A Design Method of On-Demand Type Feedback Controller Using Coprime Factorization

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***Abstract*—This paper proposes a design method of on-demand type feedback controller using coprime factorization. A controller, such as generalized minimum variance control (GMVC), generalized predictive control (GPC) and so on, can be extended by using coprime factorization. Then the extended controller has a new design parameter, and the parameter can select the characteristic of the extended controller without changing the closed-loop characteristic. Considering to design safe systems, strong stability systems can be obtained by the extended controller. Moreover, focusing on feedback signal, the extended controller can adjust the magnitude of the feedback signal. That is, the proposed controller can drive the magnitude of the feedback signal to zero if the control object was achieved. In other words the feedback signal by the proposed method can appear on demand of achieving the control object. At first step, this paper proposes on-demand type feedback controller using GMVC and coprime factorization. A numerical example is given in order to check the characteristic of the proposed method.**

I. Introduction

Generalized Minimum Variance Control (GMVC) has been proposed by Clarke and others[1]. GMVC is one of the control methods for application in industry. This control method can be designed by generalized output which is selected to make the closed-loop system stable. And the control law is derived to minimize the variance of generalized output. Once the generalized output is selected, the derived controller cannot be re-designed without changing the closed-loop system. In the case of considering the application in industry, it is desirable for both of the closed-loop system and the controller to be stable in the view point of safety. That is, even if the closed-loop characteristic has been designed, it is better that the flexibility of re-designing the controller characteristic remains because of designing safe systems. Authors have proposed the extended GMVC design method[2], [3]. The extended method introduces a new design parameter for conventional GMVC by using Youla-Kucera parameterization[4]. In the method, the poles of controller can be re-designed by its parameter and can be chosen without changing the poles of closed-loop system. Therefore a strong stability system, which means that both closed-loop system and controller are stable, can be obtained by re-designing stable controller. Although the authors have proposed such a design method[5], [6], [7], [8], [9] and a concept of strong stability rate[10], [11], [12], [13] by using coprime factorization and showed that strong stability system can be obtained, the previous researches have not focused on feedback signal clearly. Under the assumption that the controlled plant is stable, the research about strong stability rate has focused on a stable open-loop output. For example, if the value of strong stability rate becomes one, the controlled output becomes equal to reference signal in the steady state whether the feedback loop is cut or not. This situation indicates that the control object is achieved and the feedback signal is not demanded (that is, the feedback signal becomes zero) in the steady state. In other words, new concept controller, whose feedback signal emerges according to the demand to make the controlled output follow the reference signal and disappears if the controlled output becomes equal to the reference signal, can be considered by using coprime factorization and extending the controller. In the proposed method, the role and the benefit that the feedback signal disappears contribute to constructing safe systems because the output of the proposed system does not diverge even if the feedback signal becomes zero by an accident. Therefore this paper proposes on-demand type feedback controller using coprime factorization. The control method to make on-demand type feedback controller is GMVC in this paper. A numerical example is shown in order to check the characteristic of the proposed controller.

This paper is organized as follows. Section 2 describes problem statement and conventional GMVC. Section 3 extends GMVC through coprime factorization and gives the proposed controller. Section 4 shows a numerical example to check the characteristic of on-demand type feedback controller. Section 5 summarizes the result of this paper.

~~Notations~~ This paper assumes that the controlled plant is stable. z^{-1} means backward shift operator $z^{-1}y(t) = y(t-1)$. $A[z^{-1}]$ and $A(z^{-1})$ means polynomial and rational function with z^{-1} respectively. Steady state gain $A(1)$ of transfer function is calculated as $z^{-1} = 1$ under the assumption that signals such as input and output for system does not change with regard to time t.

II. Problem Statement and Conventional GMVC

A single-input single-output system is considered.

$$\begin{aligned} A[z^{-1}]y(t) &= z^{-k_m}B[z^{-1}]u(t) + C[z^{-1}]\xi(t) \\ & \quad t = 0, 1, 2 \cdots \end{aligned} \tag{1}$$

$u(t)$ and $y(t)$ are input and output respectively. k_m is time delay, $\xi(t)$ is a white Gaussian noise with zero mean. $A[z^{-1}]$, $B[z^{-1}]$ and $C[z^{-1}]$ are the polynomials with degrees n, m

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and l.

$$\begin{aligned} A[z^{-1}] &= 1 + a_1 z^{-1} + \cdots a_n z^{-n} \\ B[z^{-1}] &= b_0 + b_1 z^{-1} + \cdots b_m z^{-m} \\ C[z^{-1}] &= 1 + c_1 z^{-1} + \cdots c_l z^{-l} \end{aligned} \quad (2)$$

On the system (1) the following assumptions are hold.

