# Analysis of Bracing-Constraint Dynamics with Energy-efficient for Elbow-bracing Manipulator

X. Li[1] M. Minami[1] ∗H. Han[2] and Y. Wei[2]

(1 Okayama University 2 Harbin Engineering University)

**Abstract**– The configuration of elbow-bracing is built by imitating human's handwriting behavior that human can do accurate task with less consumption energy by bracing the elbow or hand on the task. In this paper, first, the motion equation of the elbow-bracing manipulator under constrained condition has been derived. Second, as the consumption energy is calculated based on the voltage and current of the motor, the equation of motion of the motor has been proposed. Then, a control method based on the constraint dynamics of the elbow-bracing manipulator is proposed to simultaneously control constraint force and hand's trajectory and elbow-bracing position in work space. Moreover, we focus on the energy-efficient of the elbow-bracing manipulator, and analysis the factors which have a great effect on the consumption energy, i.e. elbow-bracing position, constraint force. Finally, a simulation experiment for 5-link elbow-bracing manipulator tracking spatial trajectory has been conducted which reveals the effectiveness of energy-efficiecy and high accuracy for the elbow-bracing manipulator and presents the ralationship of the above two factors to the energy-efficiency.

**Key Words:** Elbow-bracing manipulator; Constraint dynamics; constraint force control ; Energy-efficient

## 1 Introduction

Humans can write characters accurately on a paper with less power by bracing and restricting the wrist, as shown in Fig.1. Moreover, this bracing strategy may overcome the hindrances of hyper-redundant manipulators being too heavy to spare the hand payload for desired tasks.

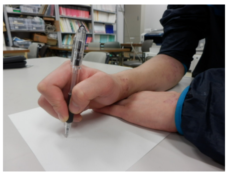

Fig. 1: Human's writing motion utilizing bracing wrist

Roy and Whitcomb [1)] categorized motions and control methods of constrained robot as (a) model based control [2)] [3)] that assume undeformable robots and deformable environments, and (b) methods based on position/velocity control [4)] that assume undeformable robots and also deformable environments. Park and Khatib [5)] [6)] proposed kinematics model of plural contact to control constraint motion in category (b). Finally, there is classification of (c) control method [7)] that assume undeformable robots and undeformable environments. Yamane and Nakamura proposed walking of humanoid robot [8)] and a concept of dynamics filter [9)] in this category. Effectiveness and accuracy of hyper-redundant manipulators subject to constraint on environments have been discussed, West and Asada [10)] proposed common contact mode of kinematics for designing position/force simultaneous controller of manipulator in constraint motion.

In this paper, considering the control method of undeformable robots and undeformable environments. Under these conditions, algebraic equation can be derived from constraint condition and equation of motion as (1).

$$\boldsymbol{A}\boldsymbol{f}_n = \boldsymbol{a} - \boldsymbol{B}\boldsymbol{\tau} \tag{1}$$

$\boldsymbol{f}_n$ is constraint force, $\boldsymbol{A}$, $\boldsymbol{a}$ and $\boldsymbol{B}$ are vector and matrices that will be defined in the next section, $\boldsymbol{\tau}$ is a vector of input torques. Eq.(1) shows an algebraic relation between input torques and constraint force when robot's hand is subject to constraint. The above equation has been derived by Hemami and Wyman [11)] in discipline of biped walking, and applied by Peng and Adachi [12)] in discipline of force/position control by robots at the beginning. Peng considered that $\boldsymbol{\tau}$ is input and $\boldsymbol{f}_n$ is output, and Eq.(1) was used as force sensor to detect $\boldsymbol{f}_n$. Despite the nature that the robot motions under a condition of (c) undeformable robot and undeformable environment be subject to the algebraic equation, Eq.(1), researches on robot force control in category (c) seems to be not based on the Eq.(1) except Peng as far as we know. In this paper, Eq.(1) is used for calculating input torque $\boldsymbol{\tau}$ to accomplish desired constraint force $\boldsymbol{f}_{nd}$ contrary to Peng's idea. Considering the hand writing motion, we know that too much pushing the wrist to table bears fatigue and meaningless, and also too less pushing makes us tired too. This suggests a hypothesis that appropriate supporting force exists, and also effective bracing position may exist.

In the past of this research, the control of constraint motion has been applied to many robots. The grinding robot that has been researching by Minami and Adachi [13)], the hand of robot is constrainted to a changing surface to grinding a target object into desired shape with force-sensorless feed-forward control. And the bracing control also be used to a mobile robot with redundant manipulator and the bracing manipulator that have been researching by Washino and Minami [14)], Kondo and Itoshima [15)] to maintaining the balance of the mobile robot or conserve energy of the redundant manipulator. Xiang Li, et al.[16)] derive the dynamical equation for humanoid robot using Extended Newton-Euler and discuss its walking models, such as slipping, bumping, surface-contacting and point-contacting of foot.

In this paper, we focus on the energy-efficient of the elbow-bracing manipulator through two factors, i.e. elbow-bracing position and constraint force. In section 2, the motion equation of elbow-bracing manipulator with constrained force and motor has been derived. In section 3, PD controller has been used to achieve the task for the elbow-bracing manipulator. In section 4, a simulation experiment for a 5-link elbow-bracing manipulator has been conducted. Finally, we give our conclusion.

# 2 Motion equation with constraint and motor for elbow-bracing manipulator

## 2.1 Constrained condition

As shown in Fig.2, the intermediate links of an n-link manipulator are contacted with the environment at $p$ points.

