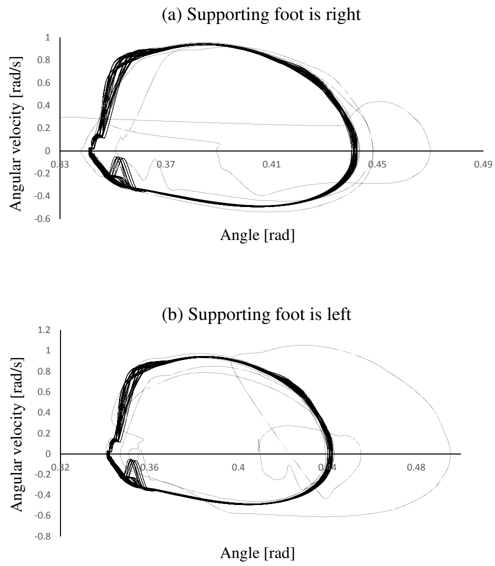

Fig. 5. Relation of q_{10} and $\dot{q}_{10}$ when first step is by right foot

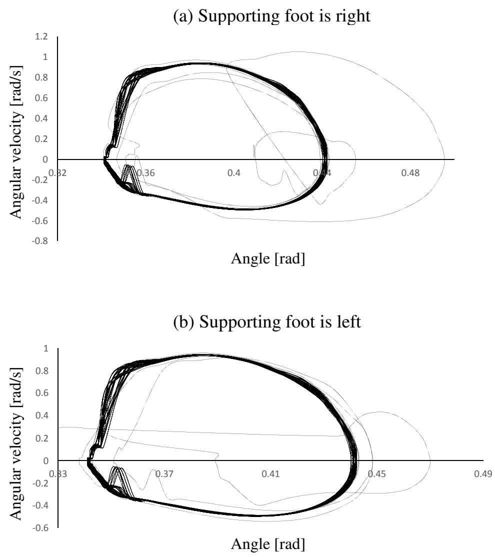

Fig. 6. Relation of q_{10} and $\dot{q}_{10}$ when first step is by left foot

7 CONCLUSION

In this paper, we showed a walking model of humanoid including slipping, bumping, surface-contacting and point-contacting of foot, which dynamical equation is derived by Newton-Euler method with constraint condition, named as ”Extended Newton-Euler Method”. For the future, we plan to extend the calculation method to the case of more than two constraint conditions, and evaluate it by simulations.

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