[A.1] The degrees n, m and l of $A[z^{-1}]$, $B[z^{-1}]$ and $C[z^{-1}]$, and the time delay k_m are known.
[A.2] The coefficients of $A[z^{-1}]$, $B[z^{-1}]$ and $C[z^{-1}]$ are known.
[A.3] The polynomials $A[z^{-1}]$ and $B[z^{-1}]$, $A[z^{-1}]$ and $C[z^{-1}]$ are coprime.
[A.4] The polynomial $C[z^{-1}]$ is stable.

The control object is to make the output $y(t)$ follow the reference signal $w(t)$. To achieve this object, performance index J averaged over the noise is minimized.

$$\begin{aligned} \Phi(t+k_m) &= P[z^{-1}]y(t+k_m) + Q[z^{-1}]u(t) \\ &\quad - R[z^{-1}]w(t) \end{aligned} \quad (3)$$

$$J = \mathrm{E}_{\mathrm{X}}[\Phi^2(t+k_m)] \quad (4)$$

$\Phi(t+k_m)$ means generalized output. $P[z^{-1}]$, $Q[z^{-1}]$ and $R[z^{-1}]$ are polynomials with degrees of n_p, n_q and n_r. These polynomials are selected to obtain stable closed-loop poles. In conventional GMVC, Diophantine equation is given for solutions $E[z^{-1}]$ and $F[z^{-1}]$.

$$P[z^{-1}]C[z^{-1}] = A[z^{-1}]E[z^{-1}] + z^{-k_m}F[z^{-1}] \quad (5)$$

where

$$E[z^{-1}] = 1 + e_1 z^{-1} + \cdots + e_{k_m-1} z^{-(k_m-1)} \quad (6)$$

$$F[z^{-1}] = f_0 + f_1 z^{-1} + \cdots + f_{n_1} z^{-n_1} \quad (7)$$

$$n_1 = \max\{n-1, n_p + l - k_m\} \quad (8)$$

The solution $E[z^{-1}]$ of Diophantine equation is used to calculate the following polynomial $G[z^{-1}]$. $T[z^{-1}]$ gives the closed-loop characteristics.

$$G[z^{-1}] = E[z^{-1}]B[z^{-1}] + C[z^{-1}]Q[z^{-1}] \quad (9)$$

$$T[z^{-1}] = P[z^{-1}]B[z^{-1}] + Q[z^{-1}]A[z^{-1}] \quad (10)$$

From (5) and (9), the generalized output and its prediction $\hat{\Phi}(t+k_m|t)$ can be given.

$$\Phi(t+k_m) = \hat{\Phi}(t+k_m|t) + E[z^{-1}]\xi(t+k_m) \quad (11)$$

$$\begin{aligned} \hat{\Phi}(t+k_m|t) &= (F[z^{-1}]y(t) + G[z^{-1}]u(t) \\ &\quad - C[z^{-1}]R[z^{-1}]w(t))/C[z^{-1}] \end{aligned} \quad (12)$$

Since $\hat{\Phi}(t+k_m|t)$ and the noise term $E[z^{-1}]\xi(t+k_m)$ have no correlation each other, the control law $u(t)$ minimizing J can be obtained by the following equation.

$$\hat{\Phi}(t+k_m|t) = 0 \quad (13)$$

Then the control law is obtained as,

$$u(t) = \frac{C[z^{-1}]R[z^{-1}]}{G[z^{-1}]}w(t) - \frac{F[z^{-1}]}{G[z^{-1}]}y(t) \quad (14)$$

The closed-loop system for (14) can be given as,

$$y(t) = \frac{z^{-k_m}B[z^{-1}]R[z^{-1}]}{T[z^{-1}]}w(t) + \frac{G[z^{-1}]}{T[z^{-1}]}\xi(t) \quad (15)$$

where $T[z^{-1}]$ is defined in (10).