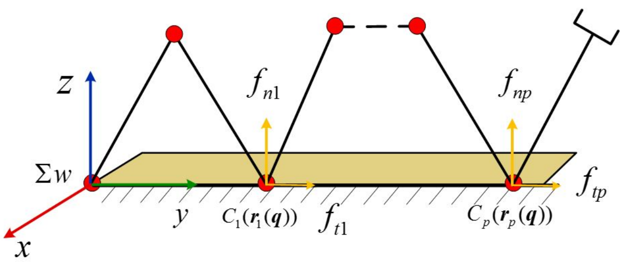


Fig. 2: Constrained model of the elbow-bracing manipulator

The constraint function is expressed as,

$$\begin{aligned}\boldsymbol{C}(\boldsymbol{r}(\boldsymbol{q})) &= [C_1(\boldsymbol{r}_1(\boldsymbol{q})), C_2(\boldsymbol{r}_2(\boldsymbol{q})), \cdots, C_p(\boldsymbol{r}_p(\boldsymbol{q}))]^{\mathrm{T}} \\ &= \boldsymbol{0} \end{aligned} \tag{2}$$

Here, $\boldsymbol{q} \in \boldsymbol{R}^n$ is joint angle vector with n joints, $\boldsymbol{r}_i \in \boldsymbol{R}^m (m < n)$ is $i$-th link position that is subject to constraint. The relation between $\boldsymbol{r}_i$ and $\boldsymbol{q}$ and the relation between $\dot{\boldsymbol{r}}_i$ and $\dot{\boldsymbol{q}}$ are expressed as,

$$\boldsymbol{r}_i = \boldsymbol{r}_i(\boldsymbol{q}) \tag{3}$$

$$\dot{\boldsymbol{r}}_i = \boldsymbol{J}_i(\boldsymbol{q})\dot{\boldsymbol{q}}, \quad \boldsymbol{J}_i(\boldsymbol{q}) = [\tilde{\boldsymbol{J}}_i(\boldsymbol{q}),\ \boldsymbol{0}]\ . \tag{4}$$

In (4), $\boldsymbol{J}_i$ is $m \times n$ matrix, $\tilde{\boldsymbol{J}}_i$ consists of $m \times i$ matrix and zero submatrix $\boldsymbol{0}$ with $m \times (n-i)$.

In the formulation of constraint motion of robot, we consider that a plural intermediate links are contacting with the environment. In Fig.2, a generalized surface can be defined with the position constraints along the tangents to this surface and force constraints along the normals. Then the unit vectors of normals, $\boldsymbol{j}_{cci}$, which represent direction of constraint forces, $\boldsymbol{f}_n = [f_{n1}, f_{n2} \ldots f_{np}]^T$, and the unit vectors of tangents, $\boldsymbol{j}_{tti}$, which represent direction of friction forces, $\boldsymbol{f}_t = [f_{t1}, f_{t2} \ldots f_{tp}]^T$, are expressed as,

$$\boldsymbol{j}_{cci} = \left(\frac{\partial \boldsymbol{C}_i}{\partial \boldsymbol{r}^{\mathrm{T}}}\right)^{\mathrm{T}} / \left\| \frac{\partial \boldsymbol{C}_i}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \tag{5}$$

$$\boldsymbol{j}_{tti} = \frac{\dot{\boldsymbol{r}}_i}{\|\dot{\boldsymbol{r}}_i\|} \tag{6}$$

## 2.2 Motion equation with constraint

As we known, the jacobian transpose maps Cartesian forces into equivalent joint torques. Then, we define that,

$$\boldsymbol{j}_{ci}^{\mathrm{T}} = \boldsymbol{J}_i(\boldsymbol{q})^{\mathrm{T}} \boldsymbol{j}_{cci} = \left(\frac{\partial C_i}{\partial \boldsymbol{q}^{\mathrm{T}}}\right)^T / \left\| \frac{\partial C_i}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \tag{7}$$

$$\boldsymbol{j}_{ti}^{\mathrm{T}} = \boldsymbol{J}_i(\boldsymbol{q})^{\mathrm{T}} \boldsymbol{j}_{tti} = \left(\frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{q}^{\mathrm{T}}}\right)^{\mathrm{T}} \frac{\dot{\boldsymbol{r}}_i}{\|\dot{\boldsymbol{r}}_i\|} \tag{8}$$

$$\boldsymbol{J}_c^{\mathrm{T}} = [\boldsymbol{j}_{c1}^{\mathrm{T}},\ \boldsymbol{j}_{c2}^{\mathrm{T}},\ \cdots,\ \boldsymbol{j}_{cp}^{\mathrm{T}}] \tag{9}$$

$$\boldsymbol{J}_t^{\mathrm{T}} = [\boldsymbol{j}_{t1}^{\mathrm{T}},\ \boldsymbol{j}_{t2}^{\mathrm{T}},\ \cdots,\ \boldsymbol{j}_{tp}^{\mathrm{T}}] \tag{10}$$

$\boldsymbol{J}_c^{\mathrm{T}}$, $\boldsymbol{J}_t^{\mathrm{T}}$ are $n \times p$ matrices, and $\boldsymbol{f}_n$, $\boldsymbol{f}_t$ are $p \times 1$ vectors. Using above definitions, equation of motion of the manipulator subject to constraints at $p$ points is expressed as

$$\begin{aligned} &\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ = \ &\boldsymbol{\tau} + \sum_{i=1}^{p} (\boldsymbol{j}_{ci}^{\mathrm{T}} f_{ni}) - \sum_{i=1}^{p} (\boldsymbol{j}_{ti}^{\mathrm{T}} f_{ti}) \\ = \ &\boldsymbol{\tau} + \boldsymbol{J}_c^{\mathrm{T}} \boldsymbol{f}_n - \boldsymbol{J}_t^{\mathrm{T}} \boldsymbol{f}_t \end{aligned} \tag{11}$$