III. Extension of GMVC through Coprime Factorization

A. Coprime Factorization of Controlled Systems

For coprime factorization, the family of stable rational functions RH_∞ is considered,

$$RH_\infty = \{G(z^{-1}) = \frac{G_n[z^{-1}]}{G_d[z^{-1}]}\} \quad (16)$$

where $G_d[z^{-1}]$ is stable polynomial. Transfer function $G_p(z^{-1})$ of the system (1) between $u(t)$ and $y(t)$ is given in the form of a ratio of rational functions in RH_∞,

$$\begin{aligned} y(t) &= \frac{z^{-k_m}B[z^{-1}]}{A[z^{-1}]}u(t) \\ &= G_p(z^{-1})u(t) \\ &= N(z^{-1})D^{-1}(z^{-1})u(t) \end{aligned} \quad (17)$$

$N(z^{-1})$ and $D(z^{-1})$ are rational functions in RH_∞ and coprime each other. This paper assumes that the controlled system $G_p(z^{-1})$ is stable.

In the next step, the following Bezout identity is considered.

$$X(z^{-1})N(z^{-1}) + Y(z^{-1})D(z^{-1}) = 1 \quad (18)$$

The solutions $X(z^{-1})$ and $Y(z^{-1})$ of Bezout identity are in RH_∞. Then all the stabilizing controller is given in Youla-Kucera parameterization[4] from (17) and (18).

$$u(t) = C_1(z^{-1})w(t) - C_2(z^{-1})y(t) \quad (19)$$

$$C_1(z^{-1}) = (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1}K(z^{-1}) \quad (20)$$

$$\begin{aligned} C_2(z^{-1}) &= (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1} \\ &\quad \cdot (X(z^{-1}) + U(z^{-1})D(z^{-1})) \end{aligned} \quad (21)$$

$U(z^{-1})$, $K(z^{-1}) \in RH_\infty$ are free parameters and $w(t)$ is reference signal. From (19), (20), (21) and (17), the closed-loop transfer function is given.

$$\begin{aligned} y(t) &= N(z^{-1})D^{-1}(z^{-1})(Y(z^{-1}) - U(z^{-1}) \\ &\quad \cdot N(z^{-1}))^{-1}K(z^{-1})w(t) - N(z^{-1})D^{-1}(z^{-1}) \\ &\quad \cdot (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1}(X(z^{-1}) \\ &\quad + U(z^{-1})D(z^{-1}))y(t) \end{aligned} \quad (22)$$

Therefore

$$\begin{aligned} &\{D(z^{-1})(Y(z^{-1}) - U(z^{-1})N(z^{-1})) + N(z^{-1}) \\ &\cdot (X(z^{-1}) + U(z^{-1})D(z^{-1}))\}y(t) \\ &= N(z^{-1})K(z^{-1})w(t) \end{aligned} \quad (23)$$

Then the closed-loop transfer function is given from (18).

$$y(t) = N(z^{-1})K(z^{-1})w(t) \quad (24)$$

If the controller is designed for settling control, the output $y(t)$ converges to $w(t)$ as time progresses. It means that the steady-state gain of closed-loop system (24) is designed to be $N(1)K(1) = 1$. Moreover the design parameter $U(z^{-1})$ in the stabilizing controller (19) does not affect (24). Therefore when closed-loop system (24) is designed to be stable and stabilizing controller (19) is also designed to be stable through $U(z^{-1})$, strong stability system can be obtained. Here it is noticed that the closed-loop system (24) is independent of design parameter $U(z^{-1})$.

B. Concept of On-demand Type Feedback Controller

In the previous research[14], the authors have proposed a design method of strong stability system and defined the selection method of design parameter $U(z^{-1})$, which can equate steady state gains of the closed-loop system and the open-loop system. Through this research, it was found that the derived closed-loop system allows that the feedback signal becomes zero in the steady state because the controller is designed to make the open-loop gain equal to the closed-loop gain. It means that the feedback signal appears (does not become zero) so as to achieve the control object, and the feedback signal becomes zero when the control object was achieved in the steady state. Therefore this paper defines such a controller as on-demand type feedback controller.