Differentiating (2) with respect to time $t$ twice, constraint condition of $\ddot{\boldsymbol{q}}$ is set up like

$$\dot{\boldsymbol{q}}^{\mathrm{T}} \left[ \frac{\partial}{\partial \boldsymbol{q}} \left( \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} + \left( \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \ddot{\boldsymbol{q}} = \boldsymbol{0} \tag{12}$$

The solution of (11) that is $\dot{\boldsymbol{q}}$ and $\boldsymbol{q}$ must satisfy (12) disregarding time $t$ that the manipulator be always subject to constraint. When the $\ddot{\boldsymbol{q}}$ satisfying (12) and the $\ddot{\boldsymbol{q}}$ in (11) is equal to each other, the solution $\boldsymbol{q}(t)$ in (11) satisfies (2) regardless of time.

Here, the relation between constraint force $\boldsymbol{f}_n$ and friction force $\boldsymbol{f}_t$ is shown in the following equation with coefficients of sliding friction.

$$\begin{gathered} \boldsymbol{f}_t = \boldsymbol{K}\boldsymbol{f}_n, \quad \boldsymbol{K} = \mathrm{diag}[K_1, K_2, \cdots, K_p] \\ 0 < K_i < 1, (i = 1, 2, \cdots, p) \end{gathered} \tag{13}$$

Therefore, Eq.(11) can be translated into the following equation.

$$\begin{aligned} & \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{D}\dot{\boldsymbol{q}} \\ = \;& \boldsymbol{\tau} + (\boldsymbol{J}_c^{\mathrm{T}} - \boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K})\boldsymbol{f}_n \end{aligned} \tag{14}$$

In order to obtain the relationship between $\boldsymbol{\tau}$ and $\boldsymbol{f}_n$, such as (1), we combine (12) and (14) to eliminate $\ddot{\boldsymbol{q}}$. First, we define

$$\boldsymbol{M}_c \triangleq (\partial \boldsymbol{C}/\partial \boldsymbol{q}^{\mathrm{T}})\boldsymbol{M}^{-1}(\partial \boldsymbol{C}/\partial \boldsymbol{q}^{\mathrm{T}})^{\mathrm{T}} \tag{15}$$

$$\boldsymbol{B} \triangleq \left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \left( \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \boldsymbol{M}^{-1} \tag{16}$$

$$\boldsymbol{a} \triangleq \boldsymbol{B}\left\{\boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} + \boldsymbol{g}\right\} - \left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \dot{\boldsymbol{q}}^{\mathrm{T}} \left[ \frac{\partial}{\partial \boldsymbol{q}} \left( \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} \tag{17}$$

$$\boldsymbol{A} \triangleq \boldsymbol{M}_c - \boldsymbol{B}\boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K} \tag{18}$$

Then,

$$\begin{aligned} & \dot{\boldsymbol{q}}^{\mathrm{T}} \left[ \frac{\partial}{\partial \boldsymbol{q}} \left( \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} + \left( \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \boldsymbol{M}^{-1}(\boldsymbol{h} \\ & + \boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K}\boldsymbol{f}_n + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{g} - \boldsymbol{\tau} - \boldsymbol{J}_c^{\mathrm{T}}\boldsymbol{f}_n) = 0 \end{aligned} \tag{19}$$

$$\begin{aligned} & \boldsymbol{M}_c\boldsymbol{f}_n \\ & = \left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \left( \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \boldsymbol{M}^{-1}(\boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K}\boldsymbol{f}_n + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{h} \\ & + \boldsymbol{g} - \boldsymbol{\tau}) - \left\| \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{r}^{\mathrm{T}}} \right\| \dot{\boldsymbol{q}}^{\mathrm{T}} \left[ \frac{\partial}{\partial \boldsymbol{q}} \left( \frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} \right) \right] \dot{\boldsymbol{q}} \\ & = \boldsymbol{B}\boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K}\boldsymbol{f}_n - \boldsymbol{B}\boldsymbol{\tau} + \boldsymbol{a} \end{aligned} \tag{20}$$

Finally, we can get (1). And we assume that $\boldsymbol{A}$ is positive definite matrix. Therefore, $\boldsymbol{f}_n$ can be expressed as,

$$\boldsymbol{f}_n = \boldsymbol{A}^{-1}(\boldsymbol{a} - \boldsymbol{B}\boldsymbol{\tau}) \tag{21}$$

### 2.3 Equation of Motor

To represent the motion of each motor, the following symbols are used.
$v_i$ is the voltage of the motor, $\boldsymbol{v} = [v_1, v_2, \cdots, v_l]^T$;
$i_i$ is the current of the motor, $\boldsymbol{i} = [i_1, i_2, \cdots, i_l]^T$;
$v_{gi}$ is the back EMF of the motor;
$R_i$ is the resistance of the motor;
$L_i$ is the inductance of the motor;
$\theta_i$ is the rotational angle of the motor;
$\tau_{gi}$ is the electromagnetic torque of the motor;
$\tau_{Li}$ is the load torque of the motor;
$I_{mi}$ is the inertia of the motor;
$K_{Ei}$ is the coefficient of the back EMF;
$K_{Ti}$ is the coefficient of the electromagnetic torque;
$d_{mi}$ is the viscous friction coefficient of the reducer;
$k_i$ is the reduction radio of the reducer;

As the DC motor is considered in this paper, the coefficients, $K_{Ei}$ and $K_{Ti}$ are equal. And we assume that

$$K_{Ti} = K_{Ei} = K_i \tag{22}$$

The relationship between the parameters of the motor can be expressed as the following equations

$$v_i(t) = L_i\dot{i}_i + R_i i_i(t) + v_{gi}(t) \tag{23}$$
$$v_{gi}(t) = K_{Ei}\dot{\theta}_i(t) \tag{24}$$
$$I_{mi}\ddot{\theta} = \tau_{gi}(t) - \tau_{Li}(t) - d_{mi}\dot{\theta}_i \tag{25}$$
$$\tau_{gi}(t) = K_{Ti}i_i(t) \tag{26}$$

As the reduction radio of the reducer is $k_i$, the following equations can be obtained.

$$\theta_i = k_i q_i \tag{27}$$
$$\tau_{Li} = \frac{\tau_i}{k_i} \tag{28}$$

Based on the above equations, we can obtain the equation of motor.

$$L_i\frac{di_i}{dt} = v_i - R_i i_i - K_i k_i \dot{q}_i \tag{29}$$
$$\tau_i = -I_{mi}k_i^2\ddot{q}_i + K_i k_i i_i - d_{mi}k_i^2\dot{q}_i \tag{30}$$

The equation of motor can be rewritten in the form of vector, as shown in the following equations.

$$\boldsymbol{L}\frac{d\boldsymbol{i}}{dt} = \boldsymbol{v} - \boldsymbol{R}_i - \boldsymbol{K}_m\dot{\boldsymbol{q}} \tag{31}$$
$$\boldsymbol{\tau} = -\boldsymbol{J}_m\ddot{\boldsymbol{q}} + \boldsymbol{K}_m\boldsymbol{i} - \boldsymbol{D}_m\dot{\boldsymbol{q}} \tag{32}$$

Where

$$\boldsymbol{R} = \mathrm{diag}[R_1, R_2, \cdots, R_l], \boldsymbol{L} = \mathrm{diag}[L_1, L_2, \cdots, L_l]$$
$$\boldsymbol{K}_m = \mathrm{diag}[K_{m1}, K_{m2}, \cdots, K_{ml}], K_{mi} = K_i k_i$$
$$\boldsymbol{J}_m = \mathrm{diag}[J_{m1}, J_{m2}, \cdots, J_{ml}], J_{mi} = I_{mi}k_i^2$$
$$\boldsymbol{D}_m = \mathrm{diag}[D_{m1}, D_{m2}, \cdots, D_{ml}], D_{mi} = d_{mi}k_i^2$$

The consumption energy can be expressed as the following equation.

$$E_i(T) = \int_0^T v_i(t)i_i(t)dt \tag{33}$$

### 2.4 Motion equation of manipulator including motor under constraint condition

By combining (12),(14) and (31), and substituting (32) into (14), we can obtain the motion equation of manipulator including motor under constraint condition, as shown in the following equation.

$$\begin{aligned} & \begin{bmatrix} \boldsymbol{M} + \boldsymbol{J}_m & -(\boldsymbol{J}_c^{\mathrm{T}} - \boldsymbol{J}_t^{\mathrm{T}}\boldsymbol{K}) & \boldsymbol{0} \\ \dfrac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{L} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \boldsymbol{f}_n \\ d\boldsymbol{i}/dt \end{bmatrix} \\ = \;& \begin{bmatrix} \boldsymbol{K}_m\boldsymbol{i} - \boldsymbol{h} - \boldsymbol{g} - (\boldsymbol{D} + \boldsymbol{D}_m)\dot{\boldsymbol{q}} \\ -\dot{\boldsymbol{q}}^{\mathrm{T}}\left[\frac{\partial}{\partial \boldsymbol{q}}\left(\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{q}^{\mathrm{T}}}\right)\right]\dot{\boldsymbol{q}} \\ \boldsymbol{v} - \boldsymbol{R}\boldsymbol{i} - \boldsymbol{K}_m\dot{\boldsymbol{q}} \end{bmatrix} \end{aligned} \tag{34}$$