In this subsection, the concept is described briefly. Assuming that the feedback signal $C_2(z^{-1})y(t)$ in the stabilizing controller (19) becomes zero, and considering the open-loop system for the closed-loop system (24), the controller (19) is given as follows.

$$u(t) = (Y(z^{-1}) - U(z^{-1})N(z^{-1}))^{-1}K(z^{-1})w(t) \quad (25)$$

From (17), the open-loop transfer function from $w(t)$ to $y(t)$ is given.

$$\begin{aligned} y(t) &= N(z^{-1})D^{-1}(z^{-1})u(t) \\ &= N(z^{-1})D^{-1}(z^{-1})(Y(z^{-1}) - U(z^{-1}) \\ &\quad \cdot N(z^{-1}))^{-1}K(z^{-1})w(t) \\ &= (Y(z^{-1})D(z^{-1}) - U(z^{-1})N(z^{-1})D(z^{-1}))^{-1} \\ &\quad \cdot N(z^{-1})K(z^{-1})w(t) \end{aligned} \quad (26)$$

Because of $Y(z^{-1})D(z^{-1}) = 1 - X(z^{-1})N(z^{-1})$, the open-loop system can be obtained as the following equation.

$$\begin{aligned} y(t) &= (1 - X(z^{-1})N(z^{-1}) - U(z^{-1})N(z^{-1}) \\ &\quad \cdot D(z^{-1}))^{-1}N(z^{-1})K(z^{-1})w(t) \\ &= \{1 - (X(z^{-1}) + U(z^{-1})D(z^{-1}))N(z^{-1})\}^{-1} \\ &\quad \cdot N(z^{-1})K(z^{-1})w(t) \end{aligned} \quad (27)$$

The steady state output $y(t)$ of the open-loop system is given.

$$\begin{aligned} y(t) &= \{1 - (X(1) + U(1)D(1))N(1)\}^{-1} \\ &\quad \cdot N(1)K(1)w(t) \end{aligned} \quad (28)$$

Moreover the design parameter $U(z^{-1}) = U(1)$ is selected as follows.

$$U(1) = -D^{-1}(1)X(1) \quad (29)$$

Then the steady state output $y(t)$ in (28) can be expressed as,

$$y(t) = N(1)K(1)w(t) \quad (30)$$

The design parameter $U(1)$ can give the poles of controller (19) without changing the closed-loop poles of (24). From (30), the steady state gain of open-loop system becomes equal to the closed-loop's one, even if the feedback signal $C_2(z^{-1})y(t)$ in (19) becomes zero. In other words the open-loop system's output becomes equal to the reference signal $w(t)$ in the steady state because $N(1)K(1)$ is designed to be 1. This means that the feedback signal of the closed-loop system becomes zero in the steady state by choosing $U(1)$ as (29). That is, on-demand type feedback controller can be obtained.

In the next step, generalized minimum variance control system with on-demand type feedback controller is designed under the assumption that the controlled plant is stable. In the case that $P[z^{-1}]$ and $Q[z^{-1}]$ in generalized output $\Phi(t+k_m)$ are chosen for $T[z^{-1}]$ to be stable, comparing transfer function (17) to (15), $N(z^{-1})$ and $D(z^{-1})$ can be chosen as follows;

$$N(z^{-1}) = \frac{z^{-k_m}B[z^{-1}]}{T[z^{-1}]} \quad (31)$$

$$D(z^{-1}) = \frac{A[z^{-1}]}{T[z^{-1}]} \quad (32)$$

Substituting (31) and (32) into Bezout equation (18) and comparing it to Diophantine equation (5), the solutions $X(z^{-1})$ and $Y(z^{-1})$ of Bezout equation are given.

$$X(z^{-1}) = \frac{F[z^{-1}]}{C[z^{-1}]} \quad (33)$$

$$Y(z^{-1}) = \frac{G[z^{-1}]}{C[z^{-1}]} \quad (34)$$

Then the control law (14) is obtained from Youla-Kucera parameterization (19), (20) and (21) by selecting the free parameters as,

$$K(z^{-1}) = R[z^{-1}] \quad (35)$$

$$U(z^{-1}) = 0 \quad (36)$$

To extend the controller (14), instead of choosing $U(z^{-1})$ as 0, on-demand type feedback controller uses $U(1) = -D^{-1}(1)X(1)$ as given in (29). Then the extended controller through $U(1)$ is obtained as follows.

$$\begin{aligned} &(G[z^{-1}]T[z^{-1}] - U(1)z^{-k_m}B[z^{-1}]C[z^{-1}])u(t) \\ &\quad = C[z^{-1}]T[z^{-1}]R[z^{-1}]w(t) - (F[z^{-1}]T[z^{-1}] \\ &\qquad + U(1)A[z^{-1}]C[z^{-1}])y(t) \end{aligned} \quad (37)$$