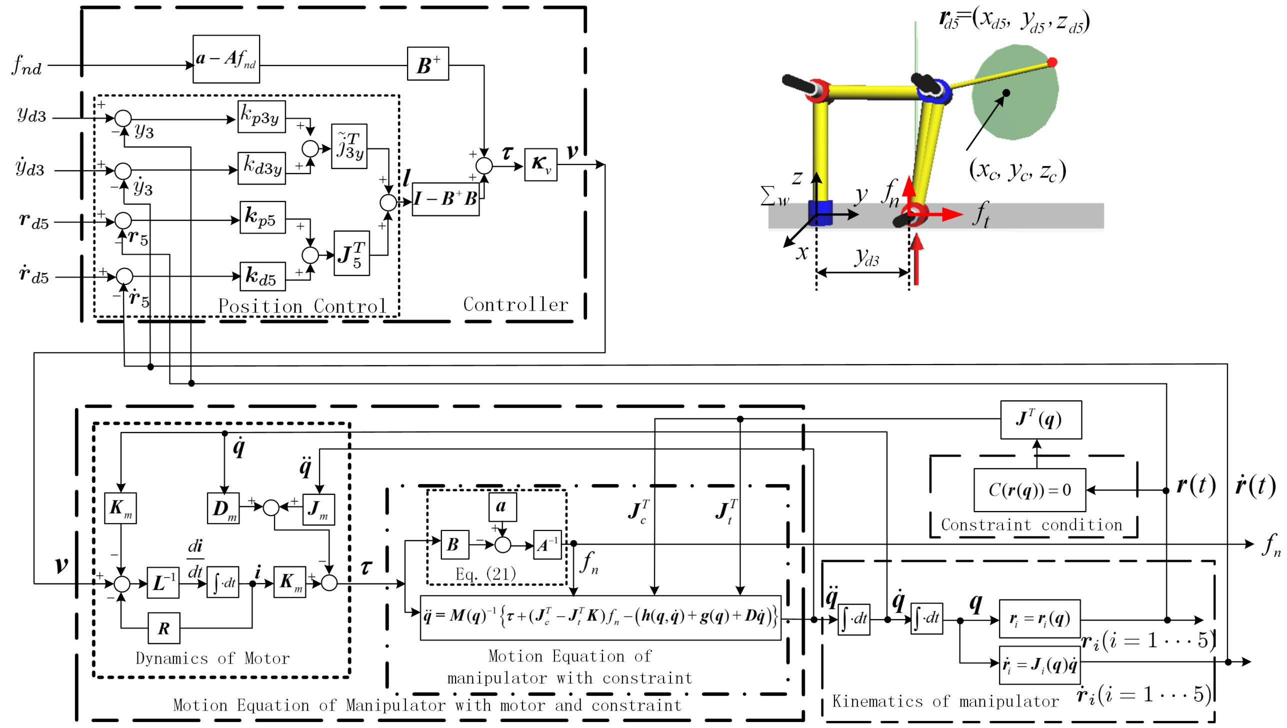


Fig. 3: Block diagram of control for 5-link elbow-bracing manipulator

## 3 Control method

Basede on (1), the control law of the torque $\boldsymbol{\tau}$ for the 4-link elbow-bracing manipulator is shown in the following equation.

$$\boldsymbol{\tau} = \boldsymbol{B}^{+}(\boldsymbol{a} - \boldsymbol{A}\boldsymbol{f}_{nd}) \tag{35}$$

Where, $\boldsymbol{B}^{+}$ is pseudo inverse matrix.

Noticed that the direction in which the position vector are constrained are complementary to the direction in which the constrained force is constrained. By dividing the woke space into two orthogonal domains, a position domains and a force domain, which are complementary to the directions of the corresponding constraints, in each of the two domains, position or force can be controlled independently and arbitrarily. Therefore, in order to simultaneously control the constraint force and position, the following equations can be obtained.

$$\boldsymbol{\tau} = \boldsymbol{B}^{+}(\boldsymbol{a} - \boldsymbol{A}\boldsymbol{f}_{nd}) + (\boldsymbol{I} - \boldsymbol{B}^{+}\boldsymbol{B})\boldsymbol{l}\ . \tag{36}$$

Where, rank$(\boldsymbol{I} - \boldsymbol{B}^{+}\boldsymbol{B})$ equals $n - p$. Because $\boldsymbol{I} - \boldsymbol{B}^{+}\boldsymbol{B}$ is non-dimensional matrix, $\boldsymbol{l}$ has dimensions of torque. Considering $\boldsymbol{l}$ to be new input, $\boldsymbol{l}$ can be used to track target trajectory of hand $\boldsymbol{r}_{d5}$ and control bracing position through null-space $\boldsymbol{I} - \boldsymbol{B}^{+}\boldsymbol{B}$ of $\boldsymbol{B}^{+}$. By the nature of pseudo inverse matrix, adding any value to $\boldsymbol{l}$ has no influence on achieving $\boldsymbol{f}_{nd}$. So, the task of tracking trajectory and the task of achieving $\boldsymbol{f}_{nd}$ can be achieved in decoupled nature.

Here, a method to determine $\boldsymbol{l}$ is discussed. In the simulation to utilize 5-link manipulator in this paper, one degree of freedom is used for force control of elbow, one degree is for contacting position control of elbow and three degrees are for three-dimensional position control of hand.

$$\begin{aligned} \boldsymbol{l} \;&=\; \tilde{\boldsymbol{j}}_{3y}^{\mathrm{T}}[K_{p3y}(y_{d3} - y_3) + K_{d3y}(\dot{y}_{d3} - \dot{y}_3)] \\ &+\; \boldsymbol{J}_5^{\mathrm{T}}[\boldsymbol{K}_{p5}(\boldsymbol{r}_{d5} - \boldsymbol{r}_5) + \boldsymbol{K}_{d5}(\dot{\boldsymbol{r}}_{d5} - \dot{\boldsymbol{r}}_5)] \end{aligned} \tag{37}$$

Here, $\tilde{\boldsymbol{j}}_{3y}^{\mathrm{T}}$ is the first column vector that comprises $\tilde{\boldsymbol{j}}_{3}^{\mathrm{T}}$ defined in Eq.(4). $K_{p3y}$ and $K_{d3y}$ are control gains of position and velocity in $y$ axis direction of 2nd link that is shown in Fig.4, $\boldsymbol{J}_5$ is Jacobian matrix defined by Eq.(4) when $i = 5$, and $\boldsymbol{K}_{p5}$ and $\boldsymbol{K}_{d5}$ are control gain matrices of position and velocity of fourth link.

Equation(36) can be realizable in the case that robots are driven by DD motors, but the input of usual DC motor is driven by voltage input. In this paper, the following equation that gives input voltage $\boldsymbol{v}$ to the DC motors is used instead of the controller of (36), where $\boldsymbol{K}_v$ is coefficient matrix to convert torque into voltage.

$$\boldsymbol{v} = \boldsymbol{K}_v\left[\boldsymbol{B}^{+}(\boldsymbol{a} - \boldsymbol{A}\boldsymbol{f}_{nd}) + (\boldsymbol{I} - \boldsymbol{B}^{+}\boldsymbol{B})\boldsymbol{l}\right] \tag{38}$$

The block diagram of the control method for 4-link elbow-bracing manipulator is shown in Fig.3. The output of the controller $\boldsymbol{v}$ is considered as the input of the motor which also includes joint angular velocity $\dot{\boldsymbol{q}}$ and angular acceleration $\ddot{\boldsymbol{q}}$ that are the outputs of the manipulator. And the output of the motor is $\boldsymbol{\tau}$ which is the input of the 4-link elbow-bracing manipulator. As the second link of the manipulator is subject to the constraint surface, the constraint force in normal direction of the surface, which is made up of $\boldsymbol{J}_c^T$ and $f_n$, and the friction force in the tangent direction of the surface, which is made up of $\boldsymbol{J}_t^T$ and $Kf_n$ are added to the motion equation of the manipulator. The formula to calculate the constraint force $f_n$ is given by (21) which guarantee that the motion of the elbow-bracing manipulator satisfies the constraint condition of (2), which is the merit of this paper.

In Fig.3, the definition of $\boldsymbol{B}$, $\boldsymbol{a}$ and $\boldsymbol{A}$ are shown in (16), (17) and (18). $\boldsymbol{B}^+$, $\boldsymbol{I}-\boldsymbol{B}^+\boldsymbol{B}$ are introduced in the former section.

# 4 Simulation for 5-link elbow-bracing manipulator

## 4.1 The model of 5-link elbow-bracing manipulator

The model of 5 links manipulator shown in Fig. 4.

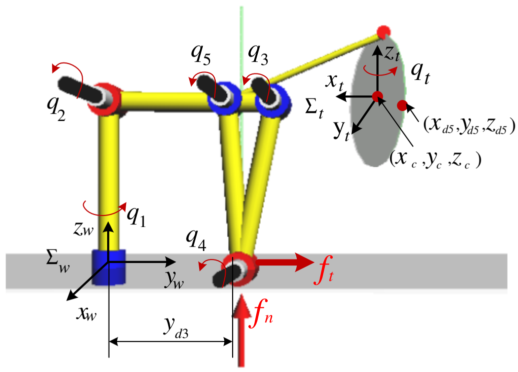


Fig. 4: Simulation model of 5-link elbow-bracing manipulator

In the initial period($t=0$), the coordinate system of $\Sigma_t$ has the same orientation of $\Sigma_w$,and hand target trajectory is given as the following equations. $T$ is the motion cycle.

$$x_d(t) = x_c \tag{39}$$

$$y_d(t) = r\cos\frac{2\pi}{T}t + y_c \tag{40}$$

$$z_d(t) = r\sin\frac{2\pi}{T}t + z_c \tag{41}$$

where, $r = 0.2[m]$, $(x_c, y_c, z_c) = (0, 0.8, 0.5)[m]$, $T = 5[s]$.

During the motion period($t > 0$), the coordinate system of $\Sigma_t$ is rotating about $z_t$-axis with rotational angle $q_t$ ($q_t = 2\pi t/T$).

## 4.2 Simulation experiment

Link's weight is $m_i = 1.0$ [kg], link's length is $l_i = 0.5$ [m], viscous friction coefficient of joint is $D_i = 2.9$ [N·m·s/rad], torque constant is $K_i = 0.2$ [N·m/A], resistance is $R_i = 0.6$ [Ω], inductance is $L_i = 0.1$[H], inertia moment of motor is $I_{mi} = 1.64\times10^{-4}$ [kg·m$^2$], reduction ratio is $k_i = 3.0$ and viscous friction coefficient of reducer is $d_{mi} = 0.1$ [N·m·s/rad]($i = 1, 2, 3, 4, 5$).

### 4.2.1 Analysis of energy-efficiency

The task for the 5-LINK manipulator is tracking spatial trajectory which is described in the former part. As three degrees of freedom are used for the three-demensional position of manipulator's end-effector to achieve spatial trajectory tracking, it has redundency for 5-link manipulator without constraint to track three-dimensional trajectory. However, in our model of 5-link elbow-bracing manipulator, the other two degrees of freedom are used to control the elbow-bracing position $y_{d3}$ and constraint force $f_{nd}$, which, simultaneously decreases the effect of link's gravity during trajectory tracking. Therefore, the tracking accuracy for manipulator with constraint is higher than the manipulator without constraint. What's more, it is more energy-efficient.

As it valid deems that the transiant responses from initial condition should be ingored, the time of which is $3s$ [17], energy consumption $E^*(T)$ after $t = T = 5[s]$ defined by following equations are utilized to evaluate the influence of elbow-bracing position and constraint force. Moreover, the constrant force is $40[N]$, and the elbow-bracing position is $0.4[m]$.

$$E_i^*(T) = \int_T^{3T} v_i(t)i_i(t)dt, \tag{42}$$

$$E^*(T) = \sum_{i=1}^{5} E_i^*(T). \tag{43}$$

Then the comparisons of energy consumption and tracking error for manipulators with bracing elbow and without bracing elbow are shown in Fig.5 and Fig.6, seprately.