To calculate this control law, the polynomial operating on $u(t)$ in the left-hand side of (37) is divided by the leading term g_0 and the remaining term.

$$\begin{aligned} &G[z^{-1}]T[z^{-1}] - U(1)z^{-k_m}B[z^{-1}]C[z^{-1}] \\ &\quad = g_0 + z^{-1}G'[z^{-1}] \end{aligned} \quad (38)$$

Therefore the control law (37) is calculated by

$$\begin{aligned} u(t) &= \frac{1}{g_0}\{C[z^{-1}]T[z^{-1}]R[z^{-1}]w(t) \\ &\quad -(F[z^{-1}]T[z^{-1}] + U(1)A[z^{-1}]C[z^{-1}])y(t) \\ &\quad -G'[z^{-1}]u(t-1)\} \end{aligned} \quad (39)$$

From (24) it is noticed that the transfer function from reference signal to output is independent of $U(z^{-1})$. And the poles of controller can be given by the following equation.

$$G[z^{-1}]T[z^{-1}] - U(1)z^{-k_m}B[z^{-1}]C[z^{-1}] = 0 \quad (40)$$

IV. Numerical Example

In this section, the numerical example is shown to check the characteristic of the proposed controller. The following controlled system described in (1) is given.

$$\begin{aligned} A[z^{-1}] &= 1 + 0.6z^{-1} + 0.7z^{-2} \\ B[z^{-1}] &= 0.5 - 1.5z^{-1} \\ C[z^{-1}] &= 1, \quad k_m = 1 \end{aligned}$$

Simulation steps are 200, the initial values of output and input are assumed to be zero. The disturbance is assumed to be $\xi(t) = 0$. In order to design the closed-loop characteristic to be stable, the generalized output is given so as to make the controlled output $y(t)$ follow the reference signal $w(t)$.

$$\Phi(t+1) \quad = \quad y(t+1) + 0.8u(t) - 0.84z^{-2} \cdot w(t)$$

The amplitude of reference signal $w(t)$ is 1 from the beginning of simulation to 100th step, and 1.5 after 101th step. The closed-loop poles are $0.3923 \pm 0.5262i$ and its absolute value is 0.6563. Therefore the derived closed-loop system is designed to be stable. In this case, the new design parameter $U(1) = -D^{-1}(1)X(1)$ is calculated to 0.4748. Then the controller's poles are $0.7736 \pm 0.58i$ and 0.5317 and their absolute values are 0.9669 and 0.5317. That is, the strong stability system can be obtained by the extended controller. If the parameter is selected as $U(1) = 0$, the controller becomes the conventional GMVC (14). Then the absolute value of controller's pole is 1.1538 although the closed-loop poles become equal to the proposed ones. It means that the conventional GMVC of this case does not make strong stability system. Therefore it finds that the new design parameter $U(1) = -D^{-1}(1)X(1)$ has the characteristic to construct a strong stability system. But it is noticed that the new design parameter does not always supply strong stability system because it depends on the given system in (1) and the conventional controller.

Fig.1 and Fig.2 show the plant outputs by the conventional method and the proposed method respectively. The dotted lines of their figures mean the reference signals $w(t)$. The solid lines of them show the plant outputs $y(t)$. From these figures it can finds that their outputs are the same although their controllers are different ((14) and (37)). Moreover, Fig.3 and Fig.4 show the control inputs $u(t)$ (upper figure of each figure), the feedforward signals (middle one), which can be expressed as $C_1(z^{-1})w(t)$ described in (19), and the feedback signals $C_2(z^{-1})y(t)$ (lower one). These figures show that the control inputs are same. On the other hand, it can find that the feedforward and the feedback signals are different. In Fig.4, the proposed controller shows that the feedback signal appears in order to follow the reference signal and disappears (becomes zero) when the control object, which means that the output becomes equal to the reference signal, was achieved. After 101th step, the reference signal was changed from 1 to 1.5. Then the feedback signal appears again, aiming to follow new reference signal. And it disappears when the control object was achieved. Therefore it can find that the proposed controller has the characteristic that its feedback signal appears according to the demand of acheving the control object and disappears when the control object was achieved. This means the proposed controller is on-demand type feedback controller.