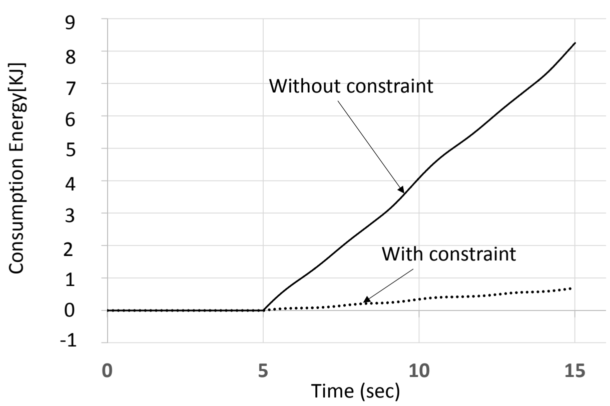


Fig. 5: Consumption energy for 5-link manipulator with 3nd link bracing and without 3nd link bracing

From Fig.5, it is known that energy consumption is reduced by 1/9 when bracing elbow in comparison

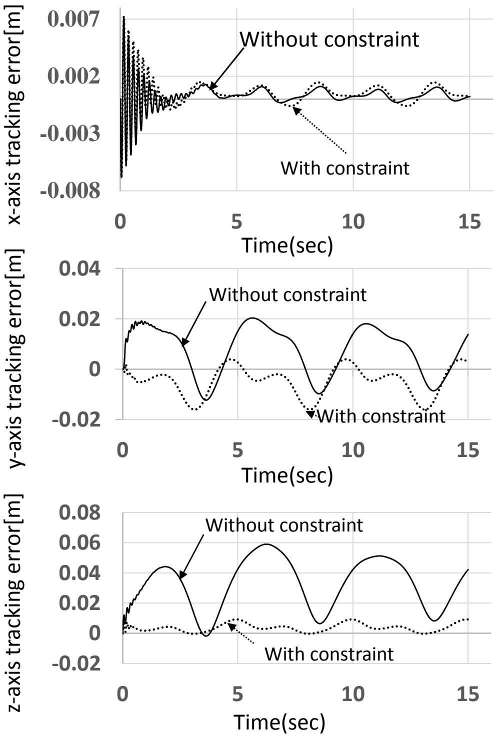


Fig. 6: Tracking error for 5-link manipuladtor with 3nd link bracing and without 3nd link bracing

with no bracing, which displays the drastic effectiveness of bracing elbow. From Fig.6, it is known that the tracking error in $y$-axis and $z$-axis for manipulator with constraint are obviously less than the tracking error for manipulator without constraint. Therefore, the model of the 5-link elbow-bracing manipulator in this paper is energy-efficient and holds high accuracy.

#### 4.2.2 Analysis of the factors which influence the consumption energy

It is known that five variables are controlled by the control law as shown in Eq.(36), and two of which are the constraint force $f_{nd}$ and elbow-bracing position $y_{d3}$. Therefore, the simulation of consumption energy for 5-link manipulator in the influence of these two factors is considered.

Fig.7 shows the range of $y_{d3}$ during the process of $q_t = 0$. Based on Fig.7, the range of $y_{d3}$ can be calculated by the following equations.

$$\begin{aligned} min(y_{d3}) &= y_c + r - \sqrt{(l_4 + l_5)^2 - z_c^2} \quad (44) \\ &\approx 0.14[m] \\ max(y_{d3}) &= \sqrt{(l_2 + l_3)^2 - l_1^2} \quad (45) \\ &\approx 0.86[m] \end{aligned}$$

Then, the simulation is conducted under the condition that the elbow-bracing position $y_{d3}$ is changing as $0.2, 0.3 \cdots 0.8[m]$, and the constraint force $f_{nd}$ is changing as $20, 25 \cdots 60[N]$, and $T = 5[s]$. The 3-D figure of the consumption energy during time $t = [T, 3T][s]$ with both elbow-bracing position and constraint force changing is shown in Fig.8.

In Fig.8, it is known that the consumption energy is

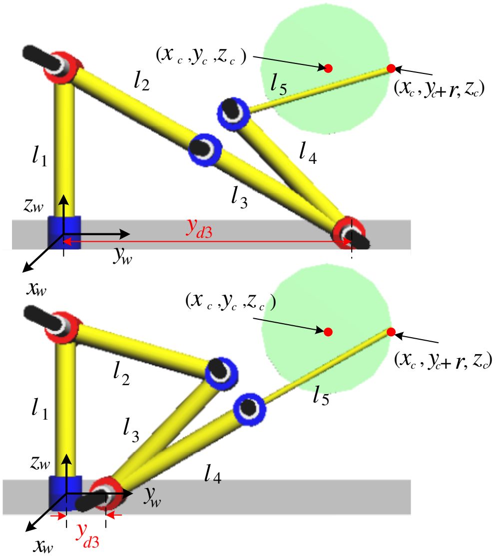


Fig. 7: Range of $y_{d3}$

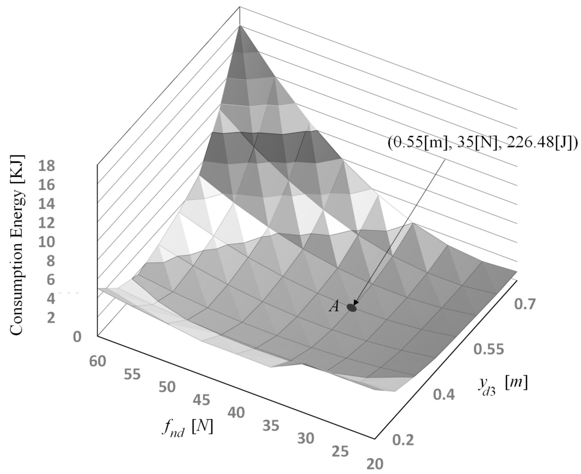


Fig. 8: Consumption energy with both elbow-bracing position and constraint force changing.(Point $A$ represents the optimal $f_{nd}$ and $y_{d3}$ in the case of the minimum consumption energy)

the function of $y_{d3}$ and $f_{nd}$. Assuming that the value of $f_{nd}$ is a constant, consumption energy is changing based on $y_{d3}$. Furthermore, the function between-consumption energy and $y_{d3}$ is parabolic form which has a minimum value. Likewise, if $y_{d3}$ is constant value, the function between consumption energy and $f_{nd}$ is parabolic form, too. Therefore, it is necessory to calculat the optimal pair for $y_{d3}$ and $f_{nd}$ to obtain the minimum consumption energy. As shown in Fig.8, at point A which means that $f_{nd} = 35[N]$ and $y_{d3} = 0.55[m]$, the consumption energy for the 5-link elbow-bracing manipulator is minimum.

## 5 Conclusion

In this paper, first, the constraint dynamics of the elbow-bracing manipulator including motor was proposed.

Next, by dividing the work space into two orthogonal domain, a position domain and a force domain, a controller whose control of constraint force and position has no interference was discussed.

Then, the consumption energy and tracking error for the 5-link manipulator with constraint and without constraint were analysed. And the simulation results reveal that the model of 5-link elbow-bracing manipulator proposed in this paper is effectiveness in the aspect of energy-efficiency and high accuracy during the process of tracking spatial trajectory.

Finally, the relationship between elbow-bracing position, constraint force and consumption energy is analysed. What's more, Based on Fig.8, it is known that the minimum consumption energy for the manipulator can be obtained at optimal values for elbow-bracing position and constraint force pair. And, in our further research, we focus on the method to obtain these values, as well as to achieve tracking rapid trajectory.

## Refences


1) J. Roy, L. L. Whitcomb, "Adaptive Force Control of Position/Velocity Controlled Robots" *Theory and Experiment, IEEE Transactions on Robotics and Automation*, Vol.18, No.2, pp.121-137, 2002.

2) B. Siciliano, L. Villani, "A passivity-based approach to force regulation and motion control of robot manipulators" *Automatica*, Vol.32, No.3, pp.443-447, 1996.

3) L. Villani, C. C. de Wit, B. Brogliato, "An exponentially stable adaptive control for force and position tracking of robot manipulators" *IEEE Trans. Automat. Contr.*, Vol44, pp.778-802, 1999.

4) J. D. Schutter, H. V. Brussel, "Compliant robot motion 2. A control approach based on external control loops" *Int. J. Robot. Res.*, Vol.7, No.4, pp.18-33, 1988.

5) J. Park, O. Khatib, "Multi-Link Multi-Contact Force Control for Manipulators" *Proc. of 2005 IEEE Int. Conf. on Robotics and Automation*, pp.3624-3629, 2005.

6) A. Petrovskaya, J. Park, O. Khatib, "Probabilistic Estimation of Whole Body Contacts for Multi-Contact Robot Control" *Proc. of IEEE International Conference on Robotics and Automation*, pp.568-573, 2007.

7) T. Yoshikawa, "Dynamic Hybrid Position/Force control of Robot Manipulators—Description of Hand Constraints and Calculation of Joint Driving Force" *IEEE J. on Robotics and Automation*, Vol.RA-3, No.5, pp.386-392, 1987.

8) K. Yamane, Y. Nakamura, "Forward Dynamics Computation of Open Kinematic Chains Based on the Principle of Virtual Work" *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp.2824-2831, 2001.

9) K. Yamane, Y. Nakamura, "Dynamics Filter - Concept and Implementation of On-Line Motion Generator for Human Figures" *IEEE Transactions on Robotics and Automation*, vol.19, no.3, pp.421-432, 2003.

10) H. West, H. Asada, "A Method for the Design of Hybrid Position/Force Controllers for Manipulators Constrained by Contact with the Environment" *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp.251-260, 1985.

11) H. Hemami, B. F. Wyman, "Modeling and Control of Constrained Dynamic Systems with Application to Biped Locomotion in the Frontal Plane" *IEEE Trans. on Automatic Control*, Vol.AC-24, No.4, pp.526-535, 1979.

12) Z. X. Peng, N. Adachi, "Position and Force Control of Manipulators without Using Force Sensors" (in Japanese) *Trans. of the Japan Society of Mechanical Engineers(C)*, Vol.57, pp.1625-1630, 1991.

13) M. Minami, K. Adachi, S. Sasaki, A. Yanou, "Improvement of Accuracy to Grind by Changing Position Control Gain for Shape-grinding" *Applied Mechanics and Materials*, Vol. 555, pp.186-191, 2014 .

14) Y. Washino, M. Minami, H. Kataoka, T. Matsuno, A. Yanou, M. Itoshima, Y. Kobayashi, "Hand- Trajectory Tracking Control with Bracing Utilization of Mobile Redundant Manipulator" *SICE Annual Conference*, pp.219-224, 2012.

15) D. Kondo, M. Itosima, M. Minami, A. Yano, "Proposal of Bracing Controller Utilizing Constraint Redundancy and Optimization of Bracing Position" *Proceedings of SICE Annual Conference 2013*, pp.2732-2737, 2013.

16) Xiang Li, Hiroki Imanishi, Mamoru Minami, Takayuki Matsuno and Akira Yanou, "Dynamical Modeling of Humanoid with Nonlinear Floor Friction", 21st International Symposium on Artificial Life and Robotics, B-Con Plaza Beppu, January 20-22, 2016

17) D. Kondo, X. Li, A. Yanou, M. Minami, "Energy-efficient and precise trajectory-tracking with bracing manipulator", *The 20th International Symposium on Artificial Life and Robotics*, OS2-7, pp.563-568, 2015.