Moreover, if the condition of new design parameter $U(1)$ is relaxed with a constant α as follows[12], [13],

$$U(1) \quad = \quad -\alpha D^{-1}(1)X(1)$$

then the magnitude of the feedback signal can be adjusted. For example, in the case of $\alpha = 0.9$, Fig.5 shows the plant output and it is found that the output is the same as the conventional GMVC shown in Fig.1. Fig.6 shows the control input (upper figure), the feedforward signal (middle one) and the feedback signal (lower one). The controller's poles are $0.7399 \pm 0.5496i$ and 0.585, and their absolute values are 0.9217 and 0.585. From this figure, it can find that the magnitude of the feedback signal become smaller than the conventional one shown in Fig.3 although the feedback signal does not become zero even if the control object was achieved. Therefore comparing with [14], the improvement of the proposed method is the flexibility for obtaining the on-demand type feedback controller through α.

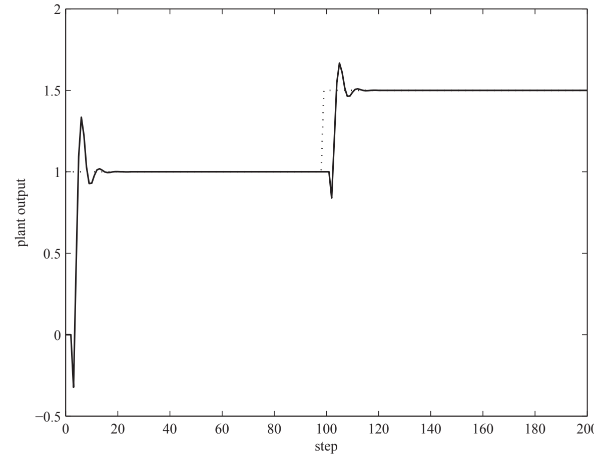

Fig. 1. Conventional method (output)

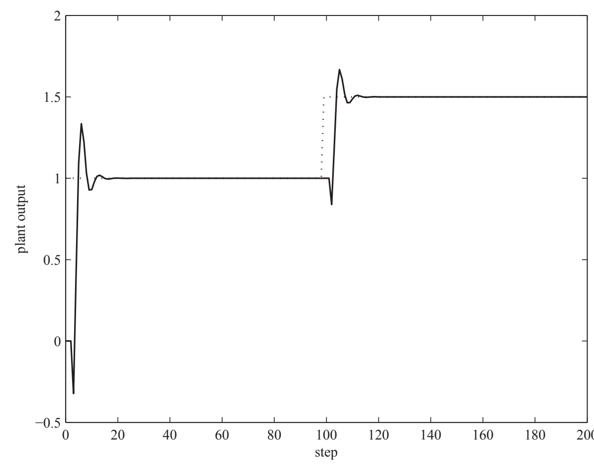

Fig. 2. Proposed method (output)

V. Conclusion

This paper proposed a design method of on-demand type feedback controller of generalized minimum variance control using coprime factorization. The numerical example was shown to check the characteristic of the proposed method, whose feedback signal emerges according to the demand to make the controlled output follow the reference signal and disappears if the controlled output becomes equal to the reference signal. As future works, there is an extension to multi-input multi-output systems using the proposed method.

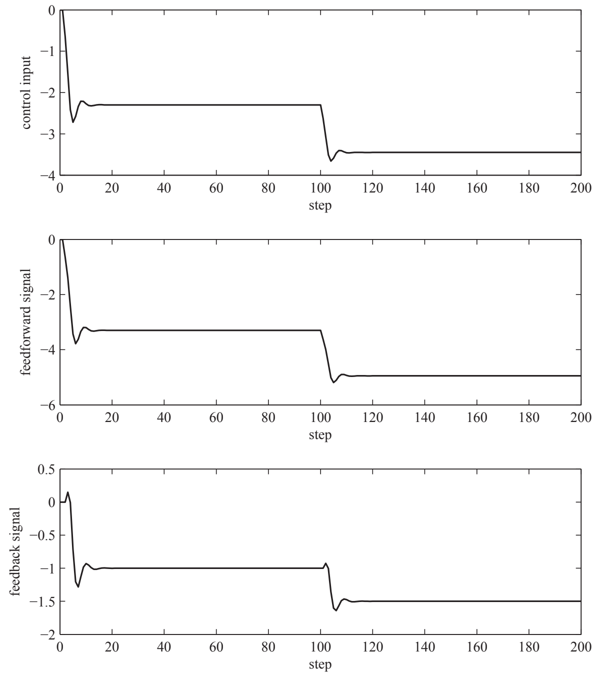

Fig. 3. Conventional method (input)

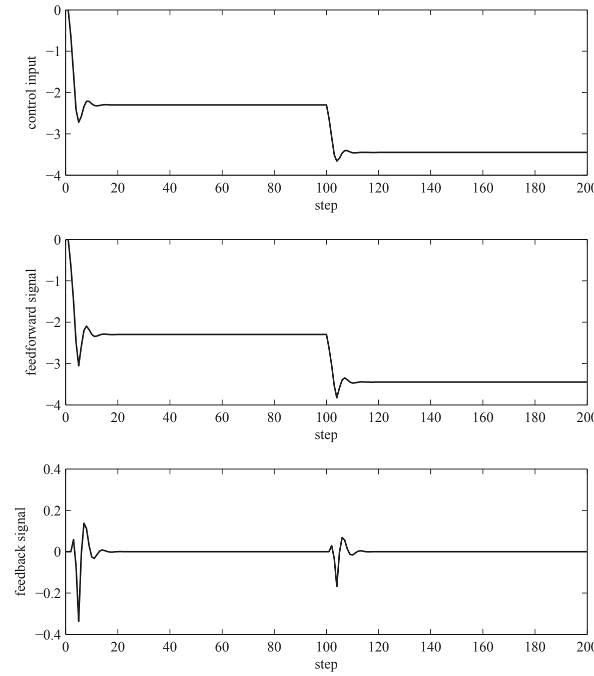

Fig. 4. Proposed method (input)

Moreover a self-tuning controller through the proposed method will be considered.

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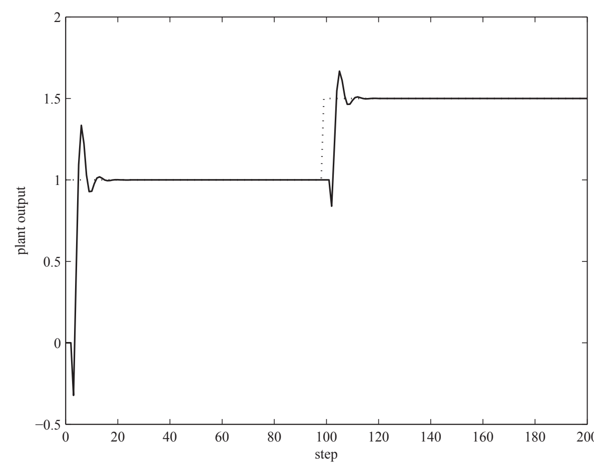

Fig. 5. Proposed method with $\alpha = 0.9$ (output)

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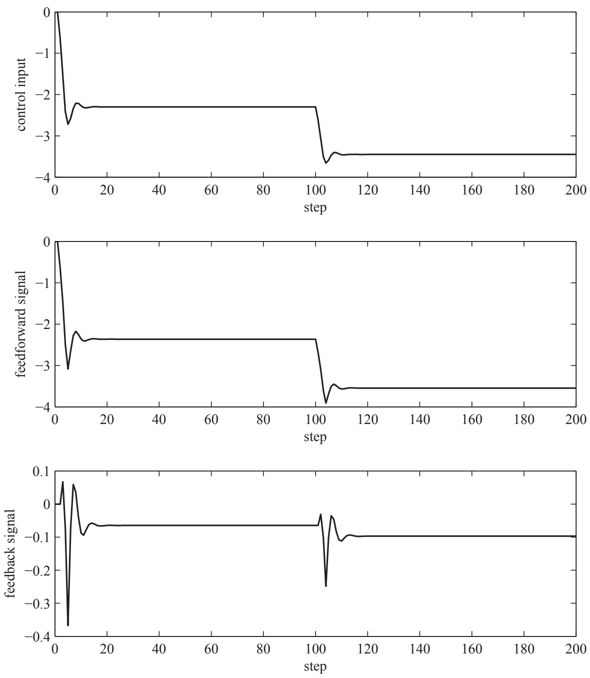

Fig. 6. Proposed method with $\alpha = 0.9$ (input)